

Simplification and Introduction Rules

| S-Rules | I-Rules |
|--|---|
| <u>Conjunctions</u> $(P \cdot Q)$ $\therefore P$ $\therefore Q$ | <u>Negated Conjunctions</u> $\sim(P \cdot Q)$ $\frac{P}{\therefore \sim Q}$ $\frac{\sim(P \cdot Q)}{\therefore \sim P}$ |
| <u>Negated Disjunctions</u> $\sim(P \vee Q)$ $\therefore \sim P$ $\therefore \sim Q$ | <u>Disjunctions</u> $(P \vee Q)$ $\frac{\sim P}{\therefore Q}$ $\frac{(P \vee Q)}{\therefore P}$ |
| <u>Negated Conditionals</u> $\sim(P \supset Q)$ $\therefore P$ $\therefore \sim Q$ | <u>Conditionals</u> $(P \supset Q)$ $\frac{P}{\therefore Q}$ $\frac{(P \supset Q)}{\therefore \sim P}$ |
| <u>Biconditionals</u> $(P \equiv Q)$ $\therefore (P \supset Q)$ $\therefore (Q \supset P)$ | |
| <u>Negated Biconditionals</u> $\sim(P \equiv Q)$ $\therefore (P \vee Q)$ $\therefore \sim(P \cdot Q)$ | |

S and I Rules: *What they mean*

The S- and I-Rules developed in our text describe inferences we can make, from the (already established or assumed) truth of some statements to the truth of some other statements. As we use them in proofs and refutations, the rules all say that if we have statements of a certain kind on one or more existing lines of the proof, we are entitled to add one or two additional lines of certain kinds.

(Note 1: These rules can only be applied to statements that occur on lines that are un-starred and un-blocked off.

Note 2: These rules say only what lines we are *permitted* to write. It is the “Proof Strategy” that tells us what line we *ought* to write next.)

S-Rules

The S-Rules all apply to a single line. That is, they tell us that if we find a certain kind of statement on one line of a proof, we are entitled to add two additional lines, if they don't already occur. (In other words, don't write something down if it already occurs on some existing un-starred, un-blocked off line.)

From a conjunction on one line of a proof, we are entitled to add a new line for each conjunct. (Star the conjunction iff (if and only if) both conjuncts are written on some line.)

From the negation of a disjunction on one line of a proof, we are entitled to add a new line for the negation of each disjunct. (Star the negated disjunction iff the negations of each disjunct are written on some line.)

From the negation of a conditional statement on one line of a proof, we are entitled to add a new line containing the antecedent of the conditional, and another containing the negation of the consequent. (Star the negated conditional iff each of these statements are written on some line.)

(For the rules applying to biconditionals and negated biconditionals, “P and “Q” stand for any statements whatsoever, whether truth-functionally atomic or truth-functionally compound.)

From a line containing the biconditional of P and Q--i.e., a statement of the form $(P \equiv Q)$, we are entitled to add a new line with the conditional $(P \supset Q)$ and another with the conditional $(Q \supset P)$. (Star the biconditional iff each of these conditional statements is written on some line.)

From a line containing the negated biconditional of P and Q--i.e., a statement of

the form $\sim(P \equiv Q)$, we are entitled to add a new line for the disjunction of P and Q -
-a statement of the form $(P \vee Q)$, and another for their negated conjunction--i.e., a
statement of the form $\sim(P \cdot Q)$. (Star the negated biconditional iff each of these
statements is written on some line.)

I-Rules

The I-Rules permit writing a new line when we already have two lines containing
statements of certain kinds.

*From the negation of a conjunction on one line and one of the conjuncts on
another line, we are entitled to add a new line containing the negation of the
other conjunct. (Star the negated conjunction after adding the new line.)*

*From a disjunction on one line and the negation of one of the disjuncts on
another line, we are entitled to add a new line containing the other disjunct. (Star
the disjunction after adding the new line.)*

*From a conditional on one line and its antecedent on another, we are entitled to
add a new line containing the consequent. From a conditional on one line and
the negation of its consequent on another, we can add a new line containing the
negation of its antecedent. (Star the conditional after adding the new line.)*