Chapter 9.1 Identity Statements

Consider the following statement:

Romeo is the lover the of Juliet.

We know that "Romeo" is a proper noun, and so is translated with a constant. And remember that since "the-so-and-so" phrases always pick out exactly one thing, "the lover of Juliet" should likewise get translated with a constant. Let's use "r" and "l" for these two constants. But how do we translate the "is" in this statement?

The word "is" has lots of uses which we won't be talking about here. The one we find in this example is sometimes called "the *is* of identity." What the statement says is that "Romeo" and " the lover of Juliet" are one and the same thing/person—i.e., that they are *identical*. Consequently, we will introduce a new symbol to our language, "=," the identity symbol, so that the above statement can be translated thus:

r=l (read "*r is identical to l*").

The introduction of the new symbol to our language takes place with the addition of an additional rule for well formed formulas:

The result of writing a small letter and then "=" and then a small letter is a wff.

As the rules states, "=" only occurs between small letters. Small letters, or "singular terms," recall, stand for individual things (Romeo, this child, the lover of Juliet), unlike capital letters, or "general terms," which stand for categories of things (being a lover, being charming, driving a Ford). So, while

Romeo is a lover.

is translated

Lr

since it says that Romeo is a member of the group or category of things which are lovers, Romeo is the lover of Juliet.

is translated

r=l

since it is saying that “Romeo” and “the lover of Juliet” stand for the very same individual thing.

Many of the statements we will be translating using the identity symbol involve negated identity statements. The way these are written might be confusing at first, but it is simply a matter of applying the rules we already have for well formed formulas. Remember our rule for “~” says that putting “~” in front of a wff results in a wff. Our new rule for the identity statement tells us that “r=l” is a wff. So, if we wanted to say that, for example that

Romeo is not the the lover of Juliet.

we would write this as

~r=l

Let’s be clear that “~r=l” does *not* say that “negative r equals l” or that “negative r is identical to l.” “r” is a constant: it stands for a name or referring expression. “Negative r” makes no more sense than “negative Romeo.” The correct way to read “~r=l,” is “It is not the case that r is identical to l,” or, more simply, “r isn’t l.” And note that we do not put parenthesis around the identity statement when constructing its negation. Neither “~(r=l)” nor “~(P)” are wffs.

With the ability to say that one thing is not the same as another, we gain the ability to translate many different English expressions. Consider:

Someone other than Romeo is rich.

What this says is that someone who isn’t Romeo is rich. We would translate this as:

(∃x)(~x=r • Rx)

i.e., there is something, call it x, such that x is not identical to Romeo, but x is rich.

Romeo alone is rich.

means that Romeo is rich, and that there is no thing that is not Romeo that is rich. Thus:

$$(Rr \cdot \sim(\exists x)(\sim x{=}r \cdot Rx))$$

Just a few more expressions to translate and we'll be done for now. Remember that the existential quantifier means "some" or "at least one." So, to translate

At least one is rich

we would simply write

$$(\exists x)Rx.$$

But what if we wanted to say that at least *two* are rich. You might think we could simply write:

$$(\exists x)(\exists y)(Rx \cdot Ry)$$

but this won't do. This says only that something, call it x, is rich and that something, call it y, is rich. As far as we know (as far as the statement explicitly says) it is possible that these two things could be the same. So, to say that at least two things are rich, we need to rule out the possibility that "x" and "y" stand for the same thing. So,

At least two are rich.

must be translated as

$$(\exists x)(\exists y)(\sim x{=}y \cdot (Rx \cdot Ry))$$

i.e., there is an x and y such that x and y are not identical and x is rich and y is rich.

Finally, it we wanted to say that exactly one is rich, we would need to say that something is rich, and then deny that there is anything which is both rich and not identical to the first thing. So

Exactly one is rich.

gets translated

$$(\exists x)(Rx \cdot \sim(\exists y)(\sim y{=}x \cdot Ry))$$

And

Exactly two are rich.

gets translated

$$(\exists x)(\exists y)((Rx \cdot Ry) \cdot \sim(\exists z)((\sim z{=}x \cdot \sim z{=}y) \cdot Rz))$$

Cool, huh?

Just for fun, try translating “At least three are rich” and “Exactly three are rich.” These translations very quickly get very long.