G. The Opacity and Hyperintensionality of That-Clauses.

We must distinguish referential from attributive positions in sentences. Concerning the referential positions I'll explain the referential opacity and transparency of their contexts in terms of the patterns of entailment already explained, namely, importation and exportation, intersubstitution, and generalization. We'll say the propositional attitudes are referentially opaque contexts for the referential positions in their that-clauses. Then, concerning attributive positions, I explain the extensionality, intensionality (with an s), and hyperintensionality of their contexts. Lots of big words to explain.

§1. Referential and attributive positions in sentences.

The positions that names and pronouns occupy in sentences are called referential positions, in contrast to the positions that adjectives and verbs occupy, which are called attributive positions or predicative positions. The sentence, “Socrates is human” consists of two positions, the referential followed by the attributive. The notation common in quantificational logic exploits the difference between capital letters and small letters; a capital letter occupies the attributive position and a small letter occupies the referential position. Let “H” abbreviate “is human” and s be Socrates. The sentence just given is reordered and abbreviated this way: “Hs.” In many languages the order of the two positions does not identify the kind of position. Here's a way of testing independently of order: Whatever predicate occupies an attributive position could be occupied by its negation, but a name, the occupant of a referential position, does not admit of a negation. Thus we might say Socrates is not human, but it does not make sense to say that [not-Socrates] is human, where attaching the “not” is supposed to yield Socrates denied. Even if one managed to make sense of this, it would have no connection to truth and falsity as genuine negation does.

Attributive (or predicative) positions are also occupied by adjectival or verbal phrases compounded of many words, some of which might be names. Thus “Plato loves” expresses a relational property (see section D§4) predicated of Socrates, and the phrase occupies an attributive position in the sentence “Plato loves Socrates.” This analysis stopped at an arbitrary point;
we could've analyzed the sentence further as having two referential positions with an intervening attributive position. The partial analysis illustrates my point, however, which is that complex predicative phrases, even those containing names, occupy attributive positions, not just single words. Every level of analysis, partial or complete, assigns contentful phrases to either referential positions or attributive positions. Finally, even the most superficial level of analysis does this assigning: Since propositions are medadic attributes, we treat the positions occupied by the \( p \) and \( q \) in the formulas of elementary logic, such as \( \text{if } p \text{ then } q \), as attributive also.

The best way to understand referential and attributive positions is relatively to a level of analysis. These sentences go from less to more analyzed:

Plato loves Socrates. (one attributive position)
Plato loves | Socrates. (two positions: attributive | referential)
Plato | loves Socrates. (two positions: referential | attributive)
Plato | loves | Socrates. (three positions: referential | attributive | referential)

Russell's method of analyzing definite descriptions has the effect of removing the appearance that a definite description (in its attributive use) occupies a referential position in the sentence in which it occurs. The method does this by dispersing the description's component general terms into obviously attributive positions in the analyzed sentence.

There are positions other than referential and attributive for noncontentful, form-indicating words; e.g., the one occupied by conjunctions to form compound sentences. Logic assigns quantifiers to still another position, although English shoehorns them into referential positions, so that there are only two sorts of position in English noncompound sentences.

With this distinction mastered, we can get down to business, first an affliction of the referential position called opacity, then an affliction of the attributive position called nonextensionality.

§2. Referential positions: opacity and transparency.

The referential positions outside the that-clauses are open to generalization and open to intersubstitution of names with the same reference, in the sense that, if we perform these operations on a true sentence, a true sentence will be the result. Thus these linguistic contexts or modes of containment of referential positions are called referentially transparent. In particular, the about-positions in de re attitudes are referentially transparent. We say the about-position belongs to the referentially transparent context to reflect the fact that a name there must function simply to refer to an object—existent objects only. It's as if one sees through the names to the things themselves.
In the that-clauses of *de re* beliefs the positions of anaphoric pronouns cross-referencing to the about-positions are also open to substitution and generalization in the places they cross-reference. So they belong to the referentially transparent context.

The rest of a *de re* belief's that-clause would be the open sentence predicated of these objects. In that-clauses we find the opacity: Since the open sentence may contain singular terms other than anaphoric pronouns, the sentence's occurrence within the context “believes that . . .” turns the positions of those singular terms into not purely referential positions. Thus “believes that . . .” is a referentially opaque context for the clause introduced by “that,” because the reports will often turn from true to false if we perform the operations of intersubstitution and wide-scope generalization on the names within the open sentence. Belia may believe about a photo, for instance, that it's of Mark Twain, but not of Samuel Clemens. Or she may think it's of Sherlock Holmes, so that it's false there is someone she thinks the photo's of. A *de dicto* belief's that-clause has no purely referential positions in it. We say a linguistic context is referentially opaque to reflect the fact that a name that fails to refer (so fails to be generalizable) can be in it and the sentence still be true. “Opaque” seems a good description of this sort of construction, for it's open to doubt whether any name in such a construction is doing its customary job of disclosure, that is, referring to a thing.

If I dichotomize referential positions into purely referential and one grade of not purely referential, then I'm supposing that, if a position is deviant with respect to one of the rules of inference, it's deviant with respect to all the others. There are theorems that go some way toward demonstrating that.² Our thesis that all attitudes are relational or notional propositional attitudes seems to undermine any counterexamples. If, for instance, you find a sentence that admits of generalization but not substitution in some referential position, as was the case with the idioms of causal explanation alluded to in section E§3, the sentence probably is not in regimented form. After being regimented the generalization will pertain to its about-position, while the failure of substitution will pertain to what's in its that-clause. Belia's belief about the photo only seems to be susceptible to generalization, not with respect to the position occupied by “Mark Twain,” but because of the implicit relational meaning of the term “photo.” There's something the photo is of, because photos are necessarily of something.

