Propositional Proof Strategy for "easier" proofs and refutations--Ch.s 7.1 and 7.2

1. **START ("*Rules for starting*"):** The premises are numbered. In the first line after the premises, write "[∴" followed by the conclusion.

(*Putting a "[" in front of a line indicates that it is "blocked off."*)

Do not number this line. After that, number each additional line consecutively. On the first new line (after the "blocked off" and un-numbered conclusion), after writing the line number, write "ASM" followed by the simpler contradictory of the conclusion.

(*Two wffs are contradictories if they are exactly alike, except that one starts with an additional "~". When two wffs are contradictories, the "simpler" one is the shorter one. So, if the conclusion was "P," you would write "ASM ~P," and if it was "~P," you would write "ASM P".*)

2. **S&I ("*Rules for continuing*"):** Find the complex wffs that aren't starred or blocked off.

(*A complex wff is any wff other than a capital letter or its negation. A simple wff is any capital letter or its negation.*)

Use these wffs and the S- and I-Rules to derive new wffs on subsequent lines. Star any wff you simplify using an S-rule (but only after you have derived both wffs), or the longer wff used in an I-rule inference.

(*You "star" a wff by placing a star, "*", to the left of the line number where that wff occurs. There are different starring rules for applying S-Rules and for applying I-Rules. When you derive a new line using an **S-Rule**, you put a star next to the line to which you applied an S-Rule. But S-Rules allow you to derive two lines from a given line. When using an S-Rule, don't star the line until (unless) you have derived both new lines.*

***I-Rules** allow you to derive a new line from two existing lines. That means you must find both of those two lines in order to apply an I-Rule. When you apply an I-Rule, put a star next to the longer of these two wffs.*)

After you derive a new line using an S- or I-Rule, *justify* this new line by writing, in the right column after the new wff, the line number(s) it was derived from. If it was derived from a single wff by an S-Rule, write that line number. If it was derived by from two previous lines by an I-Rule, list both line numbers, separated by a comma.

Keep repeating this strategy of applying S- and I-Rules, until one of the following two conditions occur ("***Rules for Stopping***"):

a. If, after doing this, you have a contradiction between any two lines, go to RAA (step 3).

(*You have a contradiction between two lines when you derive a new line that is the contradiction of a previous line.*)

b. If you have no contradiction and can't derive anything new (i.e., applying any additional S- or I-Rules will only produce wffs that you already have), go to REFUTE (step 4).

3. **RAA :** Apply the RAA rule. That is, block off all the lines starting with the assumption down to and including the line that forms the second half of a contradiction.

(*Remember, you block off a line by putting "[" in front of it. Blocked off lines can no longer be used to derive anything new.*)

After that, on a new not blocked-off line, write the simpler contradictory of the assumption. Justify this line by listing the line number of the assumption, followed by a ":", and then the lines numbers of the two halves of the contradiction, with a "&" between them (e.g., "5: 7 & 12.") Write "Valid," because you have proven the argument to be valid.

4. **REFUTE:** Construct a <u>refutation box</u> containing any simple wffs (i.e., letters or their negations) that aren't blocked off. (You may want to double-check by using these truth values to insure that it makes all the premises true and the conclusion false.) Write "Refute" under the proof, because you have refuted the claim that the argument is valid.

(*This list of simple wffs that makes up a <u>refutation box</u> will constitute a truth value assignment to the atomic components of the argument that makes the premises true and the conclusion false. By generating this truth value assignment, we have <u>refuted</u> the claim that the argument is valid.*)