1. Use the Table of Integrals on the Reference Pages to evaluate the integral $\int \sqrt{x^2+x+1}\,dx$.

2. Use the Trapezoid Rule with $n = 10$ to evaluate the integral $\int\limits_0^\pi \sin x\,dx$. Round-off the result to 6 decimal places.

3. How large should $n$ be so that the approximation of $\int\limits_0^\pi \cos(2x)\,dx$ using the Simpson's Rule is accurate to within 0.00001?

4. Use Euler's method with step size 0.1 to approximate $y(2.4)$ where $y$ is the solution of the initial value problem $y' = x^2 - xy$, $y(1) = 0$.

5. A function $y(t)$ is a solution of a differential equation $dy/dt = y^4 - 6y^3 + 5y^2$. What are the equilibrium solutions? For what values of $y$ is $y$ concave up?

6. Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{1+y^2}{y\cos x}$, $y(0) = 1$.

7. A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria. Find the number of bacteria after 4 hours.

8. One model for the spread od the rumor is that the rate of spread is proportional to the product of the fraction $y$ of the population who have heard the rumor and the fraction who have not heard the rumor. Write the differential equation that is satisfied by $y$. Solve the equation.

9. Find the limit of the sequence $a_n = \dfrac{n\cos n}{n^2+1}$.

10. Determine whether the sequence $a_n = \dfrac{4n-3}{3n+4}$ is increasing, decreasing, or not monotonic. Is it bounded?

## Solutions

1. $x^2+x+1=(x+1/2)^2+3/4$. Substitution $u=x+1/2$. Use Formula 21, with $a=\sqrt{3}/2$. Solution: $\dfrac{x+1/2}{2}\sqrt{x^2+x+1}+\dfrac{3}{4}\ln(x+\dfrac{1}{2}+\sqrt{x^2+x+1})+C$.

2. $\Delta x=\dfrac{\pi}{10}$; $T_{10}=\dfrac{\pi}{20}\left[\sin 0+2\sin\dfrac{\pi}{10}+2\sin\dfrac{2\pi}{10}+\cdots+2\sin\dfrac{9\pi}{10}+\sin\pi\right]\approx 1.983523538$. Answer: 1.983524.

3. $f(x)=\cos 2x$; $f^{(4)}(x)=16\cos 2x$; $|16\cos 2x|\leq 16$. Need $\dfrac{16(\pi-0)}{180n^4}<10^{-5}$, hence $n>\sqrt[4]{\dfrac{16\pi}{180}10^5}\approx 12.92704729$. Result: $n=13$.

4. Use $nMin=0$, $u(n)=u(n-1)+0.1$, $u(nMin)=\{1\}$, $v(n)=v(n-1)+0.1(u(n-1)^2-u(n-1)v(n-1))$, $v(nMin)=\{0\}$. Result: 1.8647.

5. Constant solutions: $dy/dt=0$. Result: $y=0$, $y=1$, $y=5$. Concave up: $d^2y/dt^2>0$. Result: $0<y<(9-\sqrt{41})/4$ and $y>(9+\sqrt{41})/4$.

6. $\dfrac{y\,dy}{1+y^2}=\sec x\,dx$. Formula 14: $\dfrac{1}{2}\ln(1+y^2)=\ln|\sec x+\tan x|+C$. General solution: $y=\pm\sqrt{e^C(\sec x+\tan x)^2-1}$. Use $x=0$, $y=1$. Answer: $y=\sqrt{2(\sec x+\tan x)^2-1}$.

7. $P(0)=500$, $dP/dt=kP$. Then $P(t)=500e^{kt}$. Use $t=3$, $P=8000$ to obtain $k=\ln(16/3)$. Then $P(4)=500e^{4\ln(16/3)}\approx 404543.2099$. Answer: 404543.

8. Denote by $T$ the total population. Then $dy/dt=ay(T-y)$ for some $a>0$. The equation $dy/dt=(aT)y(1-y/T)$ is logistic equation. Solution: $y=T/(1+Ae^{-aTt})$ where $A=(T-y(0))/y(0)$.

9. $0\leq|n\cos n/(n^2+1)|\leq n/(n^2+1)$ and $\lim_{n\to\infty}n/(n^2+1)=0$. By Sandwich Theorem $\lim_{n\to\infty}n\cos n/(n^2+1)=0$.

10. $f(x)=(4x-3)/(3x+4)$, $f'(x)=25/(3x+4)^2>0$ so $f$ is increasing, and $a_n$ is increasing. $4n-3<4$, $3n+4>3n$ so $1/(3n+4)<1/(3n)$. Thus $a_n<4n/(3n)=4/3$ and $a_n$ is bounded above by $4/3$. Both $4n-3>0$ and $3n+4>0$ so $a_n$ is bounded below by 0.