

Teaching about circuits at the introductory level: An emphasis on potential difference

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Introductory physics students often fail to develop a coherent conceptual model of electric circuits. In part, this failure occurs because the students did not develop a good understanding of the concept of electric potential. We describe an instructional approach that emphasizes the electric potential and the electric potential difference. Examples are given to illustrate this approach and how it differs from traditional treatments of these concepts. Assessment data is presented to suggest that this approach is successful in improving student understanding of electric potential and electric circuits. © 2006 American Association of Physics Teachers.
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I. INTRODUCTION

Several studies have found that students frequently fail to develop a coherent conceptual model of electric circuits in a typical introductory physics course.¹⁻³ In particular, many students do not develop a good understanding of the concept of electric potential or electric potential difference and are unable to distinguish between current and potential difference.¹⁻⁴ Some successful instructional approaches begin by developing a robust understanding of the concept of current before introducing the concept of potential difference.⁵ However, most textbooks (for example, Ref. 6) introduce the concept of potential difference first. In this article we describe an instructional approach that begins by helping students develop an understanding of the concept of potential difference before introducing the concept of current. We have used this approach successfully in the second semester of our introductory calculus-based physics sequence.

The standard beginning course introduces electric potential in the electrostatics unit. Attention is typically paid to computing the potential function for various discrete and continuous charge distributions and to using the relationship $U=QV$ to solve problems involving conservation of energy. The student often loses sight of the defining relation,

$$V(B) - V(A) = \frac{\text{work needed to move a charge } Q \text{ from } A \text{ to } B}{Q}. \quad (1)$$

The right-hand side of Eq. (1) is the definition of electromotive force (EMF); Eq. (1) expresses the fact that the EMF is identical to the potential difference for electrostatic fields.

We contend that stressing this basic definition is useful for teaching circuit theory and that the connection between Eq. (1) and circuit theory is most readily approached by a study of circuits involving only batteries and capacitors. Such purely electrostatic circuits are covered in all textbooks, but usually in a way that does not contribute to an understanding of potential differences. The reason is that typical problems involve reducing a circuit to its equivalent capacitance, which means that students do not have to think about the potential difference relations of the capacitors in the circuit. The result of this loss of emphasis is that students are usually exposed to these relationships as one of the Kirchoff circuit analysis rules (that is, the loop rule) at the same time they are

studying currents in dc circuits. They typically learn to attack problems using these rules as templates (see the discussion in Ref. 7) and, as a result, often fail to understand the basic relationships between the potential differences across different circuit elements.¹

II. POTENTIAL DIFFERENCE IN CAPACITOR CIRCUITS

As an example of the sort of instruction we have found useful, consider the problem of Fig. 1. Students do not readily understand that the voltmeter is simply reading the potential difference across capacitors C and D . Because the negative lead is close to the negative plate of capacitor A , students believe that capacitor A should somehow be involved in solving the problem. We use Eq. (1) by insisting that students think about moving a charge from the position of the voltmeter's negative lead to the position of the positive lead via any path through the circuit. The voltmeter reads the work required by this movement (per unit charge transported). Students eventually see that the path through D and C is one of several that are described by the voltmeter's reading. As usual, only telling students has limited effect; they must struggle with this sort of problem on their own or in small groups. In our courses much of this work is done during class time or assigned for homework; some examples of questions useful for this purpose are shown in the Appendix. In addition, the laboratory portion of the course has also been redesigned to allow students to focus on these issues.

The standard approach to the problem of Fig. 1 once the irrelevance of capacitor A is understood is to reduce the three capacitors to a single equivalent capacitance and solve for the common charge on each capacitor. It is useful to begin the topic of capacitors by allowing students to use only the definition of capacitance $\Delta V=Q/C$ and Eq. (1). Our goal is to help students develop a conceptual model of these circuits by doing the following:

- (1) Think of the battery as pulling positive charge from the rightmost plates of A and D (leaving them with a net negative charge) and placing that charge on the left plates of A and B .⁸
- (2) Use a field-line argument (or Gauss's law or consider how neutral conductors become charge polarized) to argue that the positive charge on the left-hand side of A is equal in magnitude to that on the right-hand side of A .

