1. Find a solution of the next initial value problem

\[ \frac{dy}{dx} = 3x^2(1 + y^2), \quad y(0) = 0. \]

Is this solution unique? Does it exist for all \( x \)? Explain.

2. A cup has been filled with coffee with initial temperature 210° F at 8:00 a.m. The coffee’s temperature was 160° F at 8:15 a.m. Room temperature is 70° F. Find the coffee’s temperature at 8:20 a.m.? (Hint: use Newton’s cooling law.)
3. Find a solution of the next initial value problem

\[ 2xy' + y = 18x, \quad y(1) = 0. \]

4. Find a general solution of the following differential equation

\[ x^2y' = xy + x^2(\cos \frac{y}{x})^2. \]
5. Solve a differential equation

\[ \frac{dx}{dt} = x^2 - 3x + 2 \]

explicitly in terms of \( t \) and \( x_0 = x(0) \). Construct a sketch showing the nature of the solutions \( x(t) \). Find all equilibrium solutions. Determine stability or instability of each equilibrium solution.

6. The time rate of change of an alligator population \( P \) in a swamp is proportional to the square of \( P \). The swamp contained 20 alligators in 1990, 40 in 2000. When will there be 100 alligators in the swamp? What happens in 2020?