Full Proof Strategy for Proofs and Refutations--up to and including Ch. 7.4.

(This strategy works for both the "easier" and "harder" proofs and refutations in Chapter 7. When this strategy differs from the "Easier" Strategy for 7.1 and 7.2, I have used a blue font.)

1. **START:** The premises are numbered. In the first line after the premises, write "[∴" followed by the conclusion. Putting a "[" in front of a line indicates that it is "blocked-off." Do not number this line. After that, number each additional line consecutively.   On the first new line, write "ASM" followed by the simpler contradictory of the conclusion. (*So, if the conclusion was "P," you would write "ASM ~P," and if it was "~P," you would write "ASM P".*) All of the remaining lines are numbered consecutively.

2. **S&I ("*Rules for continuing*"):**   Go through the complex wffs that aren't starred or blocked-off and use them to derive new wffs using S- and I-rules. (*A complex wff is any any wff other than a single capital letter, or a single capital letter with a "~" in front of it.*)

Use these wffs and the S- and I-Rules to derive new wffs on subsequent lines. Using one star ("*") for each "<u>live assumption</u>," star any wff you simplify using an S-rule (but only after you have derived both wffs), or the longer wff used in an I-rule inference.

> **Note #1:** *An assumption is <u>live</u> if it's not blocked-off. Lines that are blocked-off occur to the right of "[" or "|," and are no longer available to used for further steps.*

> **Note #2:** *From now on throughout the rest of this course, when starring, <u>you must use one star for each live assumption</u>. So, for example, if you are in a part of a proof or refutation where there are two live assumptions, you must enter two stars when applying stars.*

After you derive a new line using an S- or I-Rule, *justify* this new line by writing, in the right column after the new wff, the line number(s) it was derived from.

Keep repeating this strategy of applying S- and I-Rules, until one of the following conditions occur ("***Rules for Stopping***"):

a. If, after doing this, you have a contradiction between any two lines, go to RAA (step 3).

b. If you have no contradiction **and all the complex wffs are starred or blocked-off**, go to REFUTE (step 4).

c. If you have no contradiction, and have applied all the S- and I-Rules you can, **but still have one or more complex wff that aren't either starred or blocked-off**, examine those unstarred wffs to see whether or not they are ***broken***.

> (*The only wffs that can be considered "broken" or "unbroken" are those to which I-Rules might be applied, i.e., <u>negated conjunctions</u>, <u>disjunctions</u>, and <u>conditionals</u>. A complex wff is <u>broken</u> when <u>it is one of these three types and where you already have one side or its negation, but nothing that will allow you derive a wff that doesn't already occur on an existing un-blocked-off line</u>.)*

> **i.** If all the un-starred, un-blocked-off complex wffs are broken, go to REFUTE (step 5).
> **ii.** If there is one or more un-starred, un-blocked-off complex wff that is not broken, go to ASSUME (step 4).

3. RAA : Apply the RAA rule. That is, on a new not blocked-off line, write the simpler contradictory of the assumption. Justify this line by listing the line number of the assumption, followed by a ":", and then the lines numbers of the two halves of the contradiction, with a "&" between them (e.g., "5: 7 & 12"). After that, block off all the lines starting with the most recent live assumption down to and including the line that forms the second half of a contradiction. *If all assumptions are now blocked-off, you've proved the argument valid. Write "Valid" under the proof, and you are done. <u>Otherwise, erase star strings having more stars than the number of live assumptions (i.e., erase the stars you entered in the lines you just blocked-off)</u>, and return to step 2.*

4. ASSUME: Pick a complex wff that is not starred, not blocked-off, and not broken. If there is more than one such unbroken wff, it does not matter which one you choose.

> (*This wff will have one of these forms: "~(A • B)," "(A ∨ B)," or "(A ⊃ B)."*)

Assume either side or its negation, and return to step 2.

5. REFUTE: Construct a <u>refutation box</u> containing any simple wffs (i.e., letters or their negations) that aren't blocked-off. Write "Refute" under the proof, because you have refuted the claim that the argument is valid.