

Chapter 7 Proofs and Refutations

Test 2 Answers Fall 2019

When there were multiple possible assumptions that could be made, I have included solutions for every possible set of assumptions the rules allow you to make. When it says “[*assume X*]” after the problem number, that means that this solution uses “ASM X” as its second assumption. Star strings that should have been erased appear in parentheses and in red—(**).

1.

- *1. $(\sim(B \vee C) \supset P)$
- 2. $\sim P$
- 3. $\sim C$
- *4. $(X \supset \sim B)$
 $[\therefore \sim(B \supset X)]$
- *5. $[\text{ASM } (B \supset X)]$
- *6. $| (B \vee C)$ 1, 2
- 7. $| B$ 3, 6
- 8. $| X$ 5, 7
- 9. $[\sim B]$ 4, 8
- 10. $\sim(B \supset X)$ 5: 7 & 9

VALID

2. [Assume $(J \cdot I)$, then Assume $\sim X$]
1. Y
 2. I
 - (**)3. $((J \cdot I) \supset \sim X)$
 $[\therefore \sim(\sim X \supset (P \cdot \sim Y))]$
 - (**)(**)4. ASM $(\sim X \supset (P \cdot \sim Y))$
 - (**)5. [ASM $(J \cdot I)$ {break 3}
 6. $|\sim X$ 3, 5
 7. $|(P \cdot \sim Y)$ 4, 6
 8. $|\sim Y$ 7
 - *9. $\sim(J \cdot I)$ 5: 1, 8
 10. $\sim J$ 2, 9
 11. [ASM $\sim X$ {break 4}
 12. $|(P \cdot \sim Y)$ 4, 11
 13. $|\sim Y$ 12
 14. X 11: 1 & 13

REFUTE: Y, I, $\sim J$, X

2. *[Assume (J • I), then Assume X]*
1. Y
 2. I
 - (**)3. $((J \bullet I) \supset \sim X)$
 $[\therefore \sim(\sim X \supset (P \bullet \sim Y))]$
 - (**)4. ASM $(\sim X \supset (P \bullet \sim Y))$
 5. [ASM (J • I) {break 3}
 6. | $\sim X$ 3, 5
 7. |(P • $\sim Y$) 4, 6
 8. | $\sim Y$ 7
 - *9. $\sim(J \bullet I)$ 5: 1, 8
 10. $\sim J$ 2, 9
 11. ASM X
- REFUTE: Y, I, $\sim J$, X**

2. $[Assume (J \cdot I), then Assume (P \cdot \sim Y)]$
1. Y
 2. I
 - (**)3. $((J \cdot I) \supset \sim X)$
 $[\therefore \sim(\sim X \supset (P \cdot \sim Y))]$
 - *(**)4. $ASM (\sim X \supset (P \cdot \sim Y))$
 - (**)5. $[ASM (J \cdot I)$ {break 3}
 6. $|\sim X$ 3, 5
 7. $|(P \cdot \sim Y)$ 4, 6
 8. $|\sim Y$ 7
 - *9. $\sim(J \cdot I)$ 5: 1, 8
 10. $\sim J$ 2, 9
 11. $[ASM (P \cdot \sim Y)$ {break 4}
 12. $|\sim Y$ 11
 13. $\sim(P \cdot \sim Y)$ 11: 1 & 12
 14. X 4, 13

REFUTE: Y, I, $\sim J$, X

2. [Assume $(J \cdot I)$, then Assume $\sim(P \cdot \sim Y)$]
1. Y
 2. I
 - (**)3. $((J \cdot I) \supset \sim X)$
 $[\therefore \sim(\sim X \supset (P \cdot \sim Y))]$
 - **(**)4. ASM $(\sim X \supset (P \cdot \sim Y))$
 5. [ASM $(J \cdot I)$ {break 3}
 6. | $\sim X$ 3, 5
 7. | $(P \cdot \sim Y)$ 4, 6
 8. | $\sim Y$ 7
 - *9. $\sim(J \cdot I)$ 5: 1, 8
 10. $\sim J$ 2, 9
 11. ASM $\sim(P \cdot \sim Y)$ {break 4}
 12. X 4, 11
- REFUTE: Y, I, $\sim J$, X**

