

Consider: All A's are B's. (Examples: All bachelors are men; All dogs hate cats; All logicians suffer from OCD.) What these kinds of statements assert is that all members of the first class (A's) are included in the members of the second class (B's). In other words, for anything you like, *if* it's a member of the first class, *then* it's also a member of the second class, or $(x)(Ax \supset Bx)$.

(A philosophical parenthetical remark: How is it that "are" (in All A's are B's) becomes "if ... then" (in $(x)(Ax \supset Bx)$)? "Are," of course, is the plural form of "is" (which is a form of the verb "to be"). "Is" is a funny word. (Remember Bill Clinton's "It depends on what the meaning of the word 'is' is"?--No, I don't suppose you do. Oh well!) We tend to think of "is" as meaning "is identical to" or "is the very same thing as," as in "7 is the sum of 3 plus 4." But "is" has other uses. Consider "Sam is Italian." Obviously, we don't mean here that Sam is identical to Italian. (What in the world would that even mean?) It would be better to say here that Sam is included in the class of things that are Italian. So, while we sometimes use "is" as equivalent to the "=" in mathematics (as in " $7=3+4$ "), we also use it to indicate inclusion of the first thing within the second. It is this second sense of "is" (actually, of "are") that we are using when we say that All A's are B's. We are saying that "the A's" are included in "the B's." "Is" has lots of philosophically different uses. Can you think of any others?)

Now contrast "All A's are B's" with "Some A's are B's." If the universally quantified statement expresses "inclusion" of the first group within the second, doesn't the existentially quantified statement do so also? (Yes.) If we use a " \supset " in the first, shouldn't we also use a " \supset " in the second? (No.) We are told that "All A's are B's" becomes $(x)(Ax \supset Bx)$, but that "Some A's are B's" becomes $(\exists x)(Ax \cdot Bx)$. Why the difference?

I *could* say, "Just because." This is essentially what the author of our text does. And, if you can accept this and remember it, you'll be fine. But I would prefer that you *understand* the rules, rather than simply attempt to memorize them by rote. Consider first the existential statement, "Some A's are B's." This says that *there is something* that is both A and B. (If you like, it says that the "intersection" of the two classes is not empty.) In other words, it says that there is something that is *both A and B*, i.e., $(\exists x)(Ax \cdot Bx)$. What if we wrote, " $(\exists x)(Ax \supset Bx)$ "? That would say that there is something such that *if* it is A, then it is B. But remember the way " \supset " works—it is true unless the antecedent is true and the conclusion is false. In other words, if the antecedent is false, the conditional is true. So " $(\exists x)(Ax \supset Bx)$ " would be true even in the case where *nothing* is A. But that is not what "Some A's are B's" asserts—it says that *there is something* that is *both A and B*. So, "Some A's are B's" must be translated as " $(\exists x)(Ax \cdot Bx)$ ".

O.K., I hear you saying, but then why not use "and" in the universally quantified statement as well? Why not translate "All A's are B's" as " $(x)(Ax \cdot Bx)$ "? Well, consider what that says: it says, "Everything is such that it is *both A and B*. That is not what we want. So, we are left with using " \supset ".

Now, some of you, the really stubborn ones (that's a good thing!) are thinking, but if a conditional is true even when the antecedent is false, doesn't that mean that " $(x)(Ax \supset Bx)$ " is true even when there aren't any A's? *Yes!* Maybe that seems weird, but, recall, we noted earlier that " \supset " doesn't really capture the full sense of every "if...then" statement in English, and the same thing is true here. Is "All A's are B's" true when there *aren't any* A's? Think about it. Doesn't "All A's are B's" say the same thing as "There is nothing that is *both A and non-B*"? (Convince yourself that this is true.) But, if nothing is an A, then it is trivially true that nothing is *both A and non-B*. So, "There is nothing that is both A and non-B" is true even if nothing is A. But if "Nothing is both A and non-B" is true when nothing is A, and "Nothing is both A and non-B" says the same thing as "All A's are B's," then "All A's are B's" must likewise be true even when nothing is A.

The use of the other truth functional connectives within quantified statements is *relatively* straightforward.

Have fun!