§3. *The ideal of extensionality.*

We called the positions of singular terms in opaque constructions “not purely referential.” Stated baldly, contexts that create the not-purely-referential
positions are out-and-out illogical, since they violate two fundamental kinds of valid inference, generalization and intersubstitution. If importation and exportation of names between referential positions fail, one of the positions is not purely referential. Perhaps then, we ought to suspect the very intelligibility of locutions that make positions not purely referential. They fail to be intelligible to the fullest degree, the way mathematics and ordinary logic are intelligible. Logic and mathematics are paradigms of making intelligible the various contents to which they apply. This ideal of intelligibility, which many philosophers adhere to, especially Quine,\(^3\) posits the necessary condition for it of **extensionality**: A subject matter's intelligibility depends on the extensionality of the discourse about it, because extensional contexts permit generalization, intersubstitution, and quantifying into class abstracts (recall D§2), allowing strong proofs in set theory. Quine moderates his claim that extensionality is necessary for intelligibility because intelligibility is subject to ontological constraints. In the article just cited he says,

> There has long been discussion of revision of logic for the simplification and clarification of quantum theory, and I can conceive that extensionality might not remain immune.

But he hopes it does or needs small compromises only. For the clarification and simplification of science take precedence over the preservation of extensionality, but insofar as extensionality is compromised so is intelligibility.

However, extensionality is more multifarious than I indicated. Extensionality has two facets, (I) a facet applying to referential positions and (II) a facet applying to attributive positions. The referential facet (I) requires

(I-i) the applicability of standard logic (including substitution and generalization) to singular terms referring to things in domains of well-defined entities that the subject matter is analyzed into.

The facet applying to the attributive position also has two parts:

(II-i) the applicability of substitution to predicates expressing the same attribute, and—this next clause is distinctively Quinean—

(II-ii) a restriction on abstraction to avoid commitment to modal attributes and attributes of being the objects of a directed-toward relation in propositional attitudes.

To the extent that logic is inapplicable to reports of belief, and their content cannot be analyzed into complexes of well-defined entities, they must fall short in crisp scientific intelligibility. Thus a failure of extensionality is a bad sign for propositional attitudes.

Item (I-ii) has priority in this definition, despite its second place. It is inspired by the extensionality axiom of set theory that specifies the identity condition for sets, namely, if a set \(x\) has all the same members as a set \(y\), then
\[ x = y. \] Its converse follows from the principle of the substitutivity of identity. It settles many questions about sets, such as whether to count repetitious listing of membership (no, since the set \{a,b,b\} has all the same members as the set \{a,b\}), and how many null-sets there are (one, since \{\} has all the same members as \{\}).

The founders of set theory at the dawn of the twentieth century called the axiom \textit{Bestimmtheit}, definiteness. Quine generalized the requirement of well-definedness to all entities susceptible to scientific examination: “\textit{No entity without identity.}” That's to say, no reification, entification, or positing of entities without settling the identification or individuation of that which is being supposed real. \textbf{Identity conditions} for a thing \(x\) are procedures sufficient for settling the truth of statements of the form, \(x = y\). They may also tell us how an individual is individuated and distinguished from other things, that's to say, they may be procedures that are also sufficient for settling the falsity of identity claims. They would be stronger than the one supplied by the contrapositive of the principle of the substitutivity of identity. A pair of attributes not consisting of an attribute and its negation, the only case covered by the substitutivity principle, may yet be such that if \(x\) has one of them and \(y\) has the other, then it's not the case that \(x = y\). While difference conditions are dispensable, without conditions for the identity of a thing we may question whether it's a thing at all.

In the next two subsections we'll consider alternative identity conditions for attributes, two different ways of fleshing out facet (I-ii) as it applies to attributes. Proponents of either way may claim they're conforming to the ideal of extensionality.

\section*{§4. Attributive positions: extensional contexts.}

Let's turn to the second facet of extensionality, (II), the facet that pertains to the attributive positions in sentences. Its first clause, (II-i), requires that phrases that express the same attribute be intersubstitutable \textit{salva veritate} in attributive positions. At first blush, it's unclear why this clause should be indispensable for intelligibility. There are two alternative ways of justifying facet (II-i) of the ideal of extensionality. One is in the style of Frege; the other in the style of the early Wittgenstein.

Let's begin with the Fregean one: Facet (II-i) can be derived from facet (I) pertaining to the referential positions, if we extend the concept of identity conditions to cover attributive terms. In section D§2 we noted that by abstraction we could convert the terms in attributive positions into singular terms, which would then occupy referential positions. So in effect we already reduced this second facet of extensionality to the first. Since the
ideal of extensionality has its source of plausibility in the validity of the rules of generalization and substitutivity of identity in referential positions, this reduction is important. Those who philosophize in the style of Frege think of extensionality as extending to attributive positions only because it applies to referential positions, and some of the terms in those positions can be converted by concretion to occupants of attributive positions. Otherwise the grounds for extending extensionality to attributive positions seem flimsy to them. For those positions are not for terms that stand for things.

Leibniz proposed a condition of identity applicable to anything. It's the converse of the principle of the substitutivity of identity:

If every attribute (other than trivializing ones and irrelevant ones) that applies to \( a \) applies to \( b \) too, then \( a = b \).

The trivializing attributes are those related to identity, such as the attribute of being identical to \( a \). Obviously \( a \) has that attribute, and if \( b \) has it too, then of course \( a = b \). If we allow that attribute as an instance of the antecedent of Leibniz's identity condition, the principle is a law of logic. For the purpose of formulating an informative identity condition, we omit that attribute as trivializing. (There's good reason to think identity is only a predicate, not an attribute at all.) The relevancy condition allows us to tailor the principle to particular cases. The axiom of extensionality in set theory is an instance of this principle tailored to sets. For the identity of ordinary objects, we omit the attributes (if they are attributes) that put \( a \) or \( b \) in the content of propositional attitudes. For consider the attribute (?) of figuring in a dictum which Belia believes. The failure of \( a \) and \( b \) to share this attribute would not count against their identity. It's an empirical matter what's relevant to identity and difference. Does the occupation of distinct places at the same time suffice to differentiate two entities of the same kind? Look for the answer in the theory of that kind of entity; the theory may tolerate bilocation of numerically one entity, e.g., the theory of gods.

We can perform semantic ascent on Leibniz's proposal with the result that we get a principle of the identity of intersubstitutables:

If \( a \) and \( b \) are intersubstitutable in all relevant, nontrivializing contexts salva veritate, then \( a = b \).

