1. Use the Table of Integrals on the Reference Pages to evaluate the integral 
\[ \int \sqrt{x^2 + x + 1} \, dx. \]

2. Use the Trapezoid Rule with \( n = 10 \) to evaluate the integral \( \int_0^\pi \sin x \, dx \). Round-off the result to 6 decimal places.

3. How large should \( n \) be so that the approximation of \( \int_0^\pi \cos(2x) \, dx \) using the Simpson’s Rule is accurate to within 0.00001?

4. Use Euler’s method with step size 0.1 to approximate \( y(2.4) \) where \( y \) is the solution of the initial value problem \( y' = x^2 - xy, \; y(1) = 0 \).

5. A function \( y(t) \) is a solution of a differential equation \( \frac{dy}{dt} = y^4 - 6y^3 + 5y^2 \). What are the equilibrium solutions? For what values of \( y \) is \( y \) concave up?

6. Solve the initial value problem \( \frac{dy}{dx} = \frac{1 + y^2}{y \cos x} \), \( y(0) = 1 \).

7. A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria. Find the number of bacteria after 4 hours.

8. One model for the spread of the rumor is that the rate of spread is proportional to the product of the fraction \( y \) of the population who have heard the rumor and the fraction who have not heard the rumor. Write the differential equation that is satisfied by \( y \). Solve the equation.

9. Find the limit of the sequence \( a_n = \frac{n \cos n}{n^2 + 1} \).

10. Determine whether the sequence \( a_n = \frac{4n - 3}{3n + 4} \) is increasing, decreasing, or not monotonic. Is it bounded?
Solutions

1. \( x^2 + x + 1 = (x + 1/2)^2 + 3/4 \). Substitution \( u = x + 1/2 \). Use Formula 21, with \( a = \sqrt{3}/2 \).
   Solution: \( \frac{x + 1/2}{2} \sqrt{x^2 + x + 1} + \frac{3}{4} \ln(x + 1/2 + \sqrt{x^2 + x + 1}) + C. \)

2. \( \Delta x = \frac{\pi}{10}; T_{10} = \frac{\pi}{20} \left[ \sin 0 + 2 \sin \frac{\pi}{10} + 2 \sin \frac{2\pi}{10} + \cdots + 2 \sin \frac{9\pi}{10} + \sin \pi \right] \approx 1.983523538. \) Answer: 1.983524.

3. \( f(x) = \cos 2x; f^{(4)}(x) = 16 \cos 2x; |16 \cos 2x| \leq 16. \) Need \( \frac{16(\pi - 0)}{180n^4} < 10^{-5}, \) hence \( n > \sqrt[4]{\frac{16\pi}{180}} \approx 12.92704729. \) Result: \( n = 13. \)

4. Use \( n_{Min} = 0, u(n) = u(n - 1) + 0.1, u(n_{Min}) = \{1\}, v(n) = v(n - 1) + 0.1(u(n - 1)^2 - u(n - 1)v(n - 1)), v(n_{Min}) = \{0\}. \) Result: 1.8647.

5. Constant solutions: \( dy/dt = 0. \) Result: \( y = 0, y = 1, y = 5. \) Concave up: \( d^2y/dt^2 > 0. \) Result: \( 0 < y < (9 - \sqrt{41})/4 \) and \( y > (9 + \sqrt{41})/4. \)

6. \( \frac{ydy}{1 + y^2} = \sec x \, dx. \) Formula 14: \( \frac{1}{2} \ln(1 + y^2) = \ln |\sec x + \tan x| + C. \) General solution: \( y = \pm \sqrt{e^C(\sec x + \tan x)^2 - 1}. \) Use \( x = 0, y = 1. \) Answer: \( y = \sqrt{2(\sec x + \tan x)^2 - 1}. \)

7. \( P(0) = 500, dP/dt = kP. \) Then \( P(t) = 500e^{kt}. \) Use \( t = 3, P = 8000 \) to obtain \( k = \ln(16/3). \) Then \( P(4) = 500e^{4\ln(16/3)} \approx 404543.2099. \) Answer: 404543.

8. Denote by \( T \) the total population. Then \( dy/dt = ay(T - y) \) for some \( a > 0. \) The equation \( dy/dt = (aT)y(1 - y/T) \) is logistic equation. Solution: \( y = T/(1 + Ae^{-aTt}) \) where \( A = (T - y(0))/y(0). \)

9. \( 0 \leq |n \cos n/(n^2 + 1)| \leq n/(n^2 + 1) \) and \( \lim_{n \to \infty} n/(n^2 + 1) = 0. \) By Sandwich Theorem \( \lim_{n \to \infty} n \cos n/(n^2 + 1) = 0. \)

10. \( f(x) = (4x - 3)/(3x + 4), f'(x) = 25/(3x + 4)^2 > 0 \) so \( f \) is increasing, and \( a_n \) is increasing. \( 4n - 3 < 4, 3n + 4 > 3n \) so \( 1/(3n + 4) < 1/(3n). \) Thus \( a_n < 4n/(3n) = 4/3 \) and \( a_n \) is bounded above by \( 4/3. \) Both \( 4n - 3 > 0 \) and \( 3n + 4 > 0 \) so \( a_n \) is bounded below by \( 0. \)