

## PRACTICE GATEWAY EXAM

1. Find  $x$  to 3 decimal places if  $\log_2 x + \log_x 2 = 10/3$ .
2. Simplify  $\sqrt[3]{x^{7/2}x^{-1/3}}$ .
3. Solve for  $a$  the equation  $a^{-1} + a^{-2} = a^{-3}$ .
4. If  $\sin x/2 = a$  and  $0 \leq x \leq \pi/2$ , find  $\cos x$ .
5. If  $f(x + 3) = x^2$  find the formula for  $f(x)$ .
6. Find all solutions of  $|x - 1| < |x + 1|$ .
7. Let  $C$  be the circle with center at  $(1, 1)$  and radius 2. Find its intersection(s) with the line  $y = 3x - 2$ .
8. If  $A = (3, 1)$  and  $B = (4, -2)$  find a point  $C$  on the line segment  $AB$  so that  $AB = BC$ .
9. Factor the polynomial  $x^4 + 3x^3 - 7x^2 - 15x + 18$ .
10. The hypotenuse of a right triangle is 3 and its area is 5. Find the other 2 sides of the triangle.
11. Simplify  $\frac{a^2 - b^2}{b - a}$ .
12. Solve for  $x$ :  $2^{2x} - 3(2^x) = -2$ .
13. Find the angle between the diagonal  $AC_1$  of the cube  $ABCD A_1 B_1 C_1 D_1$  and the diagonal  $AC$  of the base  $ABCD$ .
14. A cone is sliced with a plane parallel to its base so that its height is split into 2 equal parts. What is the ratio of the volumes of 2 pieces?

## SOLUTIONS

1. Hint:  $\log_2 x = 1/\log_x 2$ . Solution:  $x = 8$  and  $x = \sqrt[3]{2}$ .
2. Solution:  $\sqrt[3]{x^{7/2-1/3}} = \sqrt[3]{x^{19/6}} = x^{19/18}$ .
3. Hint: Multiply through by  $a^3$ . Solution:  $a = (-1 \pm \sqrt{5})/2$ .
4. Hint:  $\cos 2\alpha = 1 - 2\sin^2 \alpha$ . Here:  $\cos x = 1 - 2a^2$ .
5. Hint: Replace  $x + 3$  by  $t$ , hence  $x$  by  $t - 3$ . Solution:  $f(t) = (t - 3)^2$ .
6. Hint: Raise both sides to the 2nd power. Solution:  $x > 0$ .
7. Circle:  $(x-1)^2 + (y-1)^2 = 4$ . Substitute  $y$  by  $3x-2$ . Intersection:  $(1-2/\sqrt{5}, 1-6/\sqrt{5})$  and  $(1+2/\sqrt{5}, 1+6/\sqrt{5})$ .
8. Hint: the midpoint is the average of the endpoints. Solution:  $C = (5, -5)$ .
9. Hint: it has value zero when  $x = 1$  and when  $x = 2$ , so it is divisible by  $x - 1$  and  $x - 2$ . Solution:  $(x - 1)(x - 2)(x + 3)^2$ .
10. Hint: If the sides are  $a$  and  $b$  then  $a^2 + b^2 = 9$  and  $ab = 10$ . The system has no solution.
11. Hint:  $a - b = -(b - a)$ . Solution:  $-(a + b)$ .
12. Hint: Denote  $2^x = t$ . Then  $t^2 - 3t = -2$  so  $t = 1$  or  $t = 2$ . Therefore,  $x = 0$  or  $x = 1$ .
13. Hint: If the length of the side of this cube is  $a$ , then  $AC_1 = a\sqrt{3}$  and  $AC = a\sqrt{2}$ . So, the sought angle  $\alpha$  satisfies  $\cos \alpha = (a\sqrt{2})/(a\sqrt{3}) = \sqrt{2}/\sqrt{3}$ . Using calculator:  $\alpha = 35.26$  degrees.
14. Rule: Volumes are proportional to the 3rd powers of lengths. So, the small cone has volume  $1/8$  of the volume of the big cone. Consequently, the pieces have volumes whose ratio is  $7 : 1$ .