ECE 6640
Digital Communications

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Chapter 8


1. Reed-Solomon Codes.
2. Interleaving and Concatenated Codes.
3. Coding and Interleaving Applied to the Compact Disc Digital Audio System.
4. Turbo Codes.
5. Appendix 8A. The Sum of Log-Likelihood Ratios.
Sklar’s Communications System

Notes and figures are based on or taken from materials in the course textbook:
Bernard Sklar, Digital Communications, Fundamentals and Applications,
Reed-Solomon Codes

• Nonbinary cyclic codes with symbols consisting of m-bit sequences
  – (n, k) codes of m-bit symbols exist for all n and k where
    \[ 0 < k < n < 2^m + 2 \]
  – Convenient example
    \[ (n, k) = (2^m - 1, 2^m - 1 - 2 \cdot t) \]
  – An “extended code” could use \( n = 2^m \) and become a perfect length hexidecimal or byte-length word.

• R-S codes achieve the largest possible code minimum distance for any linear code with the same encoder input and output block lengths!

\[
d_{\text{min}} = n - k + 1
t = \left\lfloor \frac{d_{\text{min}} - 1}{2} \right\rfloor = \left\lfloor \frac{n - k}{2} \right\rfloor
\]
Comparative Advantage to Binary

- For a (7,3) binary code:
  - $2^7 = 128$ n-tuples
  - $2^3 = 8$ 3-“symbol” codewords
  - $8/128 = 1/16$ of the n-tuples are codewords

- For a (7,3) R-S with 3-bit symbols
  - $(2^7)^3 = 2,097,152$ n-tuples
  - $(2^3)^3 = 512$ 3-“symbol” codewords
  - $2^9/2^{21} = 1/2^12 = 1/4,096$ of the n-tuples are codewords

- Significantly increasing hamming distances are possible!
R-S Error Probability

• Useful for burst-error corrections
  – Numerous systems suffer from burst-errors

• Error Probability

\[ P_E \approx \frac{1}{2^m - 1} \cdot \sum_{j=l+1}^{2^m-1} \binom{2^m - 1}{j} \cdot p^j \cdot (1-p)^{2^m-1-j} \]

• The bit error probability can be upper bounded by the symbol error probability for specific modulation types. For MFSK

\[ \frac{P_B}{P_E} = \frac{2^{m-1}}{2^m - 1} \]
Burst Errors

• Result in a series of bits or symbols being corrupted.
• Causes:
  – Signal fading (cell phone Rayleigh Fading)
  – Lightening or other “impulse noise” (radar, switches, etc.)
  – Rapid Transients
  – CD/DVD damage
• See Wikipedia for references:
  http://en.wikipedia.org/wiki/Burst_error

• Note that for R-S Codes, the t correction is for symbols, not just bits … therefore, t=4 implies 3-4 n-tuples of sequential errors.
R-S and Finite Fields

• R-S codes use generator polynomials
  – Encoding may be done in a systematic form
  – Operations (addition, subtraction, multiplication and division) must be defined for the m-bit symbol systems.

• Galois Fields (GF) allow operations to be readily defined
R-S Encoding/Decoding

- Done similarly to binary cyclic codes
  - GF math performed for multiplication and addition of feedback polynomial
- $U(X) = m(X) \times g(X)$ with $p(X)$ parity computed
- Syndrome computation performed
- Errors detected and corrected, but with higher complexity (a binary error calls for flipping a bit, what about an m-bit symbol?)
  - $r(X) = U(X) + e(X)$
  - Must determine error location and error value …
Reed-Solomon Summary

• Widely used in data storage and communications protocols
• You may need to know more in the future
  (systems you work with may use it)
Interleaving

• Convolutional codes are suitable for memoryless channels with random error events.

• Some errors have bursty nature:
  – Statistical dependence among successive error events (time-correlation) due to the channel memory.
    • Like errors in multipath fading channels in wireless communications, errors due to the switching noise, …

• “Interleaving” makes the channel looks like as a memoryless channel at the decoder.
Interleaving …

- Interleaving is done by spreading the coded symbols in time (interleaving) before transmission.
- The reverse is done at the receiver by deinterleaving the received sequence.
- “Interleaving” makes bursty errors look like random. Hence, Conv. codes can be used.
- Types of interleaving:
  - Block interleaving
  - Convolutional or cross interleaving
Interleaving …

- Consider a code with \( t = 1 \) and 3 coded bits.
- A burst error of length 3 cannot be corrected.

\[
\begin{array}{ccccccc}
A_1 & A_2 & A_3 & B_1 & B_2 & B_3 & C_1 & C_2 & C_3 \\
\end{array}
\]

2 errors

- Let us use a block interleaver 3x3

\[
\begin{array}{cccccccc}
A_1 & A_2 & A_3 & B_1 & B_2 & B_3 & C_1 & C_2 & C_3 \\
\end{array}
\]

Interleaver

\[
\begin{array}{cccccccc}
A_1 & B_1 & C_1 & A_2 & B_2 & C_2 & A_3 & B_3 & C_3 \\
\end{array}
\]

Deinterleaver

1 errors 1 errors 1 errors
Convolutional Interleaving

- A simple banked switching and delay structure can be used as proposed by Ramsey and Forney.
  - Interleave after encoding and prior to transmission
  - Deinterleave after reception but prior to decoding

![Diagram of Convolutional Interleaving](image-url)
Forney Reference


Fig. 1. (a) Uncorrectable erasure burst pattern. (b) Two error burst patterns that are not simultaneously correctible.

