Squaresville (50,000 points)

1.) Eight charges, $|q| = 4.00 \times 10^{-6} \text{C}$, are rigidly arranged in line with spacing $d = 10.0 \text{ cm} = 0.100 \text{ m}$ as shown. (a) Find the electric field vector, $\vec{E}_{\text{total}}$, at the center point $P$.

(b) Find the electric potential, $V$, at the center point $P$.

(c) Find the vector electric force, $\vec{F}$, acting on a charge $q = 4.00 \times 10^{-6} \text{C}$ in the center.

(d) In a constant electric field, applied everywhere, show whether these eight charges, rigidly held together, will translate in any direction or rotate about the point $P$ in the applied $E$-field, where $\vec{E} = 150 \text{ V} \hat{\phi} @ 45^\circ$. Hint: Think Free Body Diagrams and Free Rotation Diagrams.

(e) A charged piece of copper has $q = 6.61 \times 10^{-6} \text{C}$. Calculate what has to be added or removed to make this real charge from an uncharged piece of copper?

$$q = \pm Ne$$
$$q = + Ne$$
$$N = \frac{q}{e} = \frac{6.61 \times 10^{-6} \text{C}}{1.602 \times 10^{-19} \text{C}} = 4.126 \times 10^{13}$$
$$4.126 \times 10^{13} \text{ or } 41,260,000,000,000 \text{ electrons removed}$$
The Afternoon Movie on STARZ (50,000 points)

2.) (a) A spherical insulator of radius \( R \) has a total charge \( Q \) evenly distributed throughout the sphere. Use Gauss' Law to find the magnitude of the E-field at a radius \( r \), where \( r < R \). NOTE: You may evaluate the integrals by using the known equations for things like the surface area and volume of a cylinder.

\[
\begin{align*}
\rho &= \frac{Q}{4\pi R^3} ; \quad dq = \rho dV \\
\Phi_e &= \frac{\oint \vec{E} \cdot d\vec{A}}{\varepsilon_0} = \frac{q_{\text{total}}}{\varepsilon_0} \\
&= \frac{\oint E \, d\vec{A}}{\varepsilon_0} = E \oint dA = E \frac{\oint dq}{\varepsilon_0} = \frac{\rho \oint dV}{\varepsilon_0} \\
&= E A = \frac{\rho V}{\varepsilon_0} \\
&= E \left( 4\pi r^2 \right) = \frac{Q}{4\pi \varepsilon_0 R^3} r \quad \text{OR} \quad \frac{kQ}{r}
\end{align*}
\]

(b) Use direct integration to find the magnitude of \( E \) at a point \( P \) a distance \( d \) from a finite line of charge, \( L = 1.00 \text{ m} \) and \( \lambda = 1.00 \times 10^4 \text{ C/m} \), where \( d = 0.100 \text{ m} \).

\[
\vec{E} = \frac{\lambda}{2\pi \varepsilon_0} \frac{x}{x^2} \right|_{0}^{d} = \frac{\lambda}{2\pi \varepsilon_0} \frac{dx}{x} = \frac{\lambda}{2\pi \varepsilon_0} \frac{dx}{x}
\]

\[
\begin{align*}
\int_{0}^{d} \lambda \, \frac{dx}{x} &= \sqrt{\frac{d}{\sqrt{d^2 + L^2}}} \\
&= \lambda \int_{0}^{\sqrt{d^2 + L^2}} \frac{dL}{d^2 + L^2} \\
&= \lambda \left[ \frac{d + L}{d(d + L)} - \frac{d}{d(d + L)} \right] \\
&= \lambda \left( \frac{d + L}{d^2 + Ld} \right) - \frac{\lambda d}{d(d + L)} \\
&= \lambda \left( \frac{d + L}{d^2 + Ld} \right) - \lambda \frac{d}{d^2 + Ld}
\end{align*}
\]

(c) An electric field is given by \( \vec{E} = \frac{18.5N}{C} \hat{i} + (320N/C \cdot m) \hat{j} \). Find the electric potential difference \( \Delta V \) by integrating along a straight-line path from \( x_a = 0, y_a = 0 \) to \( x_b = 1.25 \text{ m}, y_b = 0 \).

\[
\begin{align*}
\vec{E} &= \frac{18.5N}{C} \hat{i} + (320N/C \cdot m) \hat{j} \\
\Delta V &= -\int \vec{E} \cdot d\vec{s} \\
&= -\int \left[ \frac{18.5N}{C} \hat{i} + (320N/C \cdot m) \hat{j} \right] \cdot d\hat{i} \\
&= -\int_{0}^{1.25m} (18.5V/m)dx = -(18.5V/m) \left[ x \right]_{0}^{1.25m} \\
&= -(18.5V/m)(1.25m - 0) = -23.13 \text{ volts}
\end{align*}
\]

(d) An electron in a television picture tube is accelerated from rest to a final kinetic energy of 37,000 eV. What accelerating potential, \( \Delta V \), is required to do this?

\[
qV = eV \\
\Delta V = 37,000 \text{ volts}
\]

(e) An electric potential is given as \( V(x,y) = A \left( x^2 + y^2 \right) \), where \( A \) is some arbitrary constant. Find the \( y \)-component of the E-field, \( E_y \).

\[
E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y} A(x^2 + y^2) \\
= -A(0 + 2y) = -2Ay
\]