Somewhere in Deep Space… (50,000 pts.)

1.) (a) Electrician’s Mate Sparks has a box of unmarked resistors. All are supposed to be $R = 100\, \Omega$. Unfortunately, someone has dropped in an identical looking 8800 \Omega resistor. “But I need a 100 \Omega resistor now! And I don’t have an ohmmeter,” the Assistant Engineer cries out. “No problem,” Sparks says, and puts together the following circuit. “This is within 20% of being right.” Show how this can be true. The resistor $R'$ may be 100 \Omega or 8800 \Omega.

![Circuit Diagram]

(b) Forty (40) identical capacitors are connected in series. The equivalent capacitance is 1386 \mu F. Find the equivalent capacitance if these same 40 capacitors were connected together all in parallel.

(c) There are two slits a distance $d$ apart on one side of an apparatus. On the other side, 2.35 m away, you can see the first bright line is 1.00 mm from the center. What is $d$?

(d) An RC circuit has a time constant of 2.00 seconds. An LC circuit with the same capacitor has an angular frequency of 4\pi rad/sec. An RL circuit with the same resistor and inductor has a time constant of 2.00 seconds. Find the capacitance $C$, if you can. If you can’t, explain why.

(e) If you put a Gaussian surface around the entire universe, what would be the net electric flux $\Phi_E$? Why?

More Star Problems! (50,000 points)

2.) \(\gamma\) (a) A spherical insulator of radius $R$ has a spherical hole in the center of radius $a$. There is a total charge $Q$ evenly distributed throughout the rest of the sphere. Use Gauss’ Law to find the magnitude of the electric field at a radius $r$, where $a < r < R$. Note: You may evaluate the integrals by using the known equations for things like the surface area and volume of a sphere.

\[
\phi = \frac{Q}{4\pi \epsilon_0} \left( \frac{1}{r} - \frac{1}{R} \right)
\]

(b) Consider an RC circuit as shown. Flip the switch $S_1$ to the left and the capacitor charges to $\pm Q_0$. For the charging capacitor, write a Kirchhoff’s Law voltage loop equation. Remember that the definition of current is $I = dq/dt$. Show that $Q(t) = Q_0 (1 - e^{-t/\tau})$.

(c) Differentiate $B = B_m \cos (kx - \omega t)$ in the wave equation for $B$, $\frac{\partial^2 B}{\partial t^2} = \frac{1}{\epsilon} \frac{\partial^2 B}{\partial x^2}$, and see if it is a solution.

(d) Consider a flat slab of glass ($n = 1.55$). Parallel light rays (in air) come in perpendicular to the surface. If the thickness of the glass is $(1000 + 1/4) \lambda_m$, would the reflections be maximized or minimized?

(e) Find the magnetic flux, $\Phi_B = \int \vec{B} \cdot d\vec{A}$, in a square area of sides $a$ in the $xy$ plane, centered on the origin, for a magnetic field $\vec{B} = b \ xy \ 2 \ 2 \ 2 \ \hat{i}$, where $b$ is an arbitrary constant.
Captain Terrible of The Space Command! (50,000 points)

3.) (a) The starship Invisible is traveling to the planet Zoltar at $\beta = 0.9999$ ($\gamma = 70.71$). From Earth, Zoltar appears to be 105 light years away. How long does Space Command on Earth think it takes them to make the trip?

(b) How long does Captain Terrible on the Invisible think that it takes?

(c) Find the relativistic K.E. of the Invisible, which has a mass of 35,500,000 kg.

(d) The Invisible needs to accelerate electrons up to a kinetic energy of $K = 50.0 \text{ KeV} = 50.0 \times 10^3 \text{ eV}$. Find the accelerating potential, $\Delta V$.

(e) These 50.0 KeV electrons are traveling in the $+x$ direction where they meet a magnetic field $\vec{B} = (1.00T) \hat{j}$. Find the magnitude and direction of the force $F_B$ acting on one of these electrons. If you can’t figure out how fast these electrons are going, use $v = 1.00 \times 10^8 \text{ m/s}$.

We’ll Have a President-Elect by Christmas for Sure… (50,000 points)

4.) (a) A sandwich has dimensions of $10.0 \text{ cm} \times 18.0 \text{ cm}$ and is $2.00 \text{ cm}$ thick on average. Since the ingredients are mostly water ($\epsilon = 80.0$), let’s assume that this sandwich forms the inside of a dielectric filled parallel plate capacitor. Find the capacitance, $C$, of the sandwich.

(b) A physics artist decorates a brand new Volkswagen Beetle by randomly gluing on 585 small glass insulating beads, each of which has a net charge $q_i = 1.37 \times 10^{-5} \text{ C}$. If the Beetle is located at the origin, find $E$ at $x = 1.00 \text{ mile} = 1.609 \text{ km}$.

(c) Two of these glass beads are 5.00 cm apart. Find the magnitude of electric force $F_E$ between them.

(d) A triangular coil is constructed as shown, with current $I = 1.25 \text{ A}$, number of loops $N = 6$, length of coil $L = 15.0 \text{ cm}$ and the length of the sides are $a = 16.0 \text{ cm}$. If the B-field in the center of the coil is as shown, which way does the current flow?

(e) For one of these equilateral triangular loops, use the result for the magnetic field of a long straight current carrying wire to determine the magnitude of the B-field at the center of the loop.