

EXAM 1

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1. Find a solution of the next initial value problem

$$\frac{dy}{dx} = e^{3x} \sin 3x, \quad y(0) = 0.$$

Is this solution unique? Does it exist for all  $x$ ? Explain.



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2. A cup has been filled with coffee with initial temperature  $210^\circ$  F at 7:00 a.m. . The coffee's temperature was  $180^\circ$  F at 7:15 a.m. Room temperature is  $70^\circ$  F. Find the coffee's temperature at 7:20 a.m. ? (*Hint:* use Newton's cooling law .)

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3. Find a solution of the next initial value problem

$$x^2y' - 2xy = x^3, \quad y(1) = 0.$$

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4. Find a general solution of the following differential equation

$$(y \cos xy + e^{-x})dx + (x \cos xy + \frac{1}{1+2y})dy = 0$$

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5. Consider a differential equation

$$\frac{dx}{dt} = x^2 - 4x + 3$$

(a) Construct a sketch showing the nature of the solutions  $x(t)$ . Find all equilibrium solutions. Determine stability or instability of each equilibrium solution.

(b) Find the solution satisfying the initial condition  $x(0) = 4$ .