State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Be Sure to Write Down Equations – Feel Free to Ask Any Questions
Use Your Time Wisely – Work on What You Can

Pumping Iron (50,000 points)

1.) A electric pump motor starts up. Attached to the motor is a large flywheel, whose purpose is to smooth out the jerky motions of the pump (don’t worry about it). The flywheel has nearly all its mass (37.5 kg) at its rim (r = 33.5 inches = 0.850 meters). It starts at rest and accelerates at \( \alpha = -53.0 \text{ rad/sec}^2 \) for 10.0 seconds. (a) What is the final angular speed \( \omega \) of the flywheel?

\[
\omega = \omega_0 + \alpha t = \omega_0 + \frac{1}{2} \alpha t^2
\]

\[
= (-53.0 \text{ rad/sec}^2)(10.0 \text{ sec}) = -530.0 \text{ rad/sec}
\]

(b) How far does the flywheel turn while it is accelerating? Answer in radians.

\[
\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 = \frac{1}{2} \alpha t^2
\]

\[
= \frac{1}{2}(-53.0 \text{ rad/sec}^2)(10.0 \text{ sec})^2 = -2650. \text{rad}
\]

(c) What torque \( \tau \) is needed to provide this angular acceleration?

\[
I_{\text{ring}} = MR^2 = (37.5 \text{kg})(0.850 \text{m})^2 = 27.09 \text{kg \cdot m}^2
\]

\[
\tau = I \alpha = (27.09 \text{kg \cdot m}^2)(-530 \text{ rad/sec}^2) = -1436 \text{N \cdot m}
\]

(d) What power did the pump motor have to provide to accelerate the flywheel? Answer in Watts.

\[
W = \tau \omega = (-1436 \text{N \cdot m})(-2650. \text{rad}) = 3,805,000 \text{J}
\]

\[
P = \frac{W}{t} = \frac{3,805,000 \text{J}}{10.0 \text{sec}} = 380,500 \text{Watts}
\]

(e) The pump pressurizes a storage tank to a gauge pressure \( (P_1 - P_2) = 235,000 \text{ Pa} \). What height \( h \) does the pipe need to be if the water in the tank, \( \rho = 1000 \text{ kg/m}^3 \), spews out the top at \( v_2 = 5.00 \text{ m/s} \)?

\[
P_1 + \rho gh + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2
\]

\[
P_1 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2
\]

\[
\rho gh_2 = P_1 - P_2 - \frac{1}{2} \rho v_2^2
\]

\[
h_2 = \frac{(P_1 - P_2) - \frac{1}{2} \rho v_2^2}{\rho g}
\]

\[
= \frac{235,000 \text{Pa} - \frac{1}{2}(1000 \text{kg/m}^3)(5.00 \text{m/s})^2}{1000 \text{kg/m}^3}(9.81 \text{m/s}^2)
\]

\[
= 22.68 \text{m}
\]
Le Cirquè Du Physiquès (50,000 points)

2.) Baby the Baby Elephant (1135 kg) stands on the left end of a balanced teeter-totter and Glamorous Glenda (50.8 kg) stands on the right end. The teeter-totter board is 12.0 meters long and has a mass of 275 kg. The teeter-totter support is located a distance D from the left end. (a) Find the vector force $F_1$ from the support on the board. You must include F.B.D.‘s and F.R.D.’s!

$$\sum F_y = F_1 - w_E - w_D - w_G = 0$$

$$F_1 = w_E + w_D + w_G = m_E g + m_D g + m_G g$$

$$= (1135 \text{kg} + 275 \text{kg} + 50.8 \text{kg})g$$

$$= (14,330 \text{N})$$

(b) Find D.

Choose a pivot point.

If choose at the support, then every other term has D.

$$\sum \tau = w_E D - w_D \left( \frac{1}{2} L - D \right) - w_G \left( L - D \right) = 0$$

If choose at left end, only one term has D. Simpler algebra.

$$\sum \tau = F_1 D - w_D \left( \frac{1}{2} L \right) - w_G L = 0$$

$$F_1 D = w_D \left( \frac{1}{2} L \right) + w_G L$$

$$D = \frac{m_D g \left( \frac{1}{2} L \right) + m_G g L}{F_1}$$

$$= \frac{(9.81 \text{m/s}^2)(12.0 \text{m})(14,330 \text{N})}{14,330 \text{N}}$$

$$= 1.547 \text{m}$$

(c) Glamorous Glenda hangs from a thick elastic strap by her teeth. The strap has $L_0 = 5.00 \text{ m}$ and a cross-sectional area $A = 0.0100 \text{ m} \times 0.0500 \text{ m} = 0.000500 \text{ m}^2$. What Young’s Modulus $Y$ must the strap have if it stretches elastically by 5.50 meters?

$$Y = \frac{F/\Delta L}{A/L_0} = \frac{w_E L_0}{A \Delta L}$$

$$= \frac{(50.8 \text{kg})(9.81 \text{m/s}^2)(5.00 \text{m})}{(0.000500 \text{m}^2)(5.50 \text{m})}$$

$$= 906,100 \text{N/m}^2$$

(d) Baby is placed inside a big pipe of diameter 7.00 meters, where he runs forward at 4.25 m/s. There is sufficient friction so that neither the elephant or the pipe slips. Find the angular velocity $\omega$ of the pipe.

$$\omega = \frac{v}{r} = \frac{4.25 \text{m/s}}{3.50 \text{m}} = 1.214 \text{rad/sec}$$

(e) Baby steps into a boat of mass 185 kg and the boat is just about to sink into water. Approximately what is the volume of the boat?

$$M = 1135 \text{kg} + 185 \text{kg} = 1320 \text{kg}$$

$$\rho_{\text{wood}} = \frac{M}{V} = \rho_{\text{water}}$$

$$V = \frac{M}{\rho_{\text{water}}} = \frac{1320 \text{kg}}{1000 \text{kg/m}^3} = 1.32 \text{m}^3$$