2. [Assume $\sim(J \cdot I)$, then $\sim X$]
1. Y
 2. I
 3. $((J \cdot I) \supset \sim X)$
[$\therefore \sim(\sim X \supset (P \cdot \sim Y))$]
 - (***)4. ASM $(\sim X \supset (P \cdot \sim Y))$
 - **5. ASM $\sim(J \cdot I)$ {break 3}
 6. $\sim J$ 2, 5
 7. [ASM $\sim X$ {break 4}
 8. |(P \cdot $\sim Y$) 4, 7
 9. [$\sim Y$ 8
 10. X 7: 1 & 9
- REFUTE: Y, I, $\sim J$, X**

2. [Assume $\sim(J \cdot I)$, then X]
1. Y
 2. I
 3. $((J \cdot I) \supset \sim X)$
[$\therefore \sim(\sim X \supset (P \cdot \sim Y))$]
 4. ASM $(\sim X \supset (P \cdot \sim Y))$
 - **5. ASM $\sim(J \cdot I)$ {break 3}
 6. $\sim J$ 2, 5
 7. ASM X {break 5}
- REFUTE: Y, I, $\sim J$, X**

2. [Assume $\sim(J \cdot I)$, then $(P \cdot \sim Y)$]
1. Y
 2. I
 3. $((J \cdot I) \supset \sim X)$
[$\therefore \sim(\sim X \supset (P \cdot \sim Y))$]
 - **4. ASM $\sim X \supset (P \cdot \sim Y)$
 - **5. ASM $\sim(J \cdot I)$ {break 3}
 6. $\sim J$ 2, 5
 7. [ASM $(P \cdot \sim Y)$ {break 5}
 8. [$\sim Y$ 7
 9. $\sim(P \cdot \sim Y)$ 7: 1 & 8
 10. X 4, 9
- REFUTE: Y, I, $\sim J$, X**

2. [Assume $\sim(J \cdot I)$, then $\sim(P \cdot \sim Y)$]
1. Y
 2. I
 3. $((J \cdot I) \supset \sim X)$
[$\therefore \sim(\sim X \supset (P \cdot \sim Y))$]
 - ***4. ASM $\sim X \supset (P \cdot \sim Y)$
 - **5. ASM $\sim(J \cdot I)$ {break 3}
 6. $\sim J$ 2, 5
 7. ASM $\sim(P \cdot \sim Y)$ {break 5}
 8. X 4, 7
- REFUTE: Y, I, $\sim J$, X**

2. *[Assume (P • ~Y)]*
1. Y
 2. I
 - *3. ((J • I) ⊃ ~X)
 [∴ ~(~X ⊃ (P • ~Y))]
 - *4. ASM (~X ⊃ (P • ~Y))
 5. [ASM (P • ~Y) {break 4}
 6. [~Y 5
 7. ~(P • ~Y) 5: 1 & 6
 8. X 4, 7
 - *9. ~(J • I) 3, 8
 10. ~J 2, 9
- REFUTE: Y, I, X, ~J**

2. *[Assume $\sim X$]*
1. Y
 2. I
 - *3. $((J \cdot I) \supset \sim X)$
 $[\therefore \sim(\sim X \supset (P \cdot \sim Y))]$
 - (**)4. ASM $(\sim X \supset (P \cdot \sim Y))$
 5. $[$ ASM $\sim X$ {break 4}
 6. $|$ (P \cdot $\sim Y$) 4, 5
 7. $|$ $\sim Y$ 6
 8. X 5: 1 & 7
 - *9. $\sim(J \cdot I)$ 3, 8
 10. $\sim J$ 2, 9
- REFUTE: Y, I, X, $\sim J$**