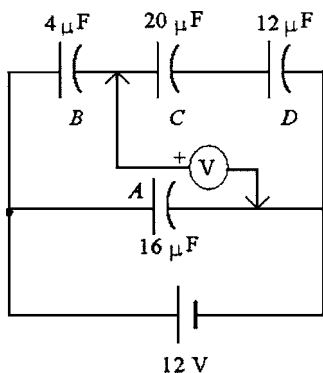


Fig. 1. Four capacitors *A*, *B*, *C*, and *D* have the capacitances shown. What does the voltmeter read?

Therefore all the positive charge that came from the right-hand side of *D* went to the left-hand side of *B*.

- (3) Use similar arguments to understand that *B*, *C*, and *D* all have the same charge Q .
- (4) Think about the work required to move a charge Q from the negative plate of *D* to the positive plate of *B* (which is what the battery did). Moving this charge from the negative plate of *D* to the positive plate of *D* requires work $Q\Delta V_D$. It requires no work to move it from the positive plate of *D* to the negative plate of *C* because this section of the circuit is an equipotential. It requires an additional $Q\Delta V_C$ to move it to the positive plate of *C*. This reasoning reinforces the idea that the potential difference across the three elements is $12V = \Delta V_B + \Delta V_C + \Delta V_D$.
- (5) We conclude that

$$12V = \frac{Q}{4\mu F} + \frac{Q}{20\mu F} + \frac{Q}{12\mu F},$$

which can be solved for Q and the sum of the potential differences across *C* and *D* is readily found.⁹

More sophisticated circuits can be handled the same way.

We do not claim that this method is the fastest way to solve purely capacitive circuits: it is usually not. But until students can carry out this kind of analysis, we claim it is counterproductive to teach them about equivalent capacitance. The point of this approach is to ensure that the concept we are teaching, the potential difference in circuits, is front and center in the solution.

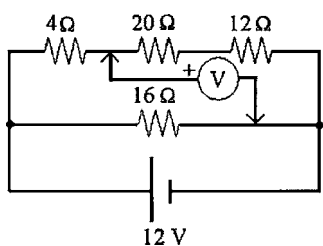


Fig. 2. Four resistors have the resistances shown. What does the voltmeter read?

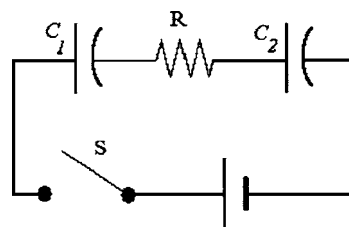
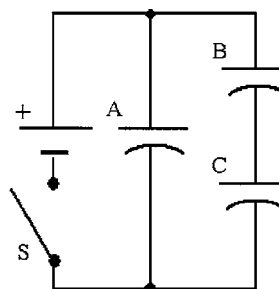


Fig. 3. At what rate is the battery doing work on the resistor at the instant when capacitor *C*₁ is charged to half of its final charge?

III. POTENTIAL DIFFERENCE RELATIONSHIPS REMAIN THE SAME

A similar solution strategy can be used for dc circuits. Replacing all the capacitors in a circuit with resistors changes nothing about the potential difference relationships. We generally reinforce this idea by having students consider dc circuits with the same geometry as the capacitive circuits they have previously considered. This strategy helps to avoid the common outcome where students think of capacitor and resistor circuits as completely different. For example, we present the problem of Fig. 2, which has the same geometrical structure as the capacitor circuit in Fig. 1. Our solution method would be identical to the one discussed earlier except that we now characterize the devices by $\Delta V = IR$ instead of $\Delta V = Q/C$. For each of these passive devices it is important to know how the potential difference across it relates to some other aspect of the circuit and then to use that knowledge to determine the circuit behavior. The constancy of current in the upper branch of Fig. 2 replaces the equality of charges in the upper branch of Fig. 1 as the condition that makes the solution possible. This approach puts the relation $\Delta V = IR$ in its proper place as an expression for the potential difference across a particular type of device, no different in principle than $\Delta V = Q/C$ for capacitors. This helps to avoid the common student perception that Ohm's law is one of the most important principles of electromagnetism.⁴

To reinforce student use of the basic potential difference relations, students are not allowed to use the shortcuts of



- A. When the switch is closed in the circuit, which capacitor do you predict will acquire the largest ΔV ? Explain.
- B. What do you predict will be the relationship between the potential differences across *A*, across *B*, and across *C*?
- C. Construct the circuit and check your prediction. Explain any differences with your predictions.
- D. What total charge passed through the battery?