(In the good old days, these two versions were called “Leibniz's Law,” and for good reason, since Leibniz often expressed pride in his discovery that there's no perfect similarity anywhere, it being impossible for two things to differ only numerically. Oh, how the terminology has become confused! So we'd better not use the name in our smug age, if we wish to be understood.)

In view of the processes of abstraction and concretion that move terms back and forth between attributive and referential positions, we can treat this principle of the identity of intersubstitutables as applying to any kind of
position, not just referential positions. Thus if proposition \( p \) is inter-
substitutable in all relevant contexts for proposition \( q \) salva veritate, then
\( p=q \). The only question is, what are the relevant contexts? (The answer we'll
eventually settle on is modal contexts.)

Given the ideal of extensionality, a question of intersubstitutability arises
concerning the things a relational belief is directed toward, namely the
incomplete propositions or attributes, and not just concerning the things the
belief is about. When is an incomplete proposition \( X \), or attribute \( X \), the
same as a proposition or attribute \( Y \), so that one can be put for the other
without affecting the truth of the report? Let's introduce some terminology
to help us think precisely about identities of this sort. We begin with the
"extension" of a term.

Consider first the monadic terms. A monadic term's **extension** is the set
of the things it's true of. Its **antiextension** is the set of the things it's false of.
Ideally a term's extension and antiextension are mutually exclusive—
"ideally" because ordinary speech has terms whose extensions and antiex-
tensions coincide. An example is the attribute of self-shaving, said of some
barber who "shaves all those, and those only, who do not shave
themselves."5 Well, the attribute of being *that* barber has null extension and
universal antiextension. Not so easily avoided is **Grelling's paradox**: The
predicate "non-self-describing" describes itself just if it does not! (Here's
another reason to restrict abstraction, because that cannot be an attribute.)

Partially defined terms also exist in ordinary speech, so that their extensions
and antiextensions are not exhaustive. Truth itself may be a partially defined
term allowing indeterminate truth value. Given the extensions and anti-
extensions of terms, we can make comparisons. The attribute of having a
heart (i.e., being cordate) and the attribute of having a kidney (i.e., being
renate) are each monadic. Suppose they're true of exactly the same things.
We then say they're **coextensive** attributes, because they have the same
extension. Generalizing to cover all terms, a polyadic attribute's extension
is the set of all the sequences it's true of. So the extension of "loves" is a set
of pairs consisting of lover and then beloved. One and the same thing can
occur nontrivially more than once in a sequence. In one pair I'm both lover
and beloved.

The changes things undergo generate paradox unless we're careful: If, for
example, caterpillars all become butterflies without destroying numerical
identity, something is a butterfly if and only if it's not a butterfly (being a
caterpillar instead). One way to dispel the paradox is to let the extension of
"butterfly at" be pairs, the first item in each being a thing that is a butterfly
and the second being a time when it's a butterfly. That thing is in the
antiextension of "butterfly at" paired with an earlier time; nothing is a
butterfly and a caterpillar at the same time. Providing thus for time, we can say a term's extension is all the sequences it is, was, or will be true of. Good, but terms that describe changes must be dyadic despite seeming monadic. Other ways to disarm the paradox don't tinker with adicity.

Every general term has extension and antiextension, even if one's the null set, which is the extension of "unicorn," since it's true of nothing, false of everything. The extension of the relation, "infinitely far from," is the null set of pairs of things. Even complete propositions, medadic attributes, have them, although we don't think of their extensions as the things they're true of, for their medadicity precludes that. So we say all true propositions have the same extension, simply the true. The extension of true propositions is that which they are true to, the actual world if you wish, rather than anything they are true of. All false propositions have the same extension, namely the false; they all point away from the world, so to speak. Propositions' antiextensions are their negations' extensions.

We prove the extensions of propositions to be truth values, truth or falsehood, thus: By manipulating a sentence's terms in ways that don't affect the extension of the whole sentence, you can produce any other sentence of like truth value. Since the sentences' extension and truth value coincide, by Ockham's razor they're identical. We use just two rules for producing one sentence from another while leaving the sentence's extension unchanged: Substitute coextensive or coreferring terms for one another in a sentence, and replace a sentence with a synonymous sentence. Substitution will leave the whole sentence's extension unchanged, because it won't change the extension of the sentence's parts. Synonymous paraphrase allows the extension of parts to change, but the synonymy certifies the whole sentence's extension won't change as a result of any changes in the extensions of its parts. Alonzo Church and Kurt Gödel made this argument explicit in 1943. Church left the argument there, expecting us to be clever enough to construct paths, preserving extensions, from any sentence to any other sentence of like truth value. I'm not so clever; I must follow a recipe. Gödel devised one (since dubbed the slingshot). This is simpler: Get a sentence \( p \) with a definite truth value, and create a description \( D_p \): "the number that's either 1, if \( p \), or 0, if \( \text{not-}p \)." It's well-behaved (cf. p. 120 above). Provably, "\( D_p=1 \)" is true if and only if \( p \) is, and neither says anything the other omits, so I may claim they're paraphrases of each other, although validity depends only on their equivalence. Get another sentence \( q \), and form a similar description \( D_q \): "the number that's 1, if \( q \), or 0, if \( \text{not-}q \)." The sentences \( q \) and "\( D_q=1 \)" are equivalent. Suppose \( p \) and \( q \) are alike in truth value; ergo \( D_p=D_q \) (i.e., \( 1=1 \), or \( 0=0 \)). Alikeness in truth value suffices for a path from \( p \) to \( q \), preserving their extensions: \( p \); \( D_p=1 \) (by equivalence with \( p \)); assume \( D_p=D_q \); \( D_q=1 \) (by
substitution); q (by equivalence). A likeness in truth value is indispensable
for the path: If \( p \) and \( q \) differ in truth value, the path is blocked for lack of
the middle equality, for \( 0=1 \) ain’t so. Truth values are extensions. Q.E.D.

