

Math 272 Final

Name _____

Show all work! June 19, 2001

(1) For the following use $\vec{a} = (2, 3, -1)$, $\vec{b} = (-1, -1, 0)$, $\vec{c} = (2, 0, -3)$, and $\vec{d} = (3, -1, 2)$.

a) $6\vec{a} - 2\vec{b} + \vec{d}$

b) $\vec{a} \times \vec{c}$

c) $\text{proj}_{\vec{b}} \vec{d}$

d) The volume of the parallelepiped spanned by $\vec{b} = (-1, -1, 0)$, $\vec{c} = (2, 0, -3)$, and $\vec{d} = (3, -1, 2)$.

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e) Find the equation of the plane containing the points $\vec{a} = (2, 3, -1)$, $\vec{b} = (-1, -1, 0)$, and $\vec{d} = (3, -1, 2)$.

f) Let A be the matrix whose columns are $\vec{b} = (-1, -1, 0)$, $\vec{c} = (2, 0, -3)$, and $\vec{d} = (3, -1, 2)$. What is the image of the unit cube $\{(x, y, z) | 0 \leq x, y, z \leq 1\}$ under the linear transformation $T(\vec{x}) = A\vec{x}$? What is the volume of the image?

(2) Consider the function

$$h(\vec{x}) = x_1^4 - 2x_1^2x_2 + x_2^3 - x_2^2.$$

a) Find the gradient and Hessian of h .

b) Find the second degree Taylor polynomial for h around $(1, 1)$.

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c) Find all critical points of h .

d) Classify all the critical points for h .

- (3) Let R be the region in the first quadrant bounded by $y = \sqrt{3}x$, $y = x/\sqrt{3}$, and $y^2 + x^2 = 4$.
- a) Set up a double integral to find the area of the region. Evaluate the integral.

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- b) Find $\iint_S \vec{F} \cdot \vec{n} \, d\sigma$ where $\vec{F} = (0, 0, x + y)$ and S is the surface above R with parametrization $\vec{f}(x, y) = (x, y, 4 - x^2 - y^2)$.

- (4) Find the work done moving a 1kg mass along $\vec{s}(t) = (t, \sqrt{t})$ from $(1, 1)$ to $(9, 3)$ in a force field $\vec{F}(x, y) = (-x + 1, -y - 2)$.