Abstract # TP-D3

Problem Solving Transfer Between Mathematics and Physics

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The Frustration

"As a HS physics and chemistry teacher, I have students that are taking AP Calculus and can not perform simple algebraic equations in Physics. In chemistry they can not do math with exponents (mole problems) or simple algebraic equations. Somewhere along the line these students have received grades such that they can take the higher level math courses but can not really do any math."

Holly Priestley, PhysLRNR Post 6/23/04
Penn State University (Great Valley Campus),
Burlington Township HS (Quoted with permission)

Transfer is Hard

- Transfer is hard to define (Leon Hsu TP-D1)
- It's not just Math to Physics transfer that is difficult
  - failure of transfer has been documented in a variety of fields (e.g., Sternberg & Frensch, 1993).
  - Physics to other disciplines is hard to document (Eric Brewe TP-D2)
- In order for transfer to occur in a particular problem context:
  - a) The solver must have a reasonable understanding of the transferable material (English, 1997; Bassok & Holyoak, 1993).
  - b) The solver must realize that the transferable material is relevant to the current problem.
  - c) The solver must successfully apply the transferable material to the current problem context.

Poster Overview

1) Do student have a reasonable understanding of mathematics?

2) Do students recognize mathematics is transferable to physics?

3) Can students successfully apply mathematics to the current problem?

4) A counter example!

Understanding Mathematics

The tremendous stability of today's [mathematics] curriculum depends on a guidance system controlled by two unwavering and outdated public assumptions:

- Mathematics is a fixed and unchanging body of facts and procedures
- To do mathematics is to calculate answers to set problems using a specific catalogue of rehearsed techniques.

(National Research Council, 1990)

"When students meet difficulties, a dominant strategy for coping is to concentrate on the procedural aspects that are usually set in examinations. Because the teacher knows that conceptual questions are rarely answered correctly, the vicious circle of procedural questions is set in motion. In deed, for those students who take an initial calculus course based on elementary procedures, there is evidence that this may have an unforeseen limiting effect on their attitudes when they take a more rigorous course at a later stage." (Tall, 1992, p. 4)

Understanding Calculus

Students have poor understanding of:

- Functions and variables
  - Many students have difficulty connecting algebraic and graphical representations of functions.
- Derivative
  - Most students could perform routine calculations, but had poor understanding of the concept of derivative.
- Integral
  - Most students could perform routine calculations, but had poor understanding of the concept of integral.

(Sabella & Redish, no date)

This is partially because most calculus courses rely on procedural aspects and do not focus on understanding (Tall, 1992).

Understanding Algebra

SIUE introductory astronomy course (no math pre-requisite), 53 students took this pretest:

You would like to mail a package that weighs 64 Newtons {the Newton is the unit of weight in the metric system} and you need the package to arrive in 4 days. How much on postage do you need to spend? The postmaster tells you that the delivery time increases by the weight of the package, but the more postage you pay, the delivery time is quicker by the amount of postage squared. As an equation this relationship is expressed as:

$$\text{Delivery time [in days]} = \frac{\text{Weight [in Newtons]}}{(\text{Postage [in dollars]})^2}$$

13% of students left the question blank

24% of students could not produce a solution

36% of students made a mistake(s)

26% of students wrote a valid solution

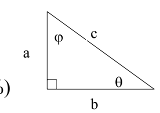
Students struggle, but they are not devoid of knowledge and skills with algebra

Understanding Trigonometry

Placement test given at beginning of introductory calc-based physics course at WMU (Fall 2003, N=130)

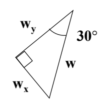
#2 The side "b" in the triangle is given by:

A. b= c tanθ (2%)
B. b = c sinφ (*55%)
C. b= c cosφ (18%)
D. b= c tanφ (2%)
E. I don't know (23%)



#3 What is the value of w_x if w = 60 N

A. 52 N (6%)
B. 34.6 N (5%)
C. 30 N (*45%)
D. 45 N (3%)
E. I don't know (38%)



Many students have a poor understanding of simple trigonometry

Realizing that the Transferable Understanding is Applicable

- Lack of transfer could be due to students' poor performance monitoring skills (Schoenfeld, 1994).
  - If students don't know where they are in a problem and where they want to go then they are not likely to select appropriate mathematics even if they fully understand the mathematical concept and how to apply it.
- Differences in surface features impede transfer. When transfer does exist it is likely to be based on similarity of superficial features. (Proctor & Dutta, 1995).

Context is Important

(from Redish, 2003; modified from Wason, 1966)

"This set of four cards satisfies the property that if there is a vowel on one side of the card, then there is an odd number on the other. How many cards do you need to turn over to be absolutely certain that the cards have been correctly chosen to satisfy this property?" [K 2 A 7] (Redish, 2003, p. 22)

"You are serving as the chaperone and bouncer at a local student bar and coffee house. Rather than standing at the door checking IDs all the time, you have occupied a table so you can do some work. When patrons come in and give their order, the servers bring you cards with the patron's order on one side and their best guess of the patron's age on the other. You then decide whether to go and check IDs. (The servers can be assumed to be trustworthy and are pretty good observers.)" [16 coke 52 Gin&Tonic] (Redish, 2003, p. 22-23)

Redish found that about 2/3 of *physicists* produce the wrong answer to the K2A7 problem after a minute or two of consideration, but that almost all were able to solve the ID problem.