Singular terms, however, don’t have extensions and antiextensions, since
they’re neither true of anything nor false of anything; they simply name.
This is the semantic counterpart of the syntactic feature of names, no adicity
(section D§4). (I recommend against collapsing naming into a kind of
unit-set extension. For what then would it be to fail to name? That’s to be
no name at all, not a name with a null extension.) The singular term’s failure
to be either true or false of something is obvious in the case of logically
proper names. The point holds also of rigid designators like “I,” “now,” and
“here” in their contexts of utterance. Considered abstractly, each of the
words is governed by a rule that determines the reference it would have if it
were uttered: Given an utterance of “I,” it refers to its utterer; given an
utterance of “now,” it refers to the time of its utterance. These rules do not
determine extensions; that would be to determine, for any utterer or time of
utterance, the unit-set having her or it as a member, as though the rules
yielded the definite descriptions, “the utterer of this utterance of ‘I’” and “the
time when this utterance of ‘now’ is uttered.” Rather, the rules yield
modally rigid references for the utterances of these pronouns. More about
this important type of name is forthcoming in sections J§1 and M§3.

Resuming our review of the semantics of attributes, coextensive attributes
are those that have identical extensions. Are they thereby identical
attributes? One view of the identity of attributes, one consistent with the
Bestimmtheit identity condition on sets in set theory, would be that
coextensive attributes are really the same attribute. Coextensivity
is sufficient for identity. Call this the extensional identity condition
for attributes.

On that view of the identity conditions of attributes, the attributes reduce to
classes. Consider monadic attributes: As with classes, we’d then expect
intersubstitutability to work for the two singular terms, “the attribute of
having a heart” and “the attribute of having a kidney,” assuming that all and
only those who have hearts have kidneys. In many propositions containing
one of these singular terms we could make the substitution, and if the
proposition were true to begin with, a true proposition would result. Yes, in
many sentences, but not in all. Belia may believe-true about the woodchuck
the attribute of having a heart and demur on the matter of its having a
kidney. Intersubstitution of coextensive attributes in a statement of her de re
beliefs can change the report from true to false. It also fails in de dicto
reports, where substitution salva veritate, as applied to complete propositions,
would mean any truth may substitute for any other truth. So this kind
of intersubstitution fails in the context of beliefs generally.

Well, it's only one view of attributes that they have extensional identity conditions. Since the identity conditions for attributes aren't settled, it's not settled why intersubstitution of coextensive attributes fails here. We'd expect this change in truth value, which results from the intersubstitution, if coextensive attributes can be different attributes, just as we'd expect truth value to change if one were to substitute “John” for “Mary” in truths about Mary. But if coextensive attributes are the same attribute, then we have one more example of a failure of the substitutivity of identity in the that-clause, while it succeeds everywhere else.

Consider now the medadic attributes: By the extensional criterion there'd be just two of them, for all true complete declarative sentences would have the same extension and all false ones would have the same extension. It follows that any truth can be substituted for any truth in a compound sentence, and the result will have the same truth value as the original. That consequence is welcome to the advocate of the ideal of extensionality. It's a commonplace of logic that sentences compounded by use of the connectives, “and,” “or,” “not,” and quantifiers permit substitution of sentences for sentences based simply on the sameness of their truth values, and the resulting compound sentences will have the same truth value as the originals. According to the Fregean way of looking at this phenomenon, which we're examining now, namely, generalizing the concept of identity to apply to attributive positions, it's to be expected, since it's an instance of the substitutivity of identity. Furthermore, other connectives, like “because,” which don't permit this truth-preserving substitutivity, definitely look misbehaved from this perspective, and so less intelligible. Extensionality requires such substitutivity.

Let's turn to the second way of justifying facet (II-i). Those who philosophize in the style of the early Wittgenstein don't account for the indispensability of extensionality's facet (II-i) for intelligibility by generalizing identity conditions. For example, most logic texts, including Quine's, don't interpret the sentential substitutivity of extensional logic as an instance of the substitutivity of identity, but rather as derived from the meaning-blindness of the connectives that logic analyzes; only the truth values of the sentences they connect matter to the compound's truth value. The contrast between this approach and the one I've been developing is this: Does substitutivity work because the things substituted are really identical, or does it work because the context of logical connectives tolerates the substitutivity of nonidenticals because they're insensitive to the differences of the sentences they connect? If it's yes to the latter alternative, it's unclear to the Fregean why extensionality should be an ideal of intelligibility. Why should people find the
connectives' insensitivity to difference a help to intelligibility? Another problem the Fregean would feel: Without the generalization of identity to attributive positions, the ideal of extensionality looks like a mere conglomerate of the four tidbits I numbered earlier in this section. What makes all the tidbits go together to form a single ideal? The Fregeans have given one reply. (They don't claim that only the generalization of identity could serve to unify them. There may be another, less obvious unity to them all; it and identity may be jointly indispensable to the intelligibility of logic and mathematics, so that extensionality is not a disparate hodgepodge of ideals.)

The other side has credible answers to these difficulties. Those who philosophize in the style of the early Wittgenstein admit that the argument for the indispensability of facet (II-i) for intelligibility is distinct from the argument for facet (I), in that facet (II-i) makes no appeal to identity conditions. Rather the argument is that to extricate logic from the content of discourse, we must strictly segregate the structural elements of our discourse from the contentful elements, and treat the structural elements autonomously. Thus all content is to be confined to atomic sentences, and all structuring of them into compounds is to be by way of contentless formal elements. The connectives such as “and,” “or,” and “not,” rightly ignore differences in the content of the propositions they connect, and that's why the truth value of compounds is unaffected by intersubstituting propositions of like truth value or predicates true of the same things.

Once this segregation is completed, the analysis of logic can be pursued autonomously from the content by analyzing just the structural features of people's discourse. All the great metatheorems about logic and mathematics, which have been proven in the past century, were tractable to proof only because of this autonomy of the structural from the contentful. For they often depend on a mathematical induction on structural features which can recur to create unendingly complex structures. Thus the Wittgensteinians made a good alternative case for the indispensability of facet (II-i) to the intelligibility of discourse and they did it without assuming attributes exist.

We'll not settle which is the better way to justify clause (II-i) of the ideal of extensionality. We'll just assume the Fregean style of speaking about this facet of extensionality in terms of identity, because it will help us understand how Carnap and Frege differ from Quine. The assumption is not intended to prejudice the case against the Wittgensteinian style.