2. *[assume X]*
1. Y
 2. I
 - **3. $((J \cdot I) \supset \sim X)$
 $[\therefore \sim(\sim X \supset (P \cdot \sim Y))]$
 4. ASM $(\sim X \supset (P \cdot \sim Y))$
 5. ASM X {break 3}
 - **6. $\sim(J \cdot I)$ 3, 5
 7. $\sim J$ 2, 6
- REFUTE: Y, I, X, $\sim J$**

3. [Assume L , then assume $(N \cdot \sim E)$]

- (**)1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)$
- (**)2. ASM $\sim(\sim(N \cdot \sim E) \cdot E)$
- | | | |
|------|------------------------|-----------|
| 3. | [ASM L | {break 1} |
| 4. | E | 1, 3 |
| 5. | $(N \cdot \sim E)$ | 2, 4 |
| 6. | $\sim E$ | 5 |
| 7. | $\sim L$ | 3: 4 & 6 |
| **8. | ASM $(N \cdot \sim E)$ | {break 2} |
| 9. | N | 8 |
| 10. | $\sim E$ | 8 |

REFUTE: $\sim L, N, \sim E$

3. [Assume L , then assume $\sim(N \cdot \sim E)$]

- (**)1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)$
- **(**)2. ASM $\sim(\sim(N \cdot \sim E) \cdot E)$
- | | | |
|------|----------------------------|-----------|
| 3. | [ASM L | {break 1} |
| 4. | E | 1, 3 |
| 5. | $(N \cdot \sim E)$ | 2, 4 |
| 6. | $\sim E$ | 5 |
| 7. | $\sim L$ | 3: 4 & 6 |
| **8. | ASM $\sim(N \cdot \sim E)$ | {break 2} |
| 9. | $\sim E$ | 2, 8 |
| 10. | $\sim N$ | 8, 9 |

REFUTE: $\sim L, \sim E, \sim N$

3. [Assume L , then assume E]
- (**)1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
- (**)(**)2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
- | | | |
|-----|-----------------------|-----------|
| 3. | [ASM L | {break 1} |
| 4. | E | 1, 3 |
| 5. | (N \cdot $\sim E$) | 2, 4 |
| 6. | \sim E | 5 |
| 7. | $\sim L$ | 3: 4 & 6 |
| 8. | [ASM E | {break 2} |
| 9. | (N \cdot $\sim E$) | 2, 8 |
| 10. | \sim E | 9 |
| 11. | $\sim E$ | 8: 8 & 10 |
- REFUTE: $\sim L, \sim E$**

3. [Assume L , then assume $\sim E$]
- (**)1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
- (**)2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
- | | | |
|----|-----------------------|-----------|
| 3. | [ASM L | {break 1} |
| 4. | E | 1, 3 |
| 5. | (N \cdot $\sim E$) | 2, 4 |
| 6. | \sim E | 5 |
| 7. | $\sim L$ | 3: 4 & 6 |
| 8. | ASM $\sim E$ | {break 2} |
- REFUTE: $\sim L, \sim E$**

3. [Assume $\sim L$, then assume $(N \cdot \sim E)$]
1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
 2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
 3. ASM $\sim L$ {break 1}
 - ***4. ASM $(N \cdot \sim E)$ {break 2}
 5. N 4
 6. $\sim E$ 4
- REFUTE: $\sim L, N, \sim E$**

3. [Assume $\sim L$, then assume $\sim(N \cdot \sim E)$]
1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
 - ***2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
 3. ASM $\sim L$ {break 1}
 4. ASM $\sim(N \cdot \sim E)$ {break 2}
 5. $\sim E$ 2, 4
- REFUTE: $\sim L, \sim E$**

3. *[Assume $\sim L$, then assume E]*
1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
 - (***)2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
 3. ASM $\sim L$ {break 1}
 4. [ASM E {break 2}
 5. |(N $\cdot \sim E$) 2, 3
 6. [$\sim E$ 5
 7. $\sim E$ 4: 4 & 6
- REFUTE: $\sim L, \sim E$**

