Formal Logic
Fall 2018: Test 3—*Answers*

Part 1: Translate the following into symbolic form. (*5 points each*.)

1. No one is dangerous, if some logician has long hair.

((∃x)(Lx • Hx) ⊃ ~(∃x)Dx) *or* **((∃x)(Lx • Hx) ⊃ (x)~Dx)**

2. Baldner isn't a logician, but he does have long hair.

(~Lb • Hb)

3. If anyone is dangerous, then Baldner is dangerous.

((∃x)Dx ⊃ Db) *or* **(x)(Dx ⊃ Db)**

4. Not all who aren't logicians have long hair.

~(x)(~Lx ⊃ Hx)

5. If not everyone is dangerous, then not everyone has long hair.

(~(x)Dx ⊃ ~(x)Hx) *or* **((∃x)~Dx ⊃ (∃x)~Hx)**

6. Not any who are logicians aren't dangerous.

~(∃x)(Lx • ~Dx) *or* **(x)(Lx ⊃ Dx)**

Part 2: Construct a proof or refutation of the following arguments, using the inference rules and strategies developed in Chapter 8 of our text. As before, when the rules tell you to erase double-star strings, circle them instead. (As before, in these answers, I will indicate erased stars in red, e.g., **.) And don't forget to construct a refutation box if you refute the argument. (*10 points each.*)

1.

```
   1.  (x)~Sx
  *2. ~(∃x)~Ex
   3. ~Hc
     [∴(∃x)~(~(~Ex v Hx) ⊃ Sx)
  *4.          ⌈ASM ~(∃x)~(~(~Ex v Hx) ⊃ Sx)
   5.          |(x)Ex                              2
   6.          |(x)(~(~Ex v Hx) ⊃ Sx)              4
   7.          |~Sc                                1
   8.          |Ec                                 5
  *9.          |~(~(~Ec v Hc) ⊃ Sc)                6
 *10.          |(~Ec v Hc)                         7, 9
  11.          ⌊Hc                                 8, 10
  12. (∃x)~(~(~Ex v Hx) ⊃ Sx)                     4: 3, 11
```

VALID

2.a [second assumption: (∃x)Jx]

```
   1. (~(∃x)Jx ⊃ (∃x)Nx)
     [∴(x)(Jx v Nx)
  *2.          ASM ~(x)(Jx v Nx)
  *3.          (∃x)~(Jx v Nx)                      2
  *4.          ~(Ja v Na)                          3
   5.          ~Ja                                 4
   6.          ~Na                                 4
 **7.               ASM (∃x)Jx                     {break 1}
   8.               Jb                             7
```

REFUTE a, b: ~Ja, ~Na, Jb

2.b [second assumption: ~(∃x)Jx]

**1.	(~(∃x)Jx ⊃ (∃x)Nx)	
	[∴(x)(Jx ∨ Nx)	
*2.	ASM ~(x)(Jx ∨ Nx)	
*3.	(∃x)~(Jx ∨ Nx)	2
*4.	~(Ja ∨ Na)	3
5.	~Ja	4
6.	~Na	4
**7.	ASM ~(∃x)Jx	{break 1}
**8.	(∃x)Nx	1, 7
9.	Nb	8
10.	(x)~Jx	7
11.	~Jb	10

REFUTE a, b: ~Ja, ~Na, Nb, ~Jb

2.c [second assumption: (∃x)Nx]

1.	(~(∃x)Jx ⊃ (∃x)Nx)	
	[∴(x)(Jx ∨ Nx)	
*2.	ASM ~(x)(Jx ∨ Nx)	
*3.	(∃x)~(Jx ∨ Nx)	2
*4.	~(Ja ∨ Na)	3
5.	~Ja	4
6.	~Na	4
**7.	ASM (∃x)Nx	{break 1}
8.	Nb	7

REFUTE a, b: ~Ja, ~Na, Nb

2.d [second assumption: ~(∃x)Nx]

**1.	(~(∃x)Jx ⊃ (∃x)Nx)	
	[∴(x)(Jx ∨ Nx)	
*2.	ASM ~(x)(Jx ∨ Nx)	
*3.	(∃x)~(Jx ∨ Nx)	2
*4.	~(Ja ∨ Na)	3
5.	~Ja	4
6.	~Na	4
**7.	ASM ~(∃x)Nx	{break 1}
**8.	(∃x)Jx	1, 7
9.	Jb	8
10.	(x)~Nx	7
11.	~Nb	10

REFUTE a, b: ~Ja, ~Na, Jb, ~Nb

3.