§5. Attributive positions: intensional contexts.

Carnap, after decades of accepting an extensional identity condition for attributes, reversed himself in the 1940's and offered an identity condition he
called “intensional.” William Hamilton, a nineteenth century logician, had coined a term “intension” (with an \(s\) instead of a \(t\)) to contrast with the word “extension.” The extension of a term is, as we said, all the things it's true of. The \textbf{intension} of a term is the property or relation (i.e., the attribute) it expresses; let's not follow Carnap in allowing inconsistent attributes which contradictory terms express.\(^{10}\) Recall Grelling's paradox. For terms with nonempty extensions, their intension is the attribute each of the things in the term's extension has that makes the term true of it. Whitehead and Russell popularized this coinage in chapter III of the Introduction to their \textit{Principia Mathematica} of 1910.\(^{11}\) Carnap followed their lead from the 1920's on. His approach yields an extensionality without clause (II-ii).

Carnap wanted to describe the contexts that were sensitive precisely to the identity conditions of attributes, and he believed that factual information was irrelevant to their identity conditions.

If, only as a matter of contingent fact, all and only those things that have one attribute have the other, the attributes are numerically distinct nevertheless. In contrast, predicates' coextensivity that's provable without appeal to contingent fact is sufficient for the singleness of the attribute they express. Call this the \textbf{intensional identity condition for attributes}.

This condition distinguishes attributes from classes.

Intensions are not \textbf{concepts}; we examine them in §§4-6. Propositions contain attributes, which are terms' intensions. Belia's thoughts contain concepts, which relate to the terms she uses in subtly diverse ways, also to be examined in section S. Just as her thought's content is a proposition, so her thought's concepts' contents are the attributes in the proposition.

Carnap defined \textbf{intensional contexts} as those that conformed to the intersubstitutability of identical attributes strictly, that is, \textit{only} identical attributes were intersubstitutable \textit{salva veritate}, and \textbf{extensional contexts} were those that allowed intersubstitutability of nonidentical attributes too, if they happened to be coextensive. When factual information was needed to “equate” the things or attributes whose names were to be intersubstituted, that was not strictly an identity.\(^{12}\) As so defined, extensional and intensional contexts are mutually exclusive.

An example of an intensional context is provided by modal logic,

\[ \text{it's necessary that Rep has a heart if and only if he has a heart.} \]

But substitution for “he has a heart” of a factually identified coextensive term does not preserve truth:

\[ \text{it's not necessary that Rep has a heart if and only if he has a kidney.} \]

So modal contexts are intensional. So, despite what the pair of sentences might suggest and despite the seeming opposition between intensional and
extensional, modal sentences once again (see section F§2) pass the test of logicality. For the substitution just performed was not one of identicals, and the resultant change in their truth value proves nothing untoward with respect to the compliance of modal idioms with the ideal of extensionality.

Here's another example of coextensive attributes, in this case each having the null-extension. The first is embedded in a true modal sentence, the second in a false one. The attributes are in brackets for easy identification:

- It's impossible for a thing to have the attribute of [being a barber who shaves all and only those who don't shave themselves]. That's demonstrably true, either because there's no such attribute, or, if there is, nothing could possess it. The next sentence is false, despite there being no planet with this description—they all do go round in one direction:
  - It's impossible for something to have the attribute of [being a planet revolving around the sun in the reverse direction to all the other planets].

The first attribute applies to nothing, simply as a matter of logic; the second just happens to apply to nothing as a matter of fact. On the extensional identity condition of attributes they'd be one and the same attribute nevertheless. For they'd be identical to their extension, and their extension is the null-set. But identity seems counter-intuitive. So again, trusting our intuitions, the change in truth value proves nothing untoward with respect to the modal idioms' compliance with the ideal of extensionality. What then of the terms' intersubstitutability in extensional contexts? On Carnap's way of seeing things, extensional contexts are not very discriminating of identity, since they allow much intersubstitution of nonidenticals. Modal sentences are the better discriminators.

Modal sentences are also the key contexts for the identity conditions for propositions. If \( p \) and \( q \) are intersubstitutable in all modal contexts salva veritate, then \( p=q \). Intuitively, propositions are possibility dividers, that is, a proposition partitions the range of possibilities into those consistent with its truth and those inconsistent with its truth. Modal contexts are just the right ones to test whether \( p \) and \( q \) create the same partition, in which case they're the same proposition. For, if there's a possibility consistent with one of them, but inconsistent with the other, there's a modal context in which their intersubstitution will fail. So propositions are intensions. Indeed they're the intensions of the sentence-radicals mentioned in section D§5.

The view that propositions, so understood, are suitable for being the objects of the attitudes' "directed-toward" relation, however, is just more of the thesis, that propositional attitudes are to be understood by analogy to the formulas of modal logic, which I objected to in the last section.

Singular terms, having no extensions, should have no intensions either.
But definite descriptions are names composed of predicates. Since the predicates have intensions, so do the definite descriptions. If by logic alone one can prove two descriptions name the same thing, then the descriptions have the same intension and express the same attribute. Intensional contexts allow their intersubstitution \textit{salva veritate}. Otherwise definite descriptions are not intersubstitutable in such contexts. Those analogs of proper names, such as numerals (expressions of standard form) which admit of logical derivation of statements of identity or nonidentity between them, also have intensions. Ordinary proper names, if they're not reducible to definite descriptions, seem not to have intensions. The fact that pairs of them that name the same thing are intersubstitutable in intensional contexts arises from the principle of the necessity of identity.

How does Carnap's distinction between intensional and extensional contexts relate to Quine's distinction between opaque and transparent contexts? On their face they're distinct, because Carnap's distinction refers to logical operations performed on attributive positions, but Quine's refers to operations on referential positions. Nonetheless, they're connected:

A context is transparent in its referential positions if it is extensional in its attributive positions.

If, however, our previous section's solution to the modal paradoxes holds true, namely, that definite descriptions are not genuine occupants of referential positions, the converse is false. Rather:

A context can be transparent in its referential positions even if it is intensional in its attributive positions.