3. *[Assume $\sim L$, then assume $\sim E$]*
1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
 2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
 3. ASM $\sim L$ {break 1}
 4. ASM $\sim E$ {break 2}
- REFUTE: $\sim L, \sim E$**

3. $[Assume (N \cdot \sim E)]$
- **1. $\sim(L \cdot \sim E)$
- $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
- **3. ASM $(N \cdot \sim E)$ {break 2}
4. N 3
5. $\sim E$ 3
6. $\sim L$ 1, 5
- REFUTE: N, $\sim E$, $\sim L$**

3. $[Assume \sim(N \cdot \sim E)]$
- **1. $\sim(L \cdot \sim E)$
- $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
- **2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
- **3. ASM $\sim(N \cdot \sim E)$ {break 2}
4. $\sim E$ 2, 3
5. $\sim N$ 3, 4
6. $\sim L$ 1, 4
- REFUTE: $\sim E$, $\sim N$, $\sim L$**

3. [assume E]
- *1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
 - (**)2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
 - 3. [ASM E {break 1, 2}
 - 4. $|(N \cdot \sim E)$ 2, 3
 - 5. $[\sim E$ 4
 - 6. $\sim E$ 3: 3 & 5
 - 7. $\sim L$ 1, 6
- REFUTE: $\sim E, \sim L$**

3. [assume $\sim E$]
- **1. $\sim(L \cdot \sim E)$
 $[\therefore (\sim(N \cdot \sim E) \cdot E)]$
 - 2. ASM $\sim (\sim(N \cdot \sim E) \cdot E)$
 - 3. ASM $\sim E$ {break 1, 2}
 - 4. $\sim L$ 1, 3
- REFUTE: $\sim E, \sim L$**

4. $[assume \sim I]$
- (**) 1. $(\sim I \supset (\sim S \cdot \sim E))$
- *2. $\sim((\sim E \supset L) \cdot I)$
- $[\therefore \sim E$
3. $[ASM E$
4. $| [ASM \sim I$ {break 1}
5. $| |(\sim S \cdot \sim E)$ 1, 4
6. $| | \sim E$ 5
7. $| I$ 4: 3 & 6
8. $| \sim(\sim E \supset L)$ 2, 7
9. $| \sim E$ 8
10. $\sim E$ 3: 3 & 10

VALID

4.	[<i>assume I</i>]	
*1.	$(\sim I \supset (\sim S \cdot \sim E))$	
**2.	$\sim((\sim E \supset L) \cdot I)$	
	[$\therefore \sim E$	
3.	[ASM E	
4.	[ASM I	{break 2}
5.	$\sim(\sim E \supset L)$	2, 4
6.	$\sim E$	5
7.	$\sim I$	4: 3 & 6
8.	(S \cdot $\sim E$)	1, 7
9.	$\sim E$	8
10.	$\sim E$	3: 3 & 9

VALID

4.	[<i>assume ($\sim S \cdot \sim E$)</i>]	
*1.	$(\sim I \supset (\sim S \cdot \sim E))$	
*2.	$\sim((\sim E \supset L) \cdot I)$	
	[$\therefore \sim E$	
3.	[ASM E	
4.	[ASM ($\sim S \cdot \sim E$)	{break 1}
5.	$\sim E$	4
6.	$\sim(\sim S \cdot \sim E)$	4: 3 & 5
7.	I	1, 6
8.	$\sim(\sim E \supset L)$	2, 7
9.	$\sim E$	8
10.	$\sim E$	3: 3 & 9

VALID

4. $[assume \sim(\sim S \cdot \sim E)]$
- (**)1. $(\sim I \supset (\sim S \cdot \sim E))$
- (**)2. $\sim((\sim E \supset L) \cdot I)$
- $[\therefore \sim E$
3. $[ASM E$
4. | $[ASM \sim(\sim S \cdot \sim E)$ $\{break 1\}$
5. | |I 1, 4
6. | | $\sim(\sim E \supset L)$ 2, 5
7. | $[\sim E$ 6
8. | $(\sim S \cdot \sim E)$ 4: 3 & 7
9. $[\sim E$ 8
10. $\sim E$ 3: 3 & 9

