A Modern Physics Smorgasbord (50,000 points)

1.) (a) Compton himself measured the scattering of X-rays off the electrons in graphite. For a scattering angle $\theta = 135^\circ$, and incoming X-rays with a wavelength $\lambda_0 = 0.0711 \text{ nm}$, find the wavelength $\lambda$ of the scattered X-rays.

$$\lambda - \lambda_0 = \frac{h}{m_c}(1 - \cos \theta)$$

$$\lambda = \lambda_0 + \frac{h}{m_c}(1 - \cos \theta)$$

$$= 0.0711 \text{ nm} + \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^9 \text{ m/s})}(1 - \cos 135^\circ)$$

$$= 7.11 \times 10^{-11} \text{ m} + (2.426 \times 10^{-12} \text{ m})(1.707) = 7.5241 \times 10^{-11} \text{ m}$$

$$= 0.07524 \text{ nm}$$

(b) Ordinary gold ($Z=79$) has 118 neutrons. A one-pound gold bar (0.454 kg = 454 grams) contains approximately how many atoms of gold?

$$Z = 79, N = 118, A = Z + N = 79 + 118 = 197$$

1 mole is 197 grams

$$n = \frac{454 \text{ grams}}{197 \text{ grams/mole}} = 2.305 \text{ moles}$$

$$N = nN_A = (2.305)(6.02 \times 10^{23}) = 1.388 \times 10^{24}$$

(c) A 1971 Cadillac Sedan de Ville, mass = 2250 kg, is driving along with a speed of 68 mph ($v = 30.4 \text{ m/s}$). Find the de Broglie wavelength of this big block V-8 carburetor powered hunk of iron.

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(2250 \text{ kg})(30.4 \text{ m/s})} = 9.693 \times 10^{-39} \text{ m}$$

(d) The kinetic energy of the ground state electron (n=1) in hydrogen is 13.6 eV. Find the de Broglie wavelength of this electron.

$$K = \frac{1}{2}mv^2 \quad \text{OR} \quad K = \frac{p^2}{2m}$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(13.6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{(9.11 \times 10^{-31} \text{ kg})}} = 2.186 \times 10^6 \text{ m/s}$$

$$p = \sqrt{2Km} = \sqrt{2(13.6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})(9.11 \times 10^{-31} \text{ kg})}$$

$$= 1.991 \times 10^{-24} \text{ kg} \cdot \text{m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.991 \times 10^{-24} \text{ kg} \cdot \text{m/s})} = 3.330 \times 10^{-10} \text{ m}$$

$$\text{OR}$$

$$mv = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(2.186 \times 10^6 \text{ m/s})} = 3.330 \times 10^{-10} \text{ m}$$

(c) The rest mass energy of an electron is 0.511 MeV (0.511 × 10^6 eV). In “pair creation”, a 1.022 MeV or higher energy gamma ray can create a pair of particles (electron $e^-$ and positron $e^+$), with any energy over the total rest mass energy of 1.022 MeV going into kinetic energy – this reaction conserves momentum, mass-energy and charge. It is thought that such a process can briefly and spontaneously occur in vacuum, by “borrowing” 1.022 MeV for a short time and then letting the electron-positron pair fall back together and annihilate each other before Nature notices that the energy is “missing”. Use the Heisenberg Uncertainty Relation to find the maximum time this event can take without anyone able to detect such a thing has occurred. Let $\Delta E = 1.022 \text{ MeV}$.

$$\Delta E \Delta t \geq \frac{h}{2}$$

$$\Delta t \geq \frac{h}{2\Delta E} = \frac{6.58 \times 10^{-34} \text{ eV} \cdot \text{s}}{2(1.022 \times 10^{20} \text{ eV})} = 3.219 \times 10^{-22} \text{ s}$$

$$\text{OR}$$

$$\frac{1.05 \times 10^{-24} \text{ J} \cdot \text{s}}{2(1.022 \times 10^{20} \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 3.211 \times 10^{-22} \text{ s}$$

So if the time is less than 3.21 × 10^{-22} seconds, you can temporarily cheat conservation of energy.

(But you have to remember to put it back, Cinderella!)
"All That Glitters is Not Crystalline Carbon" (50,000 points)

2.) Carbon \( (Z = 6) \) comes in several forms, one of which is graphite – carbon atoms are arranged in flat sheets which then stack on top of each other and slide around. The carbon-carbon bond distance – the distance between carbon atoms – is much shorter for within the sheets (1.42 Å) than between the sheets (3.41 Å). In diamond, another form of solid carbon, the bond length is 1.54 Å. (a) For a first approximation of the carbon-carbon distance, we note that in a multi-electron carbon atom, the bonding electrons are in the \( n = 2 \) orbits. So let us calculate \( r_n \) for the \( n = 2 \) orbital in the hydrogenic ion, \( \text{C}^5+ \), then multiply by 2 to get the distance between nuclei. Which of the above bond lengths is your calculation closest to?

\[
    r_n = \frac{n^2}{Z} a_0 \\
    r_2 = \frac{2^2}{6} \left(0.529 \times 10^{-10} \text{m}\right) = 3.527 \times 10^{-11} \text{m} \\
    d = 2r_2 = 2 \left(3.527 \times 10^{-11} \text{m}\right) = 0.7053 \times 10^{-10} \text{m} = 0.7053 \text{Å}
\]

What a surprise – not a good match to any of these.

(b) According to The Handbook of Physics and Chemistry, the photoelectric work function for carbon is 4.81 eV. We can try to approximate this by again using the hydrogenic carbon ion \( \text{C}^5+ \) and finding the energy \( E_2 \) of the electron from the \( n=2 \) orbital. (If the \( n=2 \) electron absorbed a photon of energy \( |E_2| \), it would be free of the carbon ion.) Compare the energy you calculate with the work function for carbon.

\[
    E_n = -\frac{Z^2}{n^2} 2.18 \times 10^{-18} \text{J} = -\frac{6^2}{2} 13.6 \text{eV} \\
    E_2 = -\frac{6^2}{2^2} 13.6 \text{eV} = -122.4 \text{eV}
\]

Gee, strike two. Another bad match.

(We shouldn’t be surprised – why would you use a hydrogenic carbon ion to simulate multi-electron carbon atoms in graphite?)

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1 NOTE: Several of these calculations involve trying to find real numbers using hydrogenic ions. Is this a good idea?