Fig. 4. A typical activity in the work on capacitors.

Table I. CSEM comparative data for Western Michigan University students taught with the new emphasis versus nationally normed data for calculus-based students (Ref. 11).

| | CSEM items 17–20 | | Complete CSEM | |
|---|------------------|--------|---------------|-------------|
| | Pre % | Post % | Pre % | Post % |
| Averages over three semesters ($N=173$) | 30.2 | 57.1 | 31 ± 0.8 | 58 ± 1.4 |
| National average for calculus-based university physics ($N=1213$) | 26.0 | 43.5 | 31 ± 0.3 | 47 ± 0.5 |

equivalent resistance or capacitance on examinations and homework until we are satisfied that they understand these fundamental relationships.

IV. CIRCUIT TRANSIENTS AND FREE ELECTRIC OSCILLATIONS

Although we do not develop the impedance concept in our introductory course, we analyze RC, RL, and LC circuits in the conventional way. A facility with potential difference relations makes the derivation of the relevant circuit equations almost obvious. We reinforce concepts by analyzing the meaning of the terms in equations such as $\Delta V_{\text{bat}} = IR + Q/C$ through nonstandard questions such as the one illustrated in Fig. 3. Here students are surprised to find that the exact form of the time dependence is irrelevant and only the well-studied relations $\Delta V = Q/C$ and $\Delta V = IR$ are needed to solve the problem.

V. ADDITIONAL CONSIDERATIONS

A. Importance of laboratory work

Our lectures are reinforced by laboratory work that approaches circuits the way we have described. These laboratories are based on an “elicit-confront-resolve” approach that asks students to first predict the behavior of a circuit before making measurements. They then make the measurements and compare with predictions. Figure 4 shows an example of one part of a laboratory activity. Students attest that these laboratories are an important element of their learning. For example, when asked on an end-of-semester course evaluation to respond to the question “The labs were designed to help you understand material in lecture. Were they generally successful in doing this?,” 86% responded affirmatively. Many of the written student comments also suggested that they found the laboratory activities to be an important part of the course.

Table II. DIRECT comparative data for Western Michigan University students taught with the new emphasis versus the nationally normed data for university (calculus and algebra-based) students. (Ref. 1).

| | DIRECT | |
|---|-------------------------------|---------------------|
| | items 6, 7, 15, 16, 28 (%) | Complete DIRECT (%) |
| One semester ($N=91$) | 65 | 60.3 ± 1.48 |
| National average for university physics ($N=681$) | 51 | 52 ± 0.56 |

B. Teach the voltmeter reading, not the loop rule

An important aspect of our teaching of potential difference relations in circuits is an emphasis on voltmeter readings. For example, in Fig. 1 the voltmeter measures the potential difference across capacitors C and D but also across the battery and capacitor B . It is usual to point out these potential difference relationships using the loop rule, but we have found enhanced student understanding if we stress voltmeter readings every time we want to point out a potential difference relationship in a circuit. This enhancement is likely due to students memorizing the loop rule without coming to grips with its implications.

C. Use ΔV instead of V

Others have suggested that the use of V instead of ΔV is confusing to students at this stage (for example, Ref. 10) and we use ΔV exclusively. Ohm’s law is written as $\Delta V = IR$. Voltmeters always read the potential difference between their leads (or better, the potential of the positive lead relative to the potential of the negative lead). This notational explicitness is especially important in our approach because we continually inquire about potential differences between two points in a circuit.

VI. STUDENT OUTCOMES

Some evidence suggests that this instructional focus helps students develop an improved understanding of electric potential and dc circuits. As part of our assessment, we used two multiple-choice tests with our introductory electricity and magnetism course: the conceptual survey in electricity and magnetism¹¹ (CSEM), and determining and interpreting resistive electric circuits concept test¹ (DIRECT). The data reported here comes from three semesters of our introductory calculus-based physics course. The course has a large lecture format with about 70 students per section and a weekly 2 h laboratory with 18–24 students per section.