*1.	~(~(∃x)~Hx ⊃ ~(∃x)Jx)	
2.	(x)(~Sx ⊃ ~Hx)	
	[∴~(x)(Jx ⊃ ~Sx)	
3.	⌈ASM (x)(Jx ⊃ ~Sx)	
*4.	|~(∃x)~Hx	1
*5.	|(∃x)Jx	1
6.	|(x)Hx	4
7.	|Ja	5
*8.	|(~Sa ⊃ ~Ha)	2
*9.	|(Ja ⊃ ~Sa)	3
10.	|~Sa	7, 9
11.	|~Ha	8, 10
12.	⌊Ha	6
13.	~ (x)(Jx ⊃ ~Sx)	3: 11, 12

VALID

4.

1.	(x)~Px	
*2.	(~(x)Jx • (x)Ex)	
	[∴~(∃x)~(~(Jx ∨ Px) • Ex)	
*3.	ASM (∃x)~(~(Jx ∨ Px) • Ex)	
*4.	~(x)Jx	2
5.	(x)Ex	2
*6.	~(~(Ja ∨ Pa) • Ea)	3
*7.	(∃x)~Jx	4
8.	~Jb	7
9.	~Pa	1
10.	~Pb	1
11.	Ea	5
*12.	(Ja ∨ Pa)	6, 11
13.	Ja	9, 12
14.	Eb	5

REFUTE a, b: ~Jb, ~Pa, ~Pb, Ja, Ea, Eb

5.a [second assumption: (x)Mx]

*1.	~(~(x)Mx • ~(∃x)~Ux)	
	[∴~(∃x)(~Mx • Ux)	
*2.	ASM (∃x)(~Mx • Ux)	
*3.	(~Ma • Ua)	2
4.	~Ma	3
5.	Ua	3
6.	⌈ASM (x)Mx	{break 1}
7.	⌊Ma	6
*8.	~(x)Mx	6: 4, 7
9.	(∃x)~Mx	8
*10.	(∃x)~Ux	1, 8
11.	~Ub	10

REFUTE a, b: ~Ma, Ua, ~Ub

5.b [second assumption: ~(x)Mx]

**1. ~(~(x)Mx • ~(∃x)~Ux)		
[∴~(∃x)(~Mx • Ux)		
*2.	ASM (∃x)(~Mx • Ux)	
*3.	(~Ma • Ua)	2
4.	~Ma	3
5.	Ua	3
**6.	ASM ~(x)Mx	{break 1}
**7.	(∃x)~Ux	1, 6
8.	~Ub	7
9.	(∃x)~Mx	6

REFUTE a, b, c: ~Ma, Ua, ~Ub

5.c [second assumption: ~(∃x)~Ux]

**1. ~(~(x)Mx • ~(∃x)~Ux)		
[∴~(∃x)(~Mx • Ux)		
*2.	ASM (∃x)(~Mx • Ux)	
*3.	(~Ma • Ua)	2
4.	~Ma	3
5.	Ua	3
6.	⌈ASM ~(∃x)~Ux	{break 1}
7.	|(x)Mx	1, 6
8.	⌊Ma	7
*9.	(∃x)~Ux	6: 4, 8
10.	~Ub	9

REFUTE a, b: ~Ma, Ua, ~Ub

5.d [second assumption: (∃x)~Ux]

