Necessary and Sufficient Conditions

Saying that one thing is a necessary condition for something else is not the same as saying that it is a sufficient condition. It is important to keep to these two notions clear. So let's look at examples of each.

Having eggs is a ***necessary condition*** for baking a cake. (I'm no baker, but I think this is true. Let's suppose that it is.) This means that without eggs, you can't bake a cake—i.e. no eggs implies no cake. But having eggs is not a sufficient condition for baking a cake—you need flour too. So, having eggs is a necessary condition but not a sufficient condition for baking a cake.

Let "P" = "I have eggs," and

Let "Q" = "I can bake a cake."

If P is a necessary condition for Q, we already established that without P, you don't have Q, so, "P is a necessary condition for Q" gets translated as "(~P ⊃ ~Q).

But if P is necessary for Q, that means that if Q is true, then P must also be true, since the truth of P is necessary for the truth of Q. So, "P is a necessary condition for Q" can also be translated as "(Q ⊃ P)."

(In general, "(A ⊃ B)" is logically equivalent to "(~B ⊃ ~A)." To say that two statements are logically equivalent is to say that they true and false in exactly the same circumstances. If you construct a truth table for "(~B ⊃ ~A)," you will see that it is true and false on exactly the same lines as "(A ⊃ B)." Hence, the two statements are logically equivalent. When doing translations on the test, any answer that is logically equivalent to the intended correct answer is also correct.)

So, let's sum up for "necessary conditions":

→ "P is a necessary condition for Q" can be translated as either "(~P ⊃ ~Q)" or "(Q ⊃ P)."

Let's move on to ***sufficient conditions***. Some nasty professors may have a very strict attendance policy. For them, if you miss one class, you fail the class. So in this case, missing one class is a sufficient condition for failing the class. To say that something is a sufficient condition for something else is to that it is, by itself, "enough" to bring about the second thing. In this case, missing one class is enough to fail the class. But it is not a necessary condition. There are other ways you can fail the class, for example, by failing all of the tests (even if you attended every class). So for this nasty professor, missing one class is a sufficient, but not a necessary condition for failing the class.

So to say that:

→ "P is a sufficient condition for Q" is to say that, "if P, then Q," i.e., "(P ⊃ Q)."

Finally, let's consider something that is both a ***necessary and a sufficient condition*** for something else. Suppose you have a class where the final grade in the class is entirely determined by what you get on the final exam—there are no other tests or papers, and no grade for attendance or participation.

Let "P" be "You get an A on the final exam," and

Let "Q" be "You get an A in the class."

In this case, getting an A on the final exam is a necessary and sufficient condition getting an A in the class. This means that both of the following are true: If you get an A on the final, you get an A in the class, and if you got an A in the class, you got an A on the final.

So, if P is a necessary and sufficient condition for Q, then

→ ((P ⊃Q) • (Q ⊃ P)), and this is logically equivalent to:

(P ≡ Q).

So, to sum up:

P is a sufficient condition for Q:	(P ⊃ Q)	
P is a necessary condition for Q:	(Q ⊃ P)	*or* (~P ⊃ ~Q)
P is a necessary and sufficient for Q:	(P ≡ Q)	*or* ((P ⊃ Q) • (Q ⊃ P))

Likewise:

Q, if P (or "If P, then Q"):	(P ⊃ Q)
Q only if P:	(Q ⊃ P)
P if and only if Q:	(P ≡ Q)

Finally, consider again "***P unless Q***." This says that P is true unless Q is true. In other words, if Q is true, then P isn't, i.e., (Q ⊃ ~P). But, given the truth table for "⊃," we know that "(Q ⊃ ~P)" is true just in case either it's antecedent is true or its consequent is false, i.e., just in case "(P ∨ Q)."

So,

→ P unless Q: (P ∨ Q) *or* (Q ⊃ ~P)

I hope this helps!