

Proof Strategy for Single Quantifier Proofs and Refutations--up to and including Ch. 8.5

1. START (“Rules for starting”): The premises are numbered. In the first line after the premises, write “[.:” followed by the conclusion.¹ Do not number this line. After that, number each additional line consecutively. On the first new line (after the “blocked off” and un-numbered conclusion), after writing the line number, write “ASM” followed by the simpler contradictory² of the conclusion.

2. S&I (“Rules for continuing”): Find the complex wffs³ that aren’t starred or blocked off. Use these wffs and our rules of inference, in the order listed below, to derive new wffs on subsequent lines. Using one star (“*”) for each live assumption,⁴ star any wff you simplify using an S-rule (but only after you have derived both wffs), or the longer wff used in an I-rule inference. After you derive a new line, *justify* this new line by writing, in the right column after the new wff, the line number(s) it was derived from.

Apply our rules of inference in the following order:

2.0 Apply S- and I-Rules: Apply whatever S- and I-Rules you can before proceeding to the rules for quantifiers.

2.1 REVERSE SQUIGGLES: For each unstarred, not-blocked-off line that begins with “~” and then a quantifier, derive a new wff using the Reverse Squiggle Rule. Justify this by writing (to the left of the new wff) the line number of the negated quantifier that you applied the rule to. Star the original line.

2.2 DROP EXISTENTIALS⁵: For each unstarred, non-blocked-off line that begins with an existential quantifier, drop the existential quantifier⁶ using the next available new constant (unless an instance already occurs in previous not-blocked-off lines). Justify this by writing (to the left of the new wff) the line number of existential statement you dropped. Star the original line. (REPEAT: DO NOT DROP an existential if you already have a not-blocked-off instance in a previous line. There’s no reason to derive a second instance if you already have a first)

2.3

a If (you still have a universal statement where you have not dropped the universal quantifier, **AND** you have an unbroken⁷ complex wff that isn’t starred or blocked-off where one side is a quantified statement or the negation of a quantified statement), **THEN** go to Step 4 and make an assumption that breaks that statement. Return to step 2.0.

2.3b If not (that is, if you still have a universal you have not dropped but do not have any unbroken wffs where one side is a quantified or negated quantified wff), **then DROP UNIVERSALS:** For each not-blocked-off line that begins with a universal quantifier, derive instances using each old constant. Justify this by writing (to the left of the new wff) the line number of the universal statement you dropped. DO NOT STAR the original line, as you may need to use it again.

Note: Drop a universal using a new letter (a constant that does not already appear) only if you've done everything else possible (making further assumptions if needed) and still have no old letters.

(Note: The key here is to drop universals only after you have dropped all the existentials you can, including any existentials that might be derived only after making a second assumption. So, if you still have an unbroken, unstarred, not blocked-off complex wff, where one side or the other is a quantified statement (or its negation), you should break that wff before dropping any universals. It is possible that after you break this wff you will be able to derive an existential. If so, this existential must be dropped before you drop any universals. So, you need to make a second assumption before dropping a universal if doing so will allow you to derive a new existential wff.)

Keep repeating this strategy of applying rules of inference until one of the following conditions occur ("**Rules for Stopping**"):

- a. If, after doing this, you have a contradiction⁸ between any two lines, go to RAA (step 3).
- b. If you have no contradiction **and all the complex wffs are starred or blocked off**, go to REFUTE (step 4).
- c. If you have no contradiction, and have applied all the inference rules you can, **but still have a complex wff that isn't either starred or blocked off**, examine those wffs to see whether or not they are ***broken***.⁹
 - i. If all the un-starred, un-blocked off complex wffs are broken, go to REFUTE (step 5).
 - ii. If there is one or more un-starred, un-blocked off complex wff that is not broken, go to ASSUME (step 4).

