Formal Logic—PHIL 3200

Fall 2019—First Test

Part 1:
For each of the following, circle "Yes" if it is a well formed formula, "No" if it is not. *1 point each*.

a. (P ≡ ~Q) **Yes**

b. ~(P ∨ Q)) **No**

c. ~(P ⊃ ~(Q ∨ (Q ≡ ~R)) **No**

d. ~~P **Yes**

e. (P ⊃ ~(Q)) **No**

Part 2:
Given the following truth value assignment:

P=1, Q=0, R=1, S=?

Determine the truth-value of the following truth-functional compounds, using the method from 6.3 and 6.4 of the text. Show your work. *5 points each*.

a. (~R ≡ ~(S ∨ ~(P • ~Q)))
(~1 ≡ ~(? ∨ ~(1 • ~0)))
(0 ≡ ~(? ∨ ~(1 • 1)))
(0 ≡ ~(? ∨ ~1))
(0 ≡ ~(? ∨ 0))
(0 ≡ ~?)
(O ≡ ?)
? **Truth Value Unknown**

Part 2: (Cont.)

Determine the truth-value of the following truth-functional compounds, using the method from 6.3 and 6.4 of the text. Show your work.

Given: **P=1, Q=0, R=1, S=?**

b. ((P • ~Q) ⊃ ~(R ∨ ~P))
((1 • ~0) ⊃ ~(1∨ ~1))
((1 • 1) ⊃ ~(1∨ 0))
(1 ⊃ ~1)
(1 ⊃ 0)
0 **Truth Value: False**

Given: **P=1, Q=0, R=1, S=?**

c. ((~Q ∨ ~S) ⊃ ~(P ∨ Q))
((~0 ∨ ~?) ⊃ ~(1 ∨ 0))
((1 ∨ ?) ⊃ ~(1 ∨ 0))
(1 ⊃ ~1)
(1 ⊃ 0)
0 **Truth Value: False**

Part 3:
Use truth tables to determine the validity of the following arguments. For each argument, I have provided a truth table with all the atomic components (the left side of the table), and columns on the right for each statement of the argument. You must fill in enough of the values for those columns to determine whether the argument is valid or invalid. **Explain why/how each truth table proves the validity or invalidity of the argument.** *5 points each*.

a.

(P ∨ Q)
(~Q ⊃ P)
∴ (P ≡ Q)

P	Q	(P ∨ Q)	(~Q ⊃ P)	(P ≡ Q)
0	0	0	0	1
0	1	**1**	**1**	**0**
1	0	**1**	**1**	**0**
1	1	1	1	1

Invalid: *Lines 2 and 3 make all premises true and conclusion false*

b.

(P ⊃ ~Q)
(~Q ⊃ R)
∴ (P ⊃ ~R)

P	Q	R	(P ⊃ ~Q)	(~Q ⊃ R)	(P ⊃ ~R)
0	0	0	1	0	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	0	1
1	0	1	**1**	**1**	**0**
1	1	0	0	1	1
1	1	1	0	1	0

Invalid: *Line 6 makes the premises true and the conclusion false.*

Part 4:

Use the truth evaluation method (i.e., the technique introduced in Ch. 6.7) to determine the validity of the following arguments. Show your work. *5 points each*.

a. $((P \bullet Q) \supset R)$ — P=?
$\sim R$ — ~R=1, so R=0
$\therefore \sim Q$ — ~Q=0, so Q=1

$((P \bullet Q) \supset R)$
$((? \bullet 1) \supset 0)$
$(? \supset 0)$
? *First premise has an unknown truth value*: **Invalid**
(*Assuming the remaining conditions for invalidity are met—namely, that the second premise is true and the conclusion is false—does not generate a truth value for "P," so we assign "?" to "P."*)

b. $((Q \bullet P) \equiv \sim R)$
R — R=1
P — P=1
$\therefore \sim Q$ — ~Q=0, so Q=1

$((Q \bullet P) \equiv \sim R)$
$((1 \bullet 1) \equiv \sim 1)$
$((1 \bullet 1) \equiv 0)$
$(1 \equiv 0)$
0 *First premise is false*: **Valid**

c. $((S \vee P) \supset (R \bullet P))$
R — R=1
$\sim S$ — ~S=1, so S=0
$\therefore P$ — P=0

$((S \vee P) \supset (R \bullet P))$
$((0 \vee 0) \supset (1 \bullet 0))$
$(0 \supset 0)$
1 *First premise is true*: **Invalid**

Part 5:
Translate the following statements into the symbolic language introduced in Ch. 6, using the following capital letters for truth functionally atomic statements (*5 points each.*):

H=*You are happy.*   **R**=*You are reliable.*   **T**=*You are tall.*   **S**=*You are smart.*

a. You are happy, if you are either reliable or tall.

((R ∨ T) ⊃ H)

b. You are both tall and smart, unless you are not reliable.

((T • S) ∨ ~R)   *or*   **(R ⊃ (T • S))**

c. If R is a necessary condition for H, then R is a sufficient condition for T.

((H ⊃ R) ⊃ (R ⊃ T))   *or*   **((~R ⊃ ~H) ⊃ (R ⊃ T))**

d. You are tall if and only if you are both happy and smart.

(T ≡ (H • S))

e. You are not tall, but you are happy only if you are reliable.

(~T • (H ⊃ R))

Part 6:

Using the S- and I-Rules introduced in Ch. 6.10-6.12, list the inferences we can make for each of the following exercises. In no inferences are justified by any of the rules, write "None." *3 points each*.

a.
$(\sim W \bullet \sim B)$
~W, ~B

b.
$(W \vee B)$
$\sim W$
B

c.
$\sim(W \bullet \sim B)$
NIL

d.
$(\sim W \supset \sim B)$
NIL

e.
$\sim(\sim W \supset B)$
~W, ~B

f.
$\sim(W \bullet B)$
$\sim W$
NIL

g.
$(W \supset B)$
$\sim W$
NIL

h.
$\sim(W \vee B)$
~W, ~B

i.
$\sim(\sim W \bullet B)$
$\sim W$
~B

j.
$(W \supset \sim B)$
B
~W