Get Out Of My Way – We're Going At 99% The Speed of Light! (50,000 points)

1.) It is 2222.2 miles (3575.5 km) on the Burlington Northern Santa Fe Railway from Chicago to Los Angeles.
(a) A conventional high speed train traveling at 99.0 m/s (222 mph) will take how long to make this trip?

$$v = \frac{d}{t}\ ;\ t = \frac{d}{v} = \frac{3{,}575{,}500m}{99.0m/s} = 36{,}120\sec$$

(b) Show that special relativity does not come into play in any meaningful way on this conventional trip.

$$\beta = \frac{v}{c} = \frac{99.0m/s}{3.00\times10^{8}m/s} = 3.300\times10^{-7}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.000\,000\,000\,000 = 1$$

Therefore there is no change due to relativity.

(c) An ultra-high speed train traveling at 99.0% the speed of light makes the same trip. Neglecting accelerations at either end (making this trip unrealistic, I suppose), find the time that an observer at Chicago Union Station says this trip takes.

$$v = 0.990c = (0.990)(3.00\times10^{8}m/s)$$
$$= 2.970\times10^{8}m/s$$
$$t = \frac{d}{v} = \frac{3{,}575{,}500m}{2.970\times10^{8}m/s} = 0.01204\sec$$

(d) Find the distance the train engineer thinks the trip covers and use that to find the time that he says this trip takes.

$$\beta = 0.990\ ;\ L_0 = 3{,}575{,}500m$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.990)^2}} = 7.089$$
$$L = \frac{L_0}{\gamma} = \frac{3{,}575{,}500m}{7.089} = 504{,}400m$$
$$t = \frac{d}{v} = \frac{504{,}400m}{2.970\times10^{8}m/s} = 0.001698\sec$$

(e) Convert the time that the observer at Chicago Union Station says this trip takes to the rest frame of the engineer. *Does this answer make sense?*

$$t' = \gamma\, t_0$$
$$t_0 = \frac{t'}{\gamma} = \frac{0.01204\sec}{7.089} = 0.001698\sec$$

1 $u = 1.66 \times 10^{-27}$ kg

"Stayed Tuned for Survivor: Alpha Centauri after the E=mc² Super Bowl…" **(50,000 points)**

2.) Suppose there are particles called *superbowliums*. Each superbowlium has a mass of 2.50 u. They are heading toward each other, each moving at $\beta = 0.999$ and $\gamma = 22.4$. with respect to the lab frame. (a) How fast is either superbowlium going in m/s and what is its relativistic kinetic energy?

$$v = 0.999c = (0.999)(3.00\times10^{8}m/s)$$
$$= 2.997\times10^{8}m/s$$
$$m = 2.50u = 2.50\left(1.66\times10^{-27}kg\right) = 4.150\times10^{-27}kg$$
$$K_{rel} = (\gamma-1)mc^2 = (21.4)\left(4.150\times10^{-27}kg\right)\left(3.00\times10^{8}m/s\right)^2$$
$$= 7.993\times10^{-9}J$$

(b) Find the speed and relativistic kinetic energy of the second superbowlium with respect to the rest frame of the first superbowlium.

$$u_x = -2.997\times10^{8}m/s\quad;\quad v = +2.997\times10^{8}m/s$$
$$u_x' = \frac{u_x - v}{1-u_x v/c^2}$$
$$= \frac{(-2.997\times10^{8}m/s)-(+2.997\times10^{8}m/s)}{1-\frac{(-2.997\times10^{8}m/s)(+2.997\times10^{8}m/s)}{\left(3.00\times10^{8}m/s\right)^2}} = \frac{-2\beta c}{1+\beta^2}$$
$$= -3.000\times10^{8}m/s\quad OR\quad \beta = 0.9999995\quad and\quad \gamma = 707.1$$
$$K_{rel} = (\gamma-1)mc^2 = (706.1)\left(4.150\times10^{-27}kg\right)\left(3.00\times10^{8}m/s\right)^2$$
$$= 2.637\times10^{-7}J$$

(c) Find the total relativistic momentum of these two superbowliums in the lab frame and the rest frame of the first superbowlium.

LAB FRAME:
$$p_1 = \gamma_1 m v_1 = +\gamma\, mv \qquad p_2 = \gamma_2 m v_2 = -\gamma\, mv$$
$$p_{total} = p_1 + p_2 = 0$$

PARTICLE 1 FRAME:
$$p_1' = 0$$
$$p_2' = -\gamma\, mv = -(707.1)(2.50)\left(1.66\times10^{-27}\,kg\right)\left(3.00\times10^{8}\,m/s\right)$$
$$= -8.803\times10^{-16}\,kg\cdot m/s$$
$$p_{total}' = 0 + p_2' = -8.803\times10^{-16}\,kg\cdot m/s$$

The two superbowliums collide in a totally inelastic collision, forming a new particle called *survivium* which has a mass of 5.00 *u*. (In real life we would expect such a collision to spray out particles and photons, but let's keep this super simple.) Using conservation of momentum, find (d) the speed of the survivium in the lab frame and the rest frame from the first superbowlium and (e) the relativistic kinetic energy of the survivium in the lab frame and the rest frame from the first superbowlium.

(d) LAB FRAME:
$$p_{before} = p_{after}$$
$$0 = \gamma\, MV$$
$$V = 0 \quad (\beta = 0,\ \gamma = 1)$$

(e) LAB FRAME:
$$V = 0 \quad (\beta = 0,\ \gamma = 1)$$
$$K = (\gamma - 1)Mc^2 = 0$$

(d) PARTICLE 1 FRAME:
$$p_{before} = p_{after}$$
$$p_{before} = \gamma\, MV$$
$$\gamma V = \frac{p_{before}}{M} = \frac{-8.803\times10^{-16}\,kg\cdot m/s}{(5.00)\left(1.66\times10^{-27}\,kg\right)} = -106{,}100{,}000{,}000\,m/s$$
$$\gamma\,\beta c = -106{,}100{,}000{,}000\,m/s$$
$$\gamma\,\beta = \frac{-106{,}100{,}000{,}000\,m/s}{3.00\times10^{8}\,m/s}$$
$$\left(\frac{1}{\sqrt{1-\beta^2}}\right)\beta = -353.7 \ ;\ let\ a = 353.7$$
$$\beta = -a\left(\sqrt{1-\beta^2}\right)$$
$$\beta^2 = a^2\left(1-\beta^2\right)$$
$$\beta^2 = a^2 - a^2\beta^2$$
$$\beta^2 + a^2\beta^2 = a^2$$
$$\beta^2\left(1+a^2\right) = a^2$$
$$\beta^2 = \frac{a^2}{\left(1+a^2\right)}$$
$$\beta = \frac{a}{\sqrt{\left(1+a^2\right)}} = \frac{353.7}{\sqrt{\left(1+(353.7)^2\right)}} = 0.999996$$
$$v = \beta c = 0.999996\left(3.00\times10^{8}\,m/s\right) = 2.999988\times10^{8}\,m/s$$

(e) PARTICLE 1 FRAME:
$$\beta = 0.999996$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 353.6 \qquad NOTE: This\ is\ half\ of\ 707.1!!$$
$$K = (\gamma-1)mc^2$$
$$= (352.6)(5.00)\left(1.66\times10^{-27}\,kg\right)\left(3.00\times10^{8}\,m/s\right)^2$$
$$= 2.634\times10^{-7}\,J = 1.644\times10^{12}\,eV = 1.644\,TeV$$