Equivalent Noise Bandwidth (for LPF)

When we use a filter, it is useful to apply the actual filter the noise, but think of the noise as being filter by a perfect rectangular filter (instead of shaping the noise). To do this, we need to define the "bandwidth" of the rectangular noise filter or generate for each filter designed and "Equivalent Noise Bandwidth".

If we know the power or magnitude spectrum of the filter, the equivalent rect function must have the same magnitude at w=0 and the same total power as the original filter. Defining what we know based on a LPF being given:

Fourier Transform $$H(f) = \int_{-\infty}^{\infty} h(t) \cdot \exp(-j \cdot 2\pi \cdot f \cdot t) \cdot dt$$

Coherent Voltage Gain of filter $$Gain_{DC} = H(0) = \int_{-\infty}^{\infty} h(t) \cdot dt$$

Total Filter Spectral Power $$Power = \int_{-\infty}^{\infty} |h(t)|^2 \cdot dt = \int_{-\infty}^{\infty} |H(f)|^2 \cdot df$$

Equivalent Power Rect $$|H(f)|^2 = \begin{cases} |H(0)|^2. & -BW_{eq} < f < BW_{eq} \\ 0, & BW_{eq} < |f| \end{cases}$$

Total Eq. Rect Power $$Power = 2 \cdot BW_{eq} \cdot |H(0)|^2$$

Equating the two powers

$$Power = 2 \cdot BW_{eq} \cdot |H(0)|^2 = \int_{-\infty}^{\infty} |h(t)|^2 \cdot dt = \int_{-\infty}^{\infty} |H(f)|^2 \cdot df$$

$$BW_{eq} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \cdot df}{2 \cdot |H(0)|^2}$$

or

$$BW_{eq} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \cdot df}{2 \cdot \left| \int_{-\infty}^{\infty} h(t) \cdot dt \right|^2}$$

Note: for a BPF use H(fmax) instead of H(0).