Transfer between example problem and target problem

Example problem: A small pipe can fill an oil tank in 12 hours and a large one can fill it in 8 hours. How long will it take to fill the tank if both pipes are used at the same time? (from Reed, 1993)

		Solution Procedure	
		Same	**Different**
Context	**Same**	Equivalent (Reasonably Good Transfer) [A small hose can fill a swimming pool in 6 hours and a large one can fill it in 3 hours. How long will it take to fill the pool if both hoses are used at the same time?]	Similar (Poor transfer – students unable to adjust to changes between the example and test problem) [A small pipe can fill a water tank in 20 hours and a large pipe can fill it in 15 hours. Water is used at a rate that would empty a full tank in 40 hours. How long will it take to fill the tank when both pipes are used at the same time, assuming that water is being used as the tank is filled?]
	Different	Isomorphic (Poor transfer – students focus on surface features and are unable to recognize that the example problem is relevant. – unless students have encoded the relevant features of the example problem or situation – see Bassok example below) [Tom can drive to Bill's house in 4 hours and Bill can drive to Tom's house in 3 hours. How long will it take them to meet if they both leave their houses at the same time and drive toward each other?]	Unrelated [An airplane can fly from city A to city B at an average speed of 250 mph in 3 hours less time than it takes to return from city B to city A at 200 mph. How many hours did it take to return?]

This has implications for problem format (David MaloneyTP-D4 and Kathleen Harper TP-D5)

Misapplying the Math

- Even if student know they need to transfer their competent math skills, they still can make what appears to be math errors
- Cognitive load could cause "simple" mistakes.
  - Students can do simple math problems (e.g., algebra, trig, calculus), but when they also have to think about the larger problem they may make mistakes because they do not have all of their resources available to focus on the math aspects.
  - This would suggest that mathematics procedures are not fully routinized or chunked (Dreyfus & Dreyfus). They still require significant cognitive effort from students.
  - In effect, it can be hard to learn new physics and reinforce math at the same time
- These simple mistakes are different than mistakes made due to poor conceptual understanding of mathematics (e.g., not understanding the concept of integration well enough to apply it in finding the electric field of a distributed charge.)

Mistakes Happen

In WMU study, 55% of students could answer the simple trig question (#2) and 45% could use similar trig reasoning in a physics context. This suggests that, for those students, their trig understanding was routinized enough to be used in a more complicated physics context.

In a laboratory study of college students studying physics (mechanics), subjects were asked to solve 19 problems. The good solvers (by definition) made fewer errors than the poor solvers (3.25 vs. 14.25). Both groups made relatively few mathematical errors (0.25 and 0.75 respectively, not statistically significant). In this study mathematical errors were defined as confusing sine and cosine, or dropping a negative sign, rather than mathematical difficulties on a conceptual level. (VanLehn & Jones, 1993).

In SIUE study, math errors were idiosyncratic. Rarely did same person make a similar class of mistake on subsequent problems.

An interesting counterexample From Bassok & Holyoak, 1993

Studied 11th grade students in a natural setting. All were enrolled in both an algebra and physics class. One group was taken from an algebra class that had studied the topic of arithmetic progression but had not yet studied the physics topic of constant acceleration. The other group was in the reverse situation.

Results: Algebra students who had learned the arithmetic progression method applied it 72% of the time to *both* new algebra problems *and* physics problems. Physics students who had learned the constant acceleration method applied it to *all* of the physics problems and *none* of the algebra problems. (They replicated these results in a lab study of high ability 9th grade students.)

Conclusions: They conclude that transfer can occur. The reason that transfer only occurred in one direction was due to the way that the initial problems were encoded in memory by the subjects. Students know that the details in math problems are irrelevant and thus do not encode them. In physics, however, certain details are thought to be relevant and are encoded. Physics embeds the formal mathematical structures in domain-specific concepts (initial speed, final speed, acceleration, distance, time) while algebra training is domain general (initial term, final term, constant difference, sum , number of terms). (Bassok, 1997)

Questions: If transfer occurs readily between math and physics why doesn't it seem that way? One aspect of the study is that the problems were isomorphic. A typical physics problem does not have the same solution procedures as a math problem, but in a different context. A typical physics problem requires students to use their understanding of math concepts to set up and then solve a physics problem.

Limitations

Much of this research has been done in short-term lab studies. Subjects are initially trained in a problem solving task and later (that day or a different day) subjects are asked to solve one or more transfer problems. The ability of the subjects to solve the transfer problems is usually compared to the ability of a control group (no training) to solve the same problems. This setting is obviously quite different from a school setting and it is not clear how well the findings would hold up in a school setting.

Since both mathematics and physics have different cultures, it is challenging to develop research teams which can bridge this divide authentically. For example all physicists seem to have an opinion about how math should be taught but few have actually explored contemporary math reform.

Conclusions

Some of the transfer problems between math and physics occur because many students have only developed an algorithmic understanding of the mathematics. They can solve standard math problems in standard math settings, but often do not have a "flexible" enough understanding to use these mathematics ideas in a different setting.

An interesting example is that most student know how to use their calculators in nearly any setting. This expertise could be an example of distributed cognition. There might be chunking of calculator skill in place of some math skills chunks. Furthermore, there might be differences between students who could use their calculators in other classes and those who were not permitted access.

Culturally it is permitted to be math illiterate, not language illiterate. This double-standard is problematic for math reform (NRC, 1990) and might help explain why reading, in most contexts, appears to transfer but math does not.

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