VALID

4.	[<i>assume</i> ($\sim E \supset L$)]	
**1.	$(\sim I \supset (\sim S \cdot \sim E))$	
**2.	$\sim((\sim E \supset L) \cdot I)$	
	[$\therefore \sim E$	
3.	[ASM E	
4.	[ASM ($\sim E \supset L$)]	{break 2}
5.	$\sim I$	2, 4
6.	$(\sim S \cdot \sim E)$	1, 5
7.	$\sim E$	6
8.	$\sim(\sim E \supset L)$	4: 3 & 7
9.	$\sim E$	8
10.	$\sim E$	3: 3 & 9

VALID

4.	[<i>assume</i> $\sim(\sim E \supset L)$]	
*1.	$(\sim I \supset (\sim S \cdot \sim E))$	
(**)	$\sim((\sim E \supset L) \cdot I)$	
	[$\therefore \sim E$	
3.	[ASM E	
4.	[ASM $\sim(\sim E \supset L)$]	{break 2}
5.	$\sim E$	4
6.	$(\sim E \supset L)$	4: 3 & 5
7.	$\sim I$	2, 6
8.	$(\sim S \cdot \sim E)$	1, 7
9.	$\sim E$	8
10.	$\sim E$	3: 3 & 9

VALID

5.

- *1. $(B \cdot P)$
- *2. $((P \supset C) \vee \sim I)$
 $[\therefore \sim((\sim P \vee I) \vee \sim B)]$
- *3. ASM $((\sim P \vee I) \vee \sim B)$
- 4. B 1
- 5. P 1
- *6. $(\sim P \vee I)$ 3, 4
- 7. I 5, 6
- *8. $(P \supset C)$ 2, 7
- 9. C 5, 8

REFUTE: B, P, I, C

6.

- *1. $(F \supset \sim(L \cdot P))$
- *2. $\sim(\sim L \vee \sim F)$
 $[\therefore (\sim P \cdot \sim(\sim M \cdot \sim L))$
- *3. $[\text{ASM } \sim(\sim P \cdot \sim(\sim M \cdot \sim L))$
4. $|L$ 2
5. $|F$ 2
- *6. $|\sim(L \cdot P)$ 1, 5
7. $|\sim P$ 4, 6
8. $|(\sim M \cdot \sim L)$ 3, 7
9. $|\sim L$ 8
10. $(\sim P \cdot \sim(\sim M \cdot \sim L))$ 3: 4 & 9

VALID

7. $[assume \sim K]$
- *1. $(\sim K \equiv U)$
- (**)*2. $(L \vee K)$
- *3. $(U \vee \sim K)$
- $[\therefore \sim(L \supset \sim U)]$
- (**)*4. $[ASM (L \supset \sim U)]$
- (**)*5. $| (\sim K \supset U)$ 1
- *6. $| (U \supset \sim K)$ 1
7. $| \quad [ASM \sim K]$ {break 5}
8. $| \quad | U$ 5, 7
9. $| \quad | L$ 2, 8
10. $| \quad | \sim U$ 4, 9
11. $| K$ 7: 8 & 10
12. $| U$ 3, 11
13. $| \sim K$ 6, 12
14. $\sim(L \supset \sim U)$ 4: 11 & 13