The CSEM is a 32 item multiple-choice test that assesses a broad range of topics typically covered in introductory electricity and magnetism. Four items (17–20) deal directly with student understanding of electric potential. Students taught using our approach performed significantly better on these items (and on the entire test) than students in courses where electric potential is taught in the traditional fashion (see Table I). DIRECT is a 29-item multiple-choice test that assesses student understanding of dc circuits. Questions 6, 7, 15, 16, and 28 directly probe student understanding of potential difference. Students taught using our approach per-

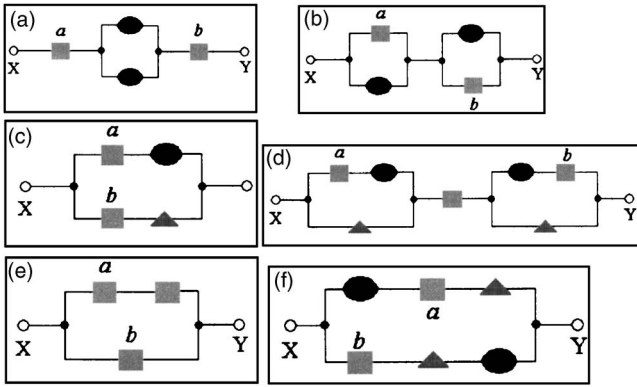


Fig. 5. Answer choices for Appendix Problem (1).

formed significantly better on these items (and better on the entire test) than students in traditionally taught courses (see Table II).

VII. SUMMARY

We have presented an instructional approach with an increased emphasis on the definition of potential difference. This emphasis helps students develop a better understanding of potential difference and dc circuits than in traditional treatments. The evidence is not conclusive because it did not result from a controlled study; however we present our approach for consideration as an alternative to the traditional treatment of circuits.

ACKNOWLEDGMENTS

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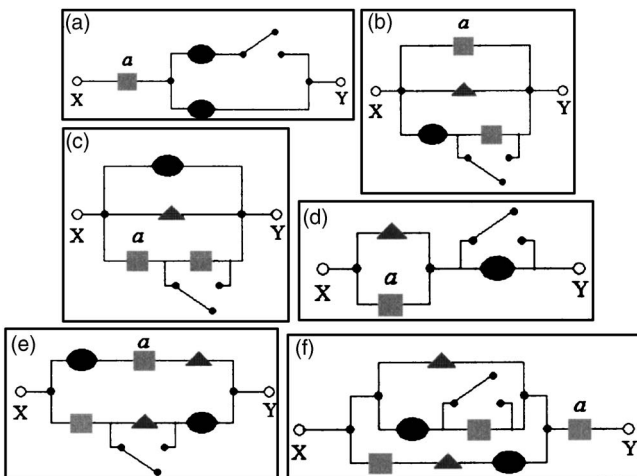


Fig. 6. Answer choices for Appendix Problems (2) and (3).

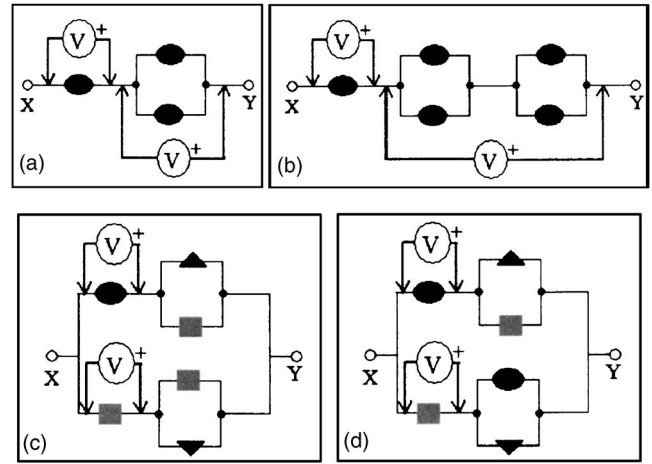


Fig. 7. Answer choices for Appendix Problem (4).

