

S- and I-Rules Explained

S-RULES

$$1) \quad \frac{(P \cdot Q)}{P, Q}$$

From a **conjunction** you can infer either conjunct. Examples:

$$\frac{(S \cdot R)}{R}$$

$$\frac{(\sim(P \supset Q) \cdot (P \vee T))}{\sim(P \supset Q)}$$

$$2) \quad \frac{\sim(P \vee Q)}{\sim P, \sim Q}$$

From the **negation of a disjunction** you can the **negation** of either disjunct. Examples:

$$\frac{\sim(S \vee R)}{\sim S}$$

$$\frac{\sim((P \cdot Q) \vee \sim(P \vee Q))}{(P \vee Q)}$$

$$3) \quad \frac{\sim(P \supset Q)}{P, \sim Q}$$

From the **negation of a conditional** you can infer either the antecedent or the **negation** of the consequent. Examples:

$$\frac{\sim(\sim P \supset \sim Q)}{Q}$$

$$\frac{\sim(P \supset (Q \supset \sim P))}{P}$$

$$4) \quad \frac{(P \equiv Q)}{(P \supset Q), (Q \supset P)}$$

From a biconditional you can infer the conditionals going in both “directions.”

$$5) \quad \frac{\sim(P \equiv Q)}{(P \vee Q), \sim(P \cdot Q)}$$

From the negation of a biconditional, you can infer the disjunction of the two components of the negated biconditional, and also the negation of the conjunction of the two components of the biconditional.

I-RULES

$$1) \quad \frac{\sim(P \cdot Q)}{P} \quad \frac{\sim(P \cdot Q)}{Q}$$
$$\frac{\quad}{\sim Q} \quad \frac{\quad}{\sim P}$$

From the **negation of a conjunction and the truth of one of its conjuncts** you can infer the **negation of the other conjunct**. Examples:

$$\frac{\sim(\sim P \cdot \sim Q)}{\sim P} \quad \frac{\sim((P \vee Q) \cdot (R \supset S))}{(P \vee Q)}$$
$$\frac{\quad}{Q} \quad \frac{\quad}{\sim(R \supset S)}$$

$$2) \quad \frac{(P \vee Q)}{\sim P} \quad \frac{(P \vee Q)}{\sim Q}$$
$$\frac{\quad}{Q} \quad \frac{\quad}{P}$$

From a **disjunction and the negation of one of the disjuncts** you can infer the truth of the other disjunct. Examples:

$$\frac{(S \vee \sim Q)}{Q} \quad \frac{((P \equiv Q) \vee (R \supset S))}{\sim(P \equiv Q)}$$
$$\frac{\quad}{S} \quad \frac{\quad}{(R \supset S)}$$

$$3) \quad \frac{(P \supset Q)}{P} \quad \frac{(P \supset Q)}{\sim Q}$$
$$\frac{\quad}{Q} \quad \frac{\quad}{\sim P}$$

From a **conditional and the truth of its antecedent** you can infer the truth of its consequent. From a **conditional and the negation of its consequent** you can infer the **negation of its antecedent**. Examples:

$$\frac{(\sim R \supset \sim S)}{\sim R} \quad \frac{((P \vee Q) \supset \sim S)}{S}$$
$$\frac{\quad}{\sim S} \quad \frac{\quad}{\sim(P \vee Q)}$$