6.7 Explained

We didn't spend as much time in class today reviewing the exercises from 6.7 as I had planned. But it looks like everyone is pretty comfortable with the material in 6.1 - 6.6. (If you are having problems with any of this, *please let me know*!)

6.7 is simply a technique for determining the validity or invalidity of an argument. But, you might say, we already learned how to do this with truth tables. Why do we need to learn another technique? Good question.

Consider the argument:

1) $(P \supset \sim(Q \equiv \sim R))$

2) P

3) $\sim R$

4) $\therefore \sim Q$

("$\therefore$," remember, just means "therefore," and indicates the conclusion of an argument.)

We could use a truth table to determine the validity of this argument, as described today in class. But note that since this argument is composed of statements having a total of three atomic components (capital letters), our truth table would need to have 8 lines. Also, since the argument consists of four statements (three premises plus the conclusion), the left side our truth table would need to have 4 columns, one for each of the premises and the conclusion. We would then have to use the "Truth Evaluation" method to determine the truth value for all 4 statements on all 8 lines. This would involve using the Truth Evaluation method ***32*** times to fill out the table. Only then could we examine the table to see if there was any line with all true premises and a false conclusion.

That is a lot of work.

The "Truth Assignment" method of 6.7 simply cuts down on some of this works. It is *simpler*, but it does take just a *little* more thinking.

So remember that *in a **valid** argument, it is **impossible** for the premises to all be true and the conclusion false.* Alternately put, *if is **possible** for the premises to be true and the conclusion false, then the argument is **invalid**.* (Print that on the insides of your eyelids!) What we do in 6.7

is try to see if it is possible to make the premise true and the conclusion false. That is, we *try* to make it invalid. If we succeed, it is invalid. If we *fail*, it is *valid*.

We do this by trying to come up with a truth value assignment (i.e., an assignment of 0's or 1's to the atomic components) that makes the premises true and the conclusion false. How do we arrive at this this truth value assignment?

Before answering this, look at the example. You will notice that the first premise is relatively complicated, but all the other statements (the rest of the premises, and the conclusion) are all more simple: they are all either atomic statements (capital letters), or the negations of atomic statements. Notice also that all of the atomic components of that first premise occur as (or as parts of) the remaining statements. ***All of the exercises in 6.7 will be like this.*** All of them will begin with a complicated first premise, followed by other premises, and a conclusion, that are always either atomic statements of the negations of atomic statements, and that always contain all the atomic components in contained in the first premise.

So, how do you come up with a truth value assignment to test (i.e., to see if leads to true premises and a false conclusion)? Start by ignoring the first premise. (This is the statement you will "solve" by plugging in truth values). Assume that the remaining premises are all true, and the conclusion is false. This will give you a truth value assignment for all the atomic components of the first premises. (Remember, I said that in all these exercises, the remaining premises and conclusion will always contain all the atomic components found in the first premise.)

In our example, we assume that premises 2) and 3) are true, and that 4), the conclusion, is false. The second premise is "P." Since we are starting with the assumption that the remaining premises (after the first one) are true, we know that P=1. The third premise is "~R." Since we are assuming that this is true (i.e., we are assuming that "R is false" is true,), we know that R=0. Finally, the conclusion is "~Q." Since we are assuming that ~Q, the conclusion, *is false* (i.e, that it is false that Q is false), we know that Q=1.

Once again, we got these values by assuming that the premises (after the first one) were all true and that the conclusion is false. This gives us an assignment of P=1, R=0, and Q=1.

We now "plug in" these values to the first premise, and use the Truth Evaluation method to see if it comes out true or false. That is, in this example, we use the Truth Evaluation method to determine the value of "(P ⊃ ~(Q ≡ ~R))," given a truth value assignment of P=1, R=0, and Q=1.

Now, I haven't worked this out myself, but that is not the point. Suppose that "(P ⊃ ~(Q ≡ ~R))" comes out true (=1) on this assignment. What do we then conclude about the validity or invalidity of the argument?

This is where students make mistakes. ***If we make that first premise come out <u>true</u>, we have proven that the argument is <u>invalid</u>.***

Remember, an argument is invalid if it is possible for the premises to be true while the conclusion is false. We arrived at our truth value assignment (for the atomic components of the first premises) by assuming that the remaining premises were all true and the conclusion false. So if the first premise comes out true when we plug in those assignments, we have arrived at a truth value assignment that makes all the premises true and the conclusion false. We have, in doing this, proven that it is possible for all the premise to be true while the conclusion is false. That is, we have proven that the argument is invalid.

So, using this method, we prove that an argument is <u>valid</u> only when the truth value of the first premise comes out <u>false</u>.

As I said, this is where students make mistakes. It is easy to think that if we have "proven" the first premise true, then we have proven the argument valid. But this is mistaken. (If you don't see this, just re-read what I wrote above. If you still don't see that, then talk with me.)

So, using this method, here is the rule:

If (creating a truth value assignment based on the assumption that the remaining premises are true and the conclusion is false) you get a value of "1" (true) for the first premise, the argument is <u>invalid</u>. If you get a value of 0, then the argument is <u>valid</u>.

Remember this and you can't go wrong!

Finally, don't forget 6.8! That's where many students get totally lost!