Given the intensional identity condition for attributes, modal statements pass the first three tests of extensionality! For, granting intensional identity conditions of attributes, statements having them as components conform to the laws of logic, and if an intensional identity condition makes attributes well-defined entities, that's all they need to conform to the ideal of extensionality. (It sounds odd, since modal contexts are intensional, and intensions contrast with extensions, but the definitions allow it.) Quine demurs; he wants to build extensional identity conditions into the ideal of extensionality, and we cunningly did not. He's willing to concede the term "attribute" as meaning an entity having the intensional identity condition, but he objects to this identity condition, because it depends on there being a scientifically respectable distinction between matters of fact and of logic, and he does not believe the distinction is a sharp one except by sheer stipulation. The dispute between Carnap and Quine over the utility of the analytic-synthetic distinction in analysis is too big for us to discuss.

Let's play the game by Quine's rules, so that to accept intensional identity conditions is to reject the ideal of extensionality by virtue of the ill-
definedness of the entities having the intensional identity conditions. This means we'll continue to wring our hands over opacity and nonextensionality. Would that compliance with the ideal of extensionality, complicated as it is, were so simple. Recall that Quine's ideal of extensionality's facet dealing with the attributive position had two parts. We only examined the first of them. The second restricted the process of abstraction (explained in section D§2). It turns out that, if you allow abstraction to such modal attributes as the attribute of being necessarily greater than seven or the attitudinal attribute of being such that Belia believes it to be a woodchuck, then these so-called "attributes" obey the extensional criterion of identity for attributes!¹³ If we let them in by abstraction, why fight to keep them out by disallowing intensional identity conditions? Keeping our minds within the pen of Quinean extensionality is turning out to be like herding cats. Well, no such abstractions, y'hear? (Quine is a bit shrill on facet (II-ii.).)

§6. The irrelevance of intensionality to propositional attitudes.

We need not spend time on the subject of intensionality, since the chief lesson for us will be that the contexts generated by propositional attitudes are not intensional contexts, in the sense that Carnap defined. As he said,

the whole belief-sentence ['John believes that . . .'] is neither extensional nor intensional with respect to the subsentence ' . . . '. Consequently, an interpretation of belief-sentences as referring either to sentences or to propositions is not quite satisfactory.¹⁴

The contexts violate the principle of substitutivity of identity even when the identities are provable without appeal to factual information. For example, it's provable by logic and some definitions that $2^3=8$, and that $3^2=9$. But Belia's command of these necessary truths used to be poor, so that Belia believed that $8 < 9$,

but, given her denials and her pencil-and-paper calculations, it wasn't the case that Belia believed that $2^3 < 3^2$.

Another example concerns Belia's earlier studies of plane geometry. At one point in her studies:

Belia believed the sides of any equilateral triangle were the same length.

But,

Belia did not believe the sides of any equiangular triangle were the same length.

It's provable that a triangle is equilateral if and only if it's equiangular. Well, her befuddlement was awhile ago. She's been studying set theory recently
and now knows the definitions of a noninductive set and a reflexive set:

Belia's convinced that the set of natural numbers is noninductive. But so far, it's not true that

Belia is convinced that the set of natural numbers is reflexive.

But mathematics alone suffices to prove the equivalence, and so the identity:

the attribute of being noninductive = the attribute of being reflexive.

The two reports would be necessarily equivalent if belief created an intensional context in its report after the “that.” Carnap's exploitation of intensional identity conditions was great for rehabilitating modal logic. Nevertheless, once again we see that the problems with propositional attitudes are different and more obstinate.

The point being made here is the flip side of the point made in section E§3, that predicates expressing attitudes cannot be used in the principle of substitutivity. We've just seen that not even when the identities are proved by logic can we apply the principle to such predicates. Here's another paradox if we were to allow such applications: Suppose A is the definite description, “the proposition expressed by ‘a=a’” and B the definite description, “the proposition expressed by ‘a=b’.” Belia knows both propositions; so they're true. In view of the necessity of identity, even if propositions are given intensional identity conditions, A=B. Surely Belia knows A a priori.

By the principle of substitutivity she'd know B a priori too. But she does not. To avoid the contradiction, section E§3 forbade an attitudinal property such as knowing from being used in the principle's consequent. The Carnapian perspective, however, is not to restrict the consequent, but rather to restrict its antecedent which affirms identity; it's not known a priori that A=B. This Carnapian strategy is hopeful, as though discoveries are to be made, whereas restricting the consequent is Quinean and fearful, as though disaster's to be avoided. So far the fearful strategy is superior: Not even intensional identity for propositions delivers proper objects of attitudes.

Carnap did not give up hope; he defined a relation of intensional isomorphism between terms if they had internal structure, so that not only did the terms have the same intension, but their parts did also and in such a way as to construct the intension of the whole terms from those of their parts. He hoped intensional isomorphism extended the idea of intension enough to capture the logical quirks of propositional attitudes. While there's sameness of intension in "equilateral triangle" and "equiangular triangle," there's lack of intensional isomorphism, he said, because the intension of the former is constructed via the intension of "sides," and the intension of the latter is constructed via the intension of "angles." This difference accounts for Belia's divergent attitudes toward the two. We'll
examine this sort of solution in section X on structured intensions, but in the meantime let me register my skepticism. I believe the example using groundhog and woodchuck (in section E§3, where truth changes to falsehood upon substitution, because of what Belia believes) shows his theory's untestability. For to make it work, you just postulate hidden structure where any counterexample supposes no structure. Thus the terms “groundhog” and “woodchuck” or “puma” and “cougar,” which I take to have unstructured meanings, would have to stand for structured meanings, like the meanings of phrases, to avoid the violations we encountered in section E. Even if that seemed to work, it would fall afoul of Benson Mates's criticism using iterated propositional attitudes. Suppose two sentences A and B are intensionally isomorphic, so that we could derive, salva veritate

Whoever believes that A believes that B from

Whoever believes that A believes that A.