Fig. 2. Periodic interleaver and corresponding deinterleaver $(B^* = B/N)$.

Fig. 3. Appearance of bursts of $B^*$ blocks separated by guard spaces of $(N - 1)B^*$ blocks in $N$ deinterleaver output streams.

Fig. 4. Appearance of bursts of $kB^*$ blocks separated by guard spaces of $(N - k)B^*$ blocks in $N$ deinterleaver output streams.
Convolutional Example

- Data fills the commutator registers
- Output sequence (in repeating blocks of 16)
  - 1 14 11 8
  - 5 2 15 12
  - 9 6 3 16
  - 13 10 7 4
  - 1 14 11 8
  - 5 2 15 12
  - 9 6 3 16
  - 13 10 7 4

Figure 8.13 Convolutional interleaver/deinterleaver example.
Concatenated codes

- A concatenated code uses two levels on coding, an inner code and an outer code (higher rate).
  - Popular concatenated codes: Convolutional codes with Viterbi decoding as the inner code and Reed-Solomon codes as the outer code
- The purpose is to reduce the overall complexity, yet achieving the required error performance.
Practical example: Compact Disc

“Without error correcting codes, digital audio would not be technically feasible.”

- Channel in a CD playback system consists of a transmitting laser, a recorded disc and a photo-detector.
- Sources of errors are manufacturing damages, fingerprints or scratches.
- Errors have bursty like nature.
- Error correction and concealment is done by using a concatenated error control scheme, called cross-interleaver Reed-Solomon code (CIRC).
CD CIRC Specifications

- Maximum correctable burst length: 4000 bits (2.5 mm track length)
- Maximum interpolatable burst length: 12,000 bit (8 mm)
- Sample interpolation rate: One sample every 10 hours at $P_B=10^{-4}$
  1000 samples/min at $P_B=10^{-3}$
- Undetected error samples (clicks): Less than one every 750 hours at $P_B=10^{-3}$
  Negligible at $P_B=10^{-3}$
- New discs are characterized by $P_B=10^{-4}$
Compact disc – cont’d

- CIRC encoder and decoder:

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Encoder

Δ interleave → C₂ encode → D* interleave → C₁ encode → D interleave

Δ deinterleave ← C₂ decode ← D* deinterleave ← C₁ decode ← D deinterleave

Decoder
```
CD Encoder Process

16-bit Left Audio
16-bit Right Audio
(24 byte frame)
RS code 8-bit symbols

RS(255, 251)
24 Used Symbols
227 Unused Symbols
Equ. RS(28, 24)

RS(255, 251)
28 Used Symbols
223 Unused Symbols
Equ. RS(32, 28)

Overall Rate 3/4
CD Decoder Process

Figure 8.17  Compact disc decoder.
Advanced Topic: Turbo Codes

- Concatenated coding scheme for achieving large coding gains
  - Combine two or more relatively simple building blocks or component codes. Often combined with interleaving.
  - For example: A Reed-Solomon outer code with a convolutional inner code
- May use soft decisions in first decoder to pass to next decoder. Multiple iterations of decoding may be used to improve decisions!
- A popular topic for research, publications, and applications.
I have been trying to run a simulation ....
- Reed Solomon Examples
- Turbo Code Examples
Turbo Code Performance

- The decoding operation can be performed multiple times or iterations.
- There is a degree of improvement as shown.

Figure 6.28  Bit-error probability as a function of $E_b/N_0$ and multiple iterations.

MATLAB Simulations

LTE Turbo-Coding

$E_b/N_0$ (dB) vs. BER

- $N = 2048$, 1 iterations
- $N = 2048$, 2 iterations
MATLAB Simulations

LTE Turbo-Coding

N = 2048, 3 iterations

N = 2048, 4 iterations
References

- [http://home.netcom.com/~chip.f/viterbi/tutorial.html](http://home.netcom.com/~chip.f/viterbi/tutorial.html)
- [http://www.eccpage.com/](http://www.eccpage.com/)
- [http://www.csee.wvu.edu/~mvalenti/turbo.html](http://www.csee.wvu.edu/~mvalenti/turbo.html)
- [http://www.eg.bucknell.edu/~kozick/elec47601/notes.html](http://www.eg.bucknell.edu/~kozick/elec47601/notes.html)
- Digital Communications I: Modulation and Coding Course, Period 3 – 2006, Sorour Falahati, Lecture 13