Chapter 9, Sections 3 and 4

You may have noticed that the LogiCola exercises for 9.3 and 9.4 are the same. So, additional translation issues are introduced in two sections, 9.3 and 9.4, but the exercises reviewing these issues are combined. Keeping that in mind, I will introduce the material from both 9.3 and 9.4 at one time, so that you are prepared to tackle the LogiCola exercises covering them. This is the final set of translation questions we will look at in this class, and they once again get more complicated than what we have previously seen. At this point, you should be more adept at thinking in the language we have introduced. You will have to use your own insight to see how more complicated English statements can be captured by this formal language.

Starting in chapter 8, we have been looking at statements with a subject-predicate form, statements like

Socrates is bald

which we have translated with a capital letter standing for the predicate, and a small letter standing for the subject, which we represented thus:

Bs

which we might read, "Baldness is true of Socrates," or "Socrates has the property of being bald."

But we are now going to consider relational statements, statements such as

Romeo loves Juliet

This statement does not claim that Romeo or Juliet have some kind of property, but rather that they standing in a certain kind of *relation* to one another. So, we will now allow capital letters to stand not only for properties that things might have (or for classes that they might belong to), but for relations that two or more things might stand in to one another. So, we will translate the statement above about the famously star-crossed lovers as:

Lrj

We might read this as, "The '__loves__' relation obtains between Romeo and Juliet."

Translating relational statements requires one final tweak or our rules for well formed formulas. Previously, these rules stated that a capital letter followed by a small letter was a wff. This is now modified as follows:

The result of writing a capital letter and then two or more small letters is a wff.

(The complete and final set of rules for wffs can be found at:

http://homepages.wmich.edu/~baldner/wffs.pdf

Just ignore the rules for modal logic, which we will not be considering this semester.)

Note that the order in which the small letters that follow the capital letter matters. It is one thing to say that Romeo loves Juliet, but quite another to say that Juliet loves Romeo. So

Lrj

says something different than

Ljr.

In this case, both statements might be true (i.e., Romeo and Juliet might each love the other), but for other relations, this cannot be the case. So

Bob is taller than Sara

which we translate

Tbs

and

Sara is taller than Bob

i.e.,

Tsb

cannot both be true. So when translating, pay attention to the order of the small letters.

You may have noticed that our new rule for wffs states that "a capital letter followed by two *or more* small letters is a wff." So, our rules for what counts as well formed formula allow for relational statements with an indefinite number of things standing in that relation. Don't get confused by this. In English, there are cases of three-placed relations, i.e., statements that describe a relation that three objects stand in to one another. The most obvious is:

Bob gave the book to Sara

or, more generally,

a gave b to c,

which we would represent as:

Gabc.

It is hard to find examples in English statements of relations involving more than 3 objects, but they are more common in mathematics. So keep in mind that the number of "*relata*," i.e., of things said to stand in a given relation, depends upon the statement being translated, and our rules for wffs allow for well formed wffs with any number of *relata*." But you needn't worry about them for the test. From here on out, we will consider only two placed relations—i.e., statement that say that one object stands in a given relation to a second object.

Relational statements can include quantifiers. If Romeo loves Juliet, then it is true, for example that

Someone loves Juliet

which we would translate as:

(∃x)Lxj

If Juliet feels the same way about Romeo as he does about her, then it follows that

There is someone Juliet loves

i.e.,

(∃x)Ljx

(If you're rusty on your Shakespeare, let me remind you that Romeo was a member of the Montague family, while Juliet was a Capulet.)

So, now we can translate various quantified statements involving relations. Here are a few from p. 213 of our text:

Some Montague loves Juliet

(∃x)(Mx • Lxj)

All Montagues love Juliet

(x)(Mx ⊃ Lxj)

(For all x, if x is a Montague, then x loves Juliet.)

Romeo loves some Capulet

(∃x)(Cx • Lrx)

Romeo loves all Capulets

(x)(Cx ⊃ Lrx)

We can also look at some more complicated examples:

Some Montague besides Romes loves Juliet

$(\exists x)(\sim x=r \bullet Lxj)$

Romeo loves all Capulets besides Juliet

$(x)((Cx \bullet \sim x=j) \supset Lrx)$

There can also be examples involving two relations. Consider:

All who know Juliet love her

$(x)(Kxj \supset Lxj)$

All who know themselves loves themselves

$(x)(Kxx \supset Lxx)$

Starting to get fun, huh?

This is where 9.3 leaves us. But let me continue here to the material in 9.4.

So, we know that if Romeo loves Juliet, then it must be true that

Someone loves Juliet

which we translated

$(\exists x)Lxj$

But if someone loves Juliet, it is also true that

Someone loves someone

which we will translate as:

$(\exists x)(\exists y)Lxy$

Note that "Someone loves someone" might be true because

Someone loves him or herself,

$(\exists x)Lxx$

or because

There is someone who loves another

$(\exists x)(\exists y)(\sim x=y \bullet Lxy)$

Now, consider the difference between the following two statements:

Everyone loves someone

and

There is someone that everyone loves.

They say different things. The first says that everyone loves someone *or other*, while the second says that there is some specific person that everyone loves. We can capture this difference with careful attention to the placement of the initial quantifiers. So,

Everyone loves someone

is translated

(x)(∃y)Lxy

(For all x, there is some y that x loves.)

While

There is someone that everyone loves

becomes

(∃x)(y)Lyx

(There is some x that all y loves)

Notice not only the changed order of the initial quantifiers, but the changed order of the variables. The first includes "Lxy," while the second includes "Lyx." There is no formal "rule" that I can give you for determining the order of the variables. You must simply look at the English statement and capture what it says. As for the initial quantifiers, note that the first quantifier in the formal statement matches the first "quantifier phrase" in the English. So, the first

Everyone loves someone

starts with "everyone," and its first quantifier is "(x)," while the second,

There is someone that everyone loves

begins with "There is someone" and its first quantifier is "(∃x)."

Just for fun, here are a few more, but don't forget to review all the examples from pp. 214-217 of your text.

There is an unloved lover.

(Note that calling someone “a lover” is understood here to mean that there is someone that this person loves.)

(∃x)(~(∃y)Lyx • (∃y)Lxy)

(For some x, it is false that there is a someone that loves x, but true that there is someone that x loves.)

Romeo loves all and only those who don’t love themselves.

(x)(Lrx ≡ ~Lxx)

(For all x, romeo loves that person if and only if that person does not love him or herself.)

All who know any person love that person.

(x)(y)(Kxy ⊃ Lxy)

Ok, so that is all the translation you need to know for this class. So, what is the value of these translation for your continued life outside this class where you won’t be concerned with formal languages? From my perspective, learning the skills involved in these kinds of translations has far more value outside this class than the proofs and refutations. Doing proofs and reputations teaches you how to take complex problems and break them down into a number of smaller, simpler steps. But learning these translations forces you to think carefully and critically about what a statement actually *says*. What are the conditions that would make it true or make it false? Unless you go on to study mathematics or computer programming, you will probably never see these kinds of formal statements again. But we are all confronted with claims trying to convince of us something—whether it be in commercial advertising, or political persuasion—and it is our task to critically examine these claims. I hope that working through this process of translating English statements into the formal language we have created will hone your skills in doing so.

Finally, you can find the various translation forms that you will need to study for the test on the “Translation Guide” I have posted on the class web page. You are now responsible for all of. The last two pages cover the forms introduced in sections 9.3 and 9.4 of our text.

And now on to our final sections on proofs and refutations involving relations and statements will multiple initial quantifiers.