1. Evaluate the integral $\int_0^1 xe^x \, dx$.

2. Evaluate the integral $\int_0^{\pi/2} \sin^3 x \, dx$.

3. Evaluate the integral $\int_1^{\sqrt{3}} \frac{\sqrt{x^2+1}}{x^4} \, dx$.

4. Evaluate the integral $\int \frac{dx}{x^3+x^2+x+1}$.

5. Use Comparison Theorem to determine whether the integral $\int_1^{\infty} \frac{dx}{x^3+1}$ is convergent or divergent.

6. The base of a certain solid is the region enclosed by $y = 1/x$, $y = 0$, $x = 1$, and $x = 4$. Every cross section of the solid taken perpendicular to the x-axis is an isosceles right triangle with its hypotenuse across the base. Find the volume of the solid.

7. If a force of 20 pounds is required to hold a spring 1 ft beyond its unstressed length, how much work does it take to stretch the spring this far?

8. Find the area of the region enclosed by the curves $y = \sin x$, $y = \cos x$, and $x = 0$.

9. Find the length of the curve $x = y^{3/2}/3 - y^{1/2}$ from $y = 1$ to $y = 9$.

10. Find the average value of $f(x) = \sin x$ on the interval $[0, \pi/2]$.

Solutions

1. Integration by parts: $u = x$, $dv = e^x\,dx$. Result: 1.

2. Substitution $u = \cos x$. Then $\sin x\,dx = -du$ and $\sin^2 x = 1 - u^2$. Obtain $\int_1^0 (1-u^2)(-du)$. Result: 2/3.

3. Substitution $x = \tan t$. Then $dx = \sec^2 t\,dt$ and $x^2 + 1 = \sec^2 t$. Obtain $\int_{\pi/4}^{\pi/3} \frac{\sqrt{\sec^2 t}}{\tan^4 t} \sec^2 t\,dt = \cdots = \int_{\pi/4}^{\pi/3} \frac{\cos t}{\sin^4 t}\,dt$. Now substitution $u = \sin t$ leads to $\int_{\sqrt{2}/2}^{\sqrt{3}/2} \frac{du}{u^4}$. Result: $\frac{8}{6\sqrt{2}} - \frac{8}{9\sqrt{3}}$.

4. Factor $x^3 + x^2 + x + 1 = (x+1)(x^2+1)$. Use partial fractions decomposition $\frac{1}{(x+1)(x^2+1)} = \frac{1/2}{x+1} + \frac{-x/2+1/2}{x^2+1}$. Result: $\frac{1}{2}\ln|x+1| - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\tan^{-1} x + C$.

5. Use the inequality $\frac{1}{x^3+1} < \frac{1}{x^3}$ and the fact that $\int_1^\infty \frac{dx}{x^3}$ converges as a p-integral with $p = 3$. Conclusion: the given integral converges.

6. The hypothenuse is $1/x$ so each leg is $1/(x\sqrt{2})$. The area of the cross section is $A(x) = 1/(4x^2)$. The volume is $\int_1^4 \frac{dx}{4x^2}$. Result: 3/16.

7. $F(x) = kx$ so $20 = k \cdot 1$ and $k = 20$. Thus, the work is $\int_0^1 20x\,dx$. Result 10 lb ft.

8. $\int_0^{\pi/4} (\cos x - \sin x)\,dx$. Result: $\sqrt{2} - 1$.

9. $\frac{dx}{dy} = \frac{1}{2}\left(y^{1/2} - y^{-1/2}\right)$. The length is $\int_1^9 \sqrt{1 + \frac{1}{4}\left(y^{1/2} - y^{-1/2}\right)^2}\,dy =$ $\int_1^9 \sqrt{\frac{1}{4}\left(y^{1/2} + y^{-1/2}\right)^2}\,dy$. Result: 32/3.

10. Average value: $\frac{2}{\pi}\int_0^{\pi/2} \sin x\,dx$. Result: $2/\pi$.