Notes for Chapter 9.2

<u>New Rules</u>

We have two new inference rules to accommodate identity statements. The first is unlike any of the existing rules, but is nevertheless very simple to understand, namely:

For any constant whatsoever, **a=a**.

When you apply this rule, write "SI" on the right to indicate that the line follows by the principle of Self Identity. There is nothing to star when you apply this rule.

This is unlike any of our other rules in that it does not justify an inference because of one or more existing lines. It isn't strictly a rule of inference, but an *axiom*. It is a logical truth that everything is identical with itself, and so it can be entered at any time during a proof, for any constant whatsoever.

This is our first rule for which there is no explicit strategic rule about when it should be applied. Since it is always permissible to write "a=a" for any constant whatsoever, we can't apply this rule whenever doing so would be possible, or we would end up with an infinitely long proof or refutation. So, you must apply this rule only when you recognize that doing so is necessary. This requires strategic "insight" on your part to recognize when you need to apply such rule in order to reach a contradiction.

The next thing to note is that the rule applies to *constants*, not to *variables*, and so it does **not** allow you to write, for example, "x=x." (You should never need to do this anyway. If it seems that you do, you have made some other sort of mistake.)

Finally, note that this rule is *useful* in only a limited class of cases. There may be cases where you may find a conditional statement (or some other statement to which an I-Rule might be applied) with an identity statement as its antecedent (or as one component). For example, if you were to find

(b=b ⊃(x)Fx)

on one line of a proof, you could use this rule to enter

b=b

on a later line, so that you could use an I-Rule to infer

(x) Fx.

Secondly, and more likely, it is possible that you find a statement such as

~c=c

on some line of a proof. If so, you could use this rule to enter

c=c

on a later line, thereby giving you a contradiction. There are cases where using this rule is the only way to derive a contradiction and allow you to apply the *Reductio Ad Absurdum* rule.

(Remember that "~c=c" is the negation of "c=c." It is **not** an identity statement between "~c" and "c.")

The second rule allows you to substitute identicals. To help understand this rule, consider the following example involving simple mathematical equations.

Suppose that you have the following mathematical equation:

47 = x+y

Suppose also that you know that

y=z.

Since you know that y=z, you know that you could replace "y" with "z" in the original equation without thereby changing it's truth or falsity, giving you

47 = x+z.

We can utilize the same principle here when we have an identity between two constants on one line and a wff that involves one of those constants on another line. We can then "substitute" the constant with another it is identical to without thereby changing the truth or falsity of the initial wff.

In the text, this rule is written:

a=b, Fa ---> Fb

To be explicit, this rule applies to any constants whatsoever (not just "a" and "b") and it applies to any wff containing one of those constants (not just to "Fa"). So, "Fa" here stands for any wff that contains one of the constants that appears in the identity statement, and "Fb" stands for the results of replacing all the instances of this constant with the second constant that appears in the identity statement. When you apply this rule, justify it by appealing to the line numbers of the identity statement and the line number that contains that constant that you replaced. Do no star any line.

That is all there is to these two new rules. There is, however, an interesting philosophical issue related to this second rule. There is a well-known limitation to cases where the second rule works. (This won't be relevant for any of the examples that appear in LogiCola or on the test. It is mentioned just because it is philosophically interesting.)

The second rule is sometimes known as the substitutivity of identicals. Consider another mathematical example. If we know that (x+2)=y, and we later learn that x=6, we can replace x with 6, giving us (6+2)=y. Since "x" and "6" are identical, replacing instances of "x" with instances of "6" won't change the truth of the equation. This principle holds throughout all of mathematics.

This kind of substitution, however, doesn't work in certain sorts of cases, and these cases tend to raise interesting philosophical questions. Suppose, for example, that we know that Dr. Jekyll is identical to Mr. Hyde. Now, if it turns out that Dr. Jekyll is 6 ft. tall, we can use this substitution rule without problem to infer that Mr. Hyde is 6 ft. tall. Since Jekyll and Hyde are the same person, whatever properties one has, the other has too.

But a problem arises in statements that involve mental states such as beliefs and desires. We know that "Jekyll is identical with Hyde" is true. Now suppose it is also true that the police *believe* that Jekyll is 6 ft. tall. Can we simply substitute "Hyde" for "Jekyll" and infer that the police believe that Hyde is 6 ft. tall? The answer is "no," because the police may not *know* (or *believe*) that Jekyll and Hyde are the same person. So, the principle fails when we are talking about the contents of our beliefs.

It seems strange that a logical principle of inference, valid throughout mathematics and applicable to the physical sciences, should turn out not to be applicable in contexts involving mental states. This is odd, in that logic is supposed to abstract from the content of statements, and look only at form. Why should it be different when we have statements about mental states? Philosophers have had lots to say on this matter.