

Parentheses and “Scope”

Consider:

Not P or Q.

What is being “*notted*,” here, i.e., what is being *negated*? Does the “not” apply to “P” or to “P or Q?”

The original statement is simply ambiguous. We can make it explicit by distinguishing between

Not P, or Q.

which we can “translate” as “ $(\sim P \vee Q)$,” and

Not either P or Q.

which we can translate as “ $\sim(P \vee Q)$.”

So, we use parentheses to indicate the “scope” of the negation. Recall the rules for well formed formulas: if ‘P’ is a wff, then so is ‘ $\sim P$.’ So, the “tilde” (the negation) applies to the well formed formula that follows it. In the first case, only ‘P’ is being negated, while in the second case, the *disjunction*, ‘ $(P \vee Q)$ ’ is being negated. The parentheses help make this clear. If the tilde occurs prior to an open paren, then what is being negated is what occurs within the parentheses—from the open paren up until the close paren that is its partner. If the tilde occurs prior to a capital letter, it is only that atomic statement that is being negated.

When we write and speak in everyday English, we are often just sloppy about this. The author will try to be more careful and use commas and the word “either” to distinguish the two cases. When a comma follows a clause, what comes before it should be understood as part of a larger claim (i.e., either a conjunction, disjunction, conditional, or bi-conditional), where what follows the comma is the second half. Similarly, the word “either” indicates that what follows is a disjunction. And so a compound statement beginning with “Not either ...” will be the negation of a disjunction.

The same principle applies to negations and conjunctions. Consider:

Not P and Q.

Does the “not” apply merely to ‘P,’ or to the whole conjunction? The English is just ambiguous. We can clarify things by distinguishing between

Not P, and Q.

which we will “translate” as “ $(\sim P \cdot Q)$,” and

Not both P and Q.

which we will translate as “ $\sim(P \cdot Q)$.” In this case, we use the word “both” in a way parallel to “either” above. We put “both” before a conjunction to make explicit that what follows is to be taken as a unit, so that whatever precedes the word “both” applies to the whole unit.