APPENDIX: EXAMPLE CONCEPTUAL PROBLEMS ON ELECTRIC POTENTIAL IN SIMPLE CIRCUITS

In the following problems each geometrical figure (square, triangle, or oval) represents an identical circuit element: capacitor, resistor, or inductor. For example, each square could represent a capacitor of capacitance $2\mu\text{F}$, each oval a resistor of resistance 3Ω , and each triangle a resistor of resistance 16Ω . The open ends marked X and Y are connected to a larger circuit, so that a nonzero potential difference $V(Y) - V(X)$ is established. Capacitors are initially uncharged before being placed in the circuit.

- (1) In which of the branches of the circuit shown in Fig. 5 will element *a* have the same potential difference across it as element *b*? Check all that apply.
- (2) In Fig. 6 all the geometrical figures are resistors (squares have one resistance, ovals another, etc.). In all of the circuit branches, the open ends X and Y are directly connected to the terminals of a 6 V battery, so that $V(Y) - V(X) = 6\text{V}$. Each branch contains a switch that forms an electrical short when closed. In which branch will the magnitude of the potential difference across element *a* increase when the switch is closed? Check all that apply.
- (3) All geometrical figures in Fig. 6 are capacitors (squares have one capacitance, ovals another, etc.) For each of the branches, the following experiment is performed:
 - (a) The open ends X and Y are not connected to anything.
 - (b) Initially uncharged capacitors are placed in the branch as shown, with the switch open.
 - (c) The ends X and Y are connected to the terminals of a battery so that $V(Y) - V(X) = 3\text{V}$.
 - (d) A measurement is made of the potential difference across capacitor *a*.
 - (e) The open ends are disconnected from the battery, all capacitors are discharged, and the switch is closed.
 - (f) The ends X and Y are reconnected to the battery as before, and a second measurement is made of the potential difference across capacitor *a*. In which of the branches in Fig. 6 will the magnitude of the second voltage reading be greater than the magnitude of the first? Check all that apply.

- (4) All of the geometrical figures in Fig. 7 are resistors and $V(Y) - V(X) = 3 \text{ V}$. In which of these figures will the two voltmeter readings be the same? Check all that apply.

¹P. V. Engelhardt and R. J. Beichner, "Students' understanding of direct current resistive electrical circuits," *Am. J. Phys.* **72**, 98–115 (2004).

²L. C. McDermott and P. S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity. Part I: Investigation of student understanding," *Am. J. Phys.* **60**, 994–1003 (1992).

³A. B. Arons, *A Guide to Introductory Physics Teaching* (Wiley, New York, 1990).

⁴E. Bagno and B. S. Eylon, "From problem solving to a knowledge structure: An example from the domain of electromagnetism," *Am. J. Phys.* **65**, 726–736 (1997).

⁵P. S. Shaffer and L. C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity. Part II: Design of instructional strategies," *Am. J. Phys.* **60**, 1003–1013 (1992).

⁶R. Serway and J. Jewett, *Physics for Scientists and Engineers*, 6th ed. (Brooks/Cole, Belmont, CA, 2004).

⁷E. Mazur, *Peer Instruction: A User's Manual* (Prentice Hall, Upper Saddle River, NJ, 1997).

⁸We tell students that it is really the negative electrons that move in a circuit but, in preparation for the study of current, it is useful to think of positive charge as moving.

⁹An interesting discussion can be generated at this point by asking students to calculate the work done by the battery in this circuit, $W = Q\Delta V$. The work done by the battery is exactly one-half of the energy stored in the capacitors. We tell students that this apparent violation of energy conservation will be explained later by considering the energy dissipated in a resistor in an RC circuit. This explanation satisfies all but the most insightful students.

¹⁰R. Knight, *Five Easy Lessons: Strategies for Successful Physics Teaching* (Addison-Wesley, San Francisco, 2002).

¹¹D. Maloney, T. O'Kuma, C. J. Hieggelke, and A. Van Heuvelen, "Surveying students' conceptual knowledge of electricity and magnetism," *Am. J. Phys.* **69**, S12–S23 (2001).



James Clerk Maxwell's silhouette in the science museum at the City of Arts and Sciences (Ciudad de las Artes y las Ciencias) in Valencia, Spain. (photo by Jay M. Pasachoff, Williams College-Hopkins Observatory, printed with permission from the Ciudad de las Artes y las Ciencias.)