1. ~(~(x)Mx • ~(∃x)~Ux)		
[∴~(∃x)(~Mx • Ux)		
*2.	ASM (∃x)(~Mx • Ux)	
*3.	(~Ma • Ua)	2
4.	~Ma	3
5.	Ua	3
**6.	ASM (∃x)~Ux	{break 1}
7.	~Ub	6

REFUTE a, b: ~Ma, Ua, ~Ub

6.a [second assumption: ~(∃x)Ex]

*1. ~((∃x)Ex • ~(x)~Qx)		
[∴~(∃x)~(Qx ⊃ ~Ex)		
*2.	⌈ASM (∃x)~(Qx ⊃ ~Ex)	
*3.	|~(Qa ⊃ ~Ea)	2
4.	|Qa	3
5.	|Ea	3
**6.	| ⌈ASM ~(∃x)Ex	{break 1}
7.	| |(x)~Ex	6
8.	| ⌊~Ea	7
9.	|(∃x)Ex	6: 5, 8
10.	|(x)~Qx	1, 9
11.	⌊~Qa	10
12. ~(∃x)~(Qx ⊃ ~Ex)		2: 4, 11

VALID

6.b [second assumption: (∃x)Ex]

**1.	~((∃x)Ex • ~(x)~Qx)	
	[∴~(∃x)~(Qx ⊃ ~Ex)	
*2.	⌈ASM (∃x)~(Qx ⊃ ~Ex)	
*3.	|~(Qa ⊃ ~Ea)	2
4.	|Qa	3
5.	|Ea	3
6.	| ⌈ASM (∃x)Ex	{break 1}
7.	| |(x)~Qx	1, 6
8.	| ⌊~Qa	7
*9.	|~(∃x)Ex	6: 4, 8
10.	|(x)~Ex	9
11.	⌊~Ea	10
12.	~(∃x)~(Qx ⊃ ~Ex)	2: 5, 11

VALID

6.c [second assumption: ~(x)~Qx]

**1.	~((∃x)Ex • ~(x)~Qx)	
	[∴~(∃x)~(Qx ⊃ ~Ex)	
*2.	⌈ASM (∃x)~(Qx ⊃ ~Ex)	
*3.	|~(Qa ⊃ ~Ea)	2
4.	|Qa	3
5.	|Ea	3
6.	| ⌈ASM ~(x)~Qx	{break 1}
**7.	| |~(∃x)Ex	1, 6
8.	| |(x)~Ex	7
9.	| ⌊~Ea	8
10.	|(x)~Qx	6: 5, 9
11.	⌊~Qa	10
12.	~(∃x)~(Qx ⊃ ~Ex)	2: 4, 11

VALID

6.d [second assumption: (x)~Qx]

```
*1. ~((∃x)Ex • ~(x)~Qx)
    [∴~(∃x)~(Qx ⊃ ~Ex)
*2.         ⌈ASM (∃x)~(Qx ⊃ ~Ex)
*3.         |~(Qa ⊃ ~Ea)                  2
 4.         |Qa                           3
 5.         |Ea                           3
 6.         |     ⌈ASM (x)~Qx             {break 1}
 7.         |     ⌊~Qa                    6
 8.         |~(x)~Qx                      6: 4, 7
*9.         |~(∃x)Ex                      1, 8
10.         |(x)~Ex                       9
11.         ⌊~Ea                          10
12. ~(∃x)~(Qx ⊃ ~Ex)                      2: 5, 11
```

VALID

7.

```
*1. (~(∃x)~Sx • (∃x)Nx)
    [∴~(x)~(Nx • Sx)
 2.         ⌈ASM (x)~(Nx • Sx)
*3.         |~(∃x)~Sx                     1
*4.         |(∃x)Nx                       1
 5.         |(x)Sx                        3
 6.         |Na                           4
*7.         |~(Na • Sa)                   2
 8.         |~Sa                          6,7
 9.         ⌊Sa                           5
10. ~(x)~(Nx • Sx)                        2: 8, 9
```

VALID