3. RAA : Apply the RAA rule. That is, block off all the lines starting with the most recent live assumption down to and including the line that forms the second half of a contradiction. After that, on a new not blocked-off line, write the simpler contradictory of the assumption. Justify this line by listing the line number of the assumption, followed by a “.”, and then the lines numbers of the two halves of the contradiction, with a “&” between them (e.g., “5: 7 & 12”). If all assumptions are now blocked off, you’ve proved the argument valid. Write “Valid” under the proof, and you are done. Otherwise, erase star strings having more stars than the number of live assumptions (i.e., erase the stars you entered in the lines you just blocked off), and return to step 2.

4. ASSUME: Pick a complex wff that is not starred, not blocked-off, and not broken. Assume either side or its negation¹⁰, and return to step 2.0.¹¹

5. REFUTE: Construct a refutation box¹² containing 1) a list of all the constants that appear on not blocked-off lines, and 2) all the not blocked-off simple wffs. This describes a “possible world” containing only the individuals referred to by these constants, and where the simple wffs in this refutation box make the premises true and the conclusion false in a “world” with just these individuals. (You may want to double-check by using these truth values to insure that it makes all the premises true and the conclusion false.) Write “Refute” under the proof, because you have refuted the claim that the argument is valid.

¹ Putting a “[“ in front of a line indicates that it is “blocked off.”

² Two wffs are contradictories if they are exactly alike, except that one starts with an additional “~”. When two wffs are contradictories, the “simpler” one is the shorter one. So, if the conclusion was “P,” you would write “ASM ~P,” and if it was “~P,” you would write “ASM P”

³ A complex wff is any wff other than a capital letter or its negation. A simple wff is any capital letter or its negation.

⁴ **Note:** An assumption is live if it’s not blocked-off. You star a wff by placing one or more stars, “*”, to the left of the line number where that wff occurs. When you derive a new wff using an **S-Rule**, star (one star for each live assumption) the wff to which you applied an S-Rule, but not until (unless) you have derived both new lines permitted by that rule. When you apply an **I-Rule**, star (one star for each live assumption) the longer of these two wffs used in applying that rule.

⁵ The Drop Existential rule may be used only when there is at least one not blocked-off assumption.

⁶ When dropping a quantifier, we start with a quantified statement--“(x)Fx” for universals and “(∃x)Fx” for existentials. We then drop the initial quantifier, and replace the remaining “Fx” with “Fa”, provided that “Fx” and “Fa” are identical except that wherever the variable occurs freely in the “Fx” the same constant occurs in “Fa”. Note: An instance of a variable occurs freely if it doesn’t occur as part of a wff that begins with a quantifier using that variable. So, only the first instance of “x” in “(Fx • (x)Gx)” occurs freely. The “x” in “Gx” is “bound” by the quantifier “(x).”

⁷ The only wffs that can be considered “broken” or “unbroken” are those to which I-Rules might be applied, i.e., negated conjunctions, disjunctions, and conditionals. A complex wff is broken when it is one of these three types and where you already have either side (it doesn’t matter which) or its negation, but nothing that will allow you derive a line that doesn’t already occur.

⁸ You have a contradiction between two lines when you derive a new line that is the contradiction of a previous line.

⁹ An unbroken wff must have one of these forms: “ $\sim(A \bullet B)$,” “ $(A \vee B)$,” or “ $(A \supset B)$.”

¹⁰ If there is more than one such wff, you may begin with any one of them that you choose. You may also choose which side, or its negation, to assume. Some of these choices may result with more lines in your proof, but none of these choices is ever “mistaken.” They will all always eventually give you the exact same results.

¹¹ At this point, you might find it useful to write “{break n}” in the right column, where n is the line number of the complex wff that you are “breaking” by assuming some simpler part of it. Doing this is **not** required, but can be a helpful reminder. Sometimes an assumption you make will “break” more than one previous un-starred complex wffs. It doesn’t matter which line number you write down. If you like, you can write down both because you are actually ‘breaking’ both.

¹² This list of simple wffs that makes up a refutation box will constitute a truth value assignment to the atomic components of the argument that makes the premises true and the conclusion false. By generating this truth value assignment, we have refuted the claim that the argument is valid.