Nevertheless, when we enclose these sentences within the context “Nobody doubts that . . .,” the substitution of B for the second occurrence of A changes the whole sentence from true to false, since philosophers doubt the adequacy of intensional isomorphism! So I'll ignore until the end of Part III (section X) these convolutions of “intensionality with an s.” We'll see the Carnapian project of reducing intentionality to intensionality is alive.18

We ignore intensionality with an s, as Carnap defined it. Others use the s-word differently, in particular Quine, who finds Carnap's definition of it indefensible. So he uses intensional to cover whatever is not extensional. In Quine's usage propositional attitudes are intensional contexts. Many others follow his example, even though they accept Carnap's sharp distinction between analytic and synthetic. They think Carnapian thoughts, but express them in Quinean language; why I don't know. More careful thinkers have recognized that, if they use the term “intensional” to talk about propositional attitudes, they need to introduce degrees of intensionality.19 Instead of that, however, I prefer Carnap's own usage, according to which propositional attitudes are neither extensional nor intensional, not because I am Carnapian, but because I want to stress the difference between modal locutions, which are intensional, and reports of propositional attitudes, which are not: Let's call them hyperintensional contexts, although we must not assume that one needs identity conditions more stringent than intensional ones. The term was introduced by Maxwell Cresswell in 1975.20 Hyperintensional contexts are contexts that are neither extensional nor intensional. They turn out to be contexts whose content is governed by the person being reported on. However close in meaning two reports may be, the person reported on settles whether they're both true, and if they are both true, whether they report the
same belief. Hyperintensionality afflicts even the veridical attitudes like knowing (recall section C§1).

I don't endorse the thought that there are entities meeting identity conditions more stringent than intensional ones. And I certainly don't wish my use of the term “hyperintensional” to imply that there are hyperintensions to go along with the extensions and intensions of general terms. The Davidsonian regimentation is very good at depriving hyperintensional contexts of their apparent ontological import. Recall that in the Davidsonian regimentation, in Rep's report a separate sentence expresses the proposition Belia believes. Then the issue is simply whether Rep has said the same thing as what Belia believes. What looks like a problem of substitution for the standard regimentation looks more like a problem of translation in the Davidsonian regimentation: How should Rep translate into his own words what Belia believed in hers? If the problem of hyperintensionality is not a matter of finding even more stringent identity conditions, and I don't think it is, then perhaps it's best not to conflate that problem with the problems associated with intensionality at all. The conflation would send us off on a goose chase to catch entities meeting even more stringent identity conditions. Another reason to keep the Davidsonian regimentation alive in our thoughts, then, is that it breaks the grip of a bad analogy on our minds. (Since translation ideally puts exact synonyms for synonyms, it yields intuitively correct results even in opaque and hyperintensional contexts. Can we exploit the concept of translation between languages to create other “salva veritate” problems even for the Davidsonian regimentation? Yes! But I'm saving that difficulty until section W§1, since there are bigger fish to fry with it than just a variant style of regimentation.)

Since we define hyperintensional contexts in terms of Carnap's two other kinds of contexts, i.e., contexts that are neither extensional nor intensional, they concern attributive positions. So the question arises how a hyperintensional context relates to an opaque context. It seems that

a context is opaque in its referential positions, if it's hyperintensional in its attributive positions.

This is another difference between intensional and hyperintensional contexts.

We can summarize our discussion of contexts with a table:

<table>
<thead>
<tr>
<th>Context Type</th>
<th>Referential Positions</th>
<th>Attributive Positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extensional</td>
<td>True</td>
<td>False</td>
</tr>
<tr>
<td>Intensional</td>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>Hyperintensional</td>
<td>False</td>
<td>False</td>
</tr>
</tbody>
</table>

These distinctions help clarify the nature of different types of contexts and their implications for the study of belief and knowledge.
### Table G-1. The contexts of referential positions (the rows), related to the contexts of attributive positions (the columns).

<table>
<thead>
<tr>
<th>Transparent</th>
<th>Intensional</th>
<th>Hyper-intensional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extensional</td>
<td>logic, math, &amp; physical science</td>
<td>modal logic</td>
</tr>
<tr>
<td>Intensional</td>
<td>causal explanation</td>
<td>that-clauses of propositional attitudes</td>
</tr>
<tr>
<td>Opaque</td>
<td>[empty]</td>
<td>[empty]</td>
</tr>
</tbody>
</table>

Quine rejected modal logic because it violated extensional identity for no purpose. He'd've rejected propositional attitudes for the same reason, if they'd not been so indispensable to people's thinking about each other. So his ideal of extensionality is at the heart of all our difficulties. If you feel the ideal of extensionality uncompelling if extensional identity conditions are required for entities to be well-defined and fearful restrictions are put on the principle of substitutivity, you're not alone. Two major figures who agree with you are Frege and Carnap. In 1892 Frege noted the seeming failure of extensionality in propositional attitudes, but he did not blame opaque constructions, i.e., modes of containment of referential positions that caused them to be not purely referential. Instead he postulated that a singular term occupying that sort of position did refer to something, but not to what it customarily refers to. This something had different identity conditions from the customary referents. In effect he was anticipating Carnap, looking to nonstandard identity conditions for the solution.

In favor of Quine's side of the dispute with Carnap, you ought to feel the value of constraining your philosophizing to conform to the ideal of extensionality. Those who do feel it, yet think propositional attitudes are logically respectable, have looked for another way to avoid the charge that the propositional attitudes are illogical, a way that does not reject extensional identity. Perhaps they'll find it. Perhaps we will, in section V.

### §7. What was Brentano's thesis really?

In 1957, Roderick Chisholm associated the thesis of the nonextensionality of reports with a definition of the mental which Franz Brentano had defended in 1874. The association was anachronistic, for it attributed to Brentano the idea of semantic ascent, which he seems to have been innocent of, and also the view that attitudes are directed toward propositions, which
he seems to have been innocent of also. Although part of Brentano's explication of intentionality implied the opaqueness of reports, he never proposed a one-to-one correspondence of intentionality of attitudes at the level of the phenomenon to the nonextensionality of reports at the level of semantic ascent. Nevertheless, Chisholm defended this recommendation and even dubbed it “Brentano's thesis.” Although it is Chisholm's thesis and at one point he was careful to call it only “a thesis resembling that of Brentano,” the name “Brentano's thesis” unfortunately stuck to this linguistic thesis. I'll not use the name. Chisholm's linguistic thesis is false; the failure of extensionality is only a generic trait of the reports of attitudes, since many nonmental idioms fail to be extensional. Examples: “. . . lacks . . .” (the landscape lacks unicorns), “. . . resembles . . .” (she resembles a mermaid), and “. . . prevents . . .” (atmospheric conditions prevent class 10 hurricanes, the idea being there are no class 10 hurricanes any more than there are mermaids or unicorns). Even “p because q” seems not fully extensional, in that substitution of truths for truths fails to be *salva veritate*.