VALID

7.	<i>[assume K]</i>	
*1.	$(\sim K \equiv U)$	
2.	$(L \vee K)$	
*3.	$(U \vee \sim K)$	
	$[\therefore \sim(L \supset \sim U)]$	
*4.	$[\text{ASM } (L \supset \sim U)]$	
*5.	$ \ (\sim K \supset U)$	1
(**)6.	$ (U \supset \sim K)$	1
7.	$ \ [\text{ASM } K]$	{break 6}
8.	$ \ \sim U$	6, 7
9.	$ \ \lceil \sim K$	3, 8
10.	$ \ \sim K$	7: 7 & 9
11.	$ U$	5, 10
12.	$ \sim L$	4, 11
13.	$\lceil K$	3, 12
14.	$\sim(L \supset \sim U)$	3: 10 & 13

VALID

7. [assume U]
- *1. $(\sim K \equiv U)$
 - (**)2. $(L \vee K)$
 - *3. $(U \vee \sim K)$
 - $[\therefore \sim(L \supset \sim U)]$
 - (**)4. [ASM $(L \supset \sim U)$
 - *5. $(\sim K \supset U)$ 1
 - (**)6. $(U \supset \sim K)$ 1
 - 7. [ASM U] {break 6}
 - 8. $\sim K$ 6, 7
 - 9. L 2, 8
 - 10. $\sim U$ 4, 9
 - 11. $\sim U$ 7: 7 & 10
 - 12. K 5, 11
 - 13. U 3, 12
 - 14. $\sim(L \supset \sim U)$ 4: 11 & 13

VALID

7.	[<i>assume</i> $\sim U$]	
*1.	$(\sim K \equiv U)$	
*2.	$(L \vee K)$	
(**)3.	$(U \vee \sim K)$	
	[$\therefore \sim(L \supset \sim U)$]	
*4.	[ASM $(L \supset \sim U)$]	
5.	$(\sim K \supset U)$	1
*(**)6.	$(U \supset \sim K)$	1
7.	[ASM $\sim U$]	{break 3}
8.	K	3, 7
9.	U	6, 8
10.	U	7: 7 & 9
11.	$\sim K$	10, 6
12.	L	2, 11
13.	$\sim U$	4, 12
14.	$\sim(L \supset \sim U)$	4: 10 & 13

VALID

7. [assume L]

- *1. $(\sim K \equiv U)$
- *2. $(L \vee K)$
- *(**)3. $(U \vee \sim K)$
- $[\because \sim(L \supset \sim U)]$
- (**)4. [ASM $(L \supset \sim U)$
- (**)5. | $(\sim K \supset U)$ 1
- *6. | $(U \supset \sim K)$ 1
- 7. | [ASM L {break 4}
- 8. | | $\sim U$ 4, 7
- 9. | | K 5, 8
- 10. | [U 3, 9
- 11. | $\sim L$ 7: 8 & 10
- 12. | K 2, 11
- 13. | $\sim U$ 6, 12
- 14. [$\sim K$ 3, 13
- 15. $\sim(L \supset \sim U)$ 4: 12 & 14

VALID

7.	$[assume \sim L]$	
	1. $(\sim K \equiv U)$	
	(**)2. $(L \vee K)$	
	*(**)3. $(U \vee \sim K)$	
	$[\therefore \sim(L \supset \sim U)]$	
	*4. $[ASM (L \supset \sim U)]$	
	*5. $ (\sim K \supset U)$	1
	(**)6. $ (U \supset \sim K)$	1
	7. $ [ASM \sim L]$	{break 2}
	8. $ K$	2, 7
	9. $ U$	3, 8
	10. $ \sim K$	6, 9
	11. $ L$	7: 8 & 10
	12. $ \sim U$	4, 11
	13. $ K$	5, 12
	14. $ U$	3, 13
	15. $\sim(L \supset \sim U)$	4: 12 & 14
	VALID	

8.