What's the real Brentano's thesis? I referred obliquely to it in the Introduction §2 when I explained the term, “intentionality,” a term whose currency is traceable to Brentano. I demurred then from the real Brentano's thesis, that intentionality is uniquely mental, agreeing to it as a verbal convention only. Really, minds emerge from mere uptake and exploitation of ambient information, along a scale reaching up to human intentionality.

We've now completed the regimentation of formats and their entailments. It's been a long haul from A to G only to prepare our data. The standardizing of our ways of reporting beliefs should not strike you as excessive tampering with our data, but rather as rendering our idioms less ambiguous. Perhaps our motive of sharpening a distinction between relational and notional attitudes could be reversed, and someone might regiment the distinction so that it's less sharp. Perhaps it could be regimented so as not to seem to violate the extensional ideal of intelligibility. Setting that possibility aside until section L, however, so far we've just been preparing our data for study, mindful of the fact that we may have prejudiced the data against some theories. If so, we'll have to reconsider the regimentation.

Let's now take up a theory of the about-position. It will be a component of a general theory of the attitudes. Sections H through O concern this component of the general theory. Later sections cover the other component of the general theory, i.e., the theory of the directed-towardness.
Notes

1. Quine created the concept in his “Reference and Modality,” in his *From a Logical Point of View* (1953) 142. A formal statement of it is in *Word and Object*, 144.

2. Kaplan lays out a proof of one theorem: if substitution fails, then generalization must fail. See his “Opacity” in L. Hahn and P. Schilpp, *The Philosophy of W. V. Quine* (1986; second, expanded edition 1998) 234. Quine, claiming the proof fails, offers a repair in his “Reply to David Kaplan,” 291. Quine does concede, however, that one can stipulate a new sense to generalization where substitution fails, if one wishes to allow such generalization.


4. See note 13 of section D. The terms “identical” and “exist” have extensions in each possible world. If that's an intension, not all intensions would be attributes.

5. Bertrand Russell “Philosophy of Logical Atomism,” (1918) lecture VII (p. 261 of *Logic and Knowledge*). To prove a priori there's no such barber, ask whether he shaves himself. Show that either answer leads to contradiction. For Kurt Grelling's paradox, see the bibliography in Quine, *The Ways of Paradox and Other Essays*, enlarged edition (1976).

6. Tarski’s formal definition of absolute truth makes it a species of truth-of. A sentence true of every sequence is true; a sentence true of no sequence is false.


8. The axiom is an instance of Leibniz's general identity condition, where the only relevant attributes describe a set's membership. Since about midcentury the axiom has come to be called Extensionality in English.

9. Wittgenstein, *Tractatus*. §5.54 states facet II of extensionality, and §5.55 states facet I. The text states an interpretation of the sections between these two. In the Introduction to the second edition of *Principia Mathematica* (1927), Whitehead and Russell note a use of extensionality pointing to this way of justifying extensionality (xiv), and in Appendix C of that edition, they announce the principle of extensionality. See Russell's *My Philosophical Development* (ch. 10) about this use.

10. Rudolf Carnap, *Meaning and Necessity*, 2nd ed. (1956), §4. An alternative to inconsistent properties is for the intension of contradictory terms to be every consistent property whatsoever, many of which cannot be coinstantiated. A contradictory sentence would express every consistent proposition rather than a single inconsistent one. C. I. Lewis, “The Modes of Meaning” (1944), recommended this alternative. I do too.
13. If we confine all modal and attitudinal phrases within attributes, we put the contexts created by those phrases in isolation, quarantining them from quantifying-in. Then the attributes can only appear in extensional contexts, and only their coextensiveness counts toward preserving the truth of the context when they are intersubstituted.
15. For the curious: Abraham Fraenkel, Abstract Set Theory, 2nd ed. (1961), pp. 28ff. A nonempty set I is noninductive if there's no positive integer n such that I has just n members. A nonempty set is reflexive if it has a proper subset that's equivalent to it, that is, there's a one-to-one mapping from all and only the things in the set to all and only the things in the subset. For the purposes of the equivalence, we leave the empty set out of consideration. Note 4 in section F referred to Hilbert's hotel. The set of its rooms was described as noninductive. But the trick of providing for more occupants without adding rooms depended on the set being reflexive.
18. Particularly in the work of Cresswell, D. Lewis (e.g., “General Semantics”), and J. Katz.
21. Frege, “On Sense and Reference” in P. Geach and M. Black, Translations from the Philosophical Writings of Gottlob Frege (1960). Quine contrasts his approach with Frege's in Word and Object, 151, and with Church's, 203.
22. Roderick Chisholm, Perceiving: A Philosophical Study (1957) ch. 11, “Intentional Inexistence.” Brentano's thesis, as Chisholm states it:

Let us say (1) that we do not need to use intentional sentences when we describe nonpsychological phenomena; we can express all of our beliefs about what is merely “physical” in sentences which are not intentional. . . . But (2) when we wish to describe perceiving, assuming, believing, knowing, wanting, hoping, and other such attitudes, then (a) we must use sentences which are intentional or (b) we must use terms we do not need to use when we describe nonpsychological phenomena. (172)

By “intentional sentence” Chisholm means an opaque construction exhibiting all the failures of extensionality. For some examples of opaque constructions used of nonmental things, see Stalnaker's Inquiry (1984) 11f. Chisholm offers those I cite in the text in his article, “Intentionality” in Paul Edwards, ed., Encyclopedia of Philosophy (1967). Chisholm would have to say they're all idioms we could give up without loss of power to express physical truth. See Brentano's own words in his Psychology From an Empirical Standpoint (1874) p. 88 of the English edition.