1. L
2. P
- *3. $(L \supset J)$
- *4. $(N \supset \sim(\sim P \cdot X))$
- *5. $(\sim J \vee N)$
- [$\therefore X$
6. ASM $\sim X$
7. J 1, 3
8. N 5, 7
9. $\sim(\sim P \cdot X)$ 4, 8

REFUTE: L, P, $\sim X$, J, N

9. [assume G]
- *(**)1. $\sim(G \cdot N)$
 - 2. $\sim(N \cdot \sim X)$
 - 3. $\sim(X \cdot G)$
 - $[\therefore \sim(G \equiv N)]$
 - *4. ASM ($G \equiv N$)
 - (**)**5.** ($G \supset N$) 4
 - 6. ($N \supset G$) 4
 - 7. [ASM G {break 2}
 - 8. | N 5, 7
 - 9. | $\sim N$ 1, 7
 - 10. $\sim G$ 7: 8 & 9
 - 11. $\sim N$ 1, 10
- REFUTE: $\sim G, \sim N$**

9. [assume $\sim G$]
- 1. $\sim(G \cdot N)$
 - 2. $\sim(N \cdot \sim X)$
 - 3. $\sim(X \cdot G)$
 - $[\therefore \sim(G \equiv N)]$
 - *4. ASM ($G \equiv N$)
 - 5. ($G \supset N$) 4
 - ****6.** ($N \supset G$) 4
 - 7. ASM $\sim G$ {break 6}
 - 8. $\sim N$ 6, 7
- REFUTE: $\sim G, \sim N$**

9. [assume N]
- (**)1. $\sim(G \cdot N)$
 2. $\sim(N \cdot \sim X)$
 3. $\sim(X \cdot G)$
 - [$\therefore \sim(G \equiv N)$]
 - *4. ASM ($G \equiv N$)
 - *5. ($G \supset N$) 4
 - (**)6. ($N \supset G$) 4
 7. [ASM N {break 6}
 8. | G 6, 7
 9. | $\sim N$ 1, 8
 10. $\sim N$ 7: 7 & 9
 11. $\sim G$ 5, 10
- REFUTE: $\sim N, \sim G$**

9. [Assume $\sim N$]
1. $\sim(G \cdot N)$
 2. $\sim(N \cdot \sim X)$
 3. $\sim(X \cdot G)$
 - [$\therefore \sim(G \equiv N)$]
 - *4. ASM ($G \equiv N$)
 - **5. ($G \supset N$) 4
 6. ($N \supset G$) 4
 7. ASM $\sim N$ {break 5}
 8. $\sim G$ 5, 7
- REFUTE: $\sim N, \sim G$**

9. [assume X]
1. $\sim(G \cdot N)$
 2. $\sim(N \cdot \sim X)$
 - **3. $\sim(X \cdot G)$
[$\therefore \sim(G \equiv N)$]
 - *4. ASM ($G \equiv N$)
 5. $(G \supset N)$ 4
 - **6. $(N \supset G)$ 4
 7. ASM X {break 3}
 8. $\sim G$ 3, 7
 9. $\sim N$ 6, 8
- REFUTE: $X, \sim G, \sim N$**

9. [assume $\sim X$]
1. $\sim(G \cdot N)$
 - **2. $\sim(N \cdot \sim X)$
 3. $\sim(X \cdot G)$
[$\therefore \sim(G \equiv N)$]
 - *4. ASM ($G \equiv N$)
 - **5. $(G \supset N)$ 4
 6. $(N \supset G)$ 4
 7. ASM $\sim X$ {break 2}
 8. $\sim N$ 2, 7
 9. $\sim G$ 5, 8
- REFUTE: $\sim N, \sim G$**

10.

- *1. $(B \cdot H)$
- *2. $\sim(Z \cdot K)$
- *3. $\sim(\sim K \cdot Z)$
 - [$\therefore \sim(\sim(Z \vee \sim H) \supset \sim B)$
- *4. [ASM $\sim(\sim(Z \vee \sim H) \supset \sim B)$
- 5. |B 1
- 6. |H 1
- *7. |(Z \vee $\sim H$) 4, 6
- 8. |Z 6, 7
- 9. | $\sim K$ 2, 8
- 10. [$\sim Z$ 3, 9
- 11. $\sim(\sim(Z \vee \sim H) \supset \sim B)$ 4: 8 & 10

VALID