

**Short-Sales Constraints and Stock Return Asymmetry:  
Evidence from the Chinese Stock Markets**

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**Abstract**

The Chinese stock markets provide a great example for testing the *Differences-of-Opinion* theory proposed by Hong and Stein, because the difficulty of short-selling stocks in these markets fits the key assumption of the theory. Using daily data of the Shanghai and Shenzhen composite indexes from June 1995 to August 2001, we find evidence that supports the Hong-Stein theory: higher trading volume today, which proxies for the differences of opinion, predicts a more negatively skewed distribution for the stock returns on the next trading day.

JEL Classification: C51, G12.

Keywords: differences of opinion; asymmetric generalized- $t$  distribution; autoregressive conditional density models.

## **Introduction**

Based on investor heterogeneity, Hong and Stein (2003) propose the *Differences-of-Opinion* theory in an attempt to explain the mechanism that generates the well-documented asymmetric distribution of stock returns. Their theory states that when differences of opinion are high, those bearish investors who are subject to a short-sales constraint will sit out of the market and their information is not revealed. This accumulated hidden information tends to come out during market declines. Therefore, the Hong-Stein model predicts that negative skewness in returns will be most pronounced after periods of heavy trading, because trading volume proxies for the differences of opinion.

Motivated by the Hong-Stein model, Chen, Hong, and Stein (2001) develop a series of cross-sectional regressions in an attempt to forecast asymmetry in the daily returns for U.S. individual stocks. Their empirical results support the Hong-Stein model in that negative skewness is most pronounced in stocks that have experienced an increase in trading volume during the prior six months. Hueng and McDonald (2005), however, find that the prediction of the Hong-Stein model is not supported in the time-series analysis with the U.S. aggregate market data. Their empirical results show that higher prior trading volume does not have a statistically significant predicting power on market skewness.

One possible explanation for Hueng and McDonald's finding is that, given the liquid derivative markets in the United States, the short-sales constraints may not be binding at the aggregate market level. The literature on options trading argues that introducing futures and options trading can potentially reduce or even eliminate the informational effect of short-sales constraints. For example, Figlewski and Webb (1993) present empirical evidence that trading in options allows investors who face short-sales constraints to take equivalent option positions.

The purpose of this paper is to test the robustness of the Hong-Stein model under a time-series framework using a different sample that is most likely to abide by the model's key assumption, that is, a market where the short-sales constraint is most likely to be binding. The

Chinese stock markets provide such an example. China's two exchanges (Shanghai and Shenzhen) were launched in 1990. Due to the lack of an institutional investor base dealing with stock lending mechanisms, it is very difficult to short-sell the Chinese stocks. They are not only expensive to borrow, but also in short supply and subject to frequent recalls. In addition, even though the Chinese financial community has discussed index derivatives since 2001, such instruments have never taken off. An eruption of speculation and corruption in the Chinese government bond future market in the mid-1990 has kept the Chinese government wary of legalizing derivatives. Not until recently have steps been taken toward the launch of stock index futures.

Using daily data of the Shanghai and Shenzhen composite indexes from June 1995 to August 2001, we find that higher trading volume today predicts a more negatively skewed distribution for tomorrow's stock return. Thus, the Hong-Stein model is supported in the Chinese stock markets.

The remainder of the paper is organized as follows. Section I introduces the time-series model we use to test the Hong-Stein theory. We present and discuss the empirical results in Section II. Section III concludes the paper.

## **I. Model**

To test the effects of prior trading volume on the skewness of stock returns, Chen, Hong, and Stein (2001) conduct a cross-sectional regression where the sample is divided into several non-overlapping six-month periods. For each six-month period they calculate the sample skewness of the returns and the average of the trading volumes for each individual stock. The positive and significant estimated coefficients from the OLS regression of negative skewness on past trading volumes provide evidence that the Hong-Stein model is supported in their cross-sectional analysis.

Chen, Hong, and Stein’s cross-sectional setup, however, is not feasible for testing the Hong-Stein model in the aggregate market because there would not be enough non-overlapping six-month periods in the sample. Hueng and McDonald (2005) propose a time-series test for the Hong-Stein model by using an “autoregressive conditional density (ARCD) model” suggested by Hansen (1994). It is a parametric model in which a parameter that measures skewness is changing everyday and depends on the daily available information on forecasting variables. By specifying the skewness parameter as a function of the past trading volume, the estimated coefficient from the Maximum Likelihood estimation provides the effect of past trading volume on market skewness.

The model used in this paper follows closely to the time-series setup in Hueng and McDonald, with several modifications suggested by previous studies on the Chinese stock markets. The conditional mean of stock returns follows an AR(m)-GARCH-in-Mean process including lagged returns:

$$y_t = \mu + \sum_{i=1}^m \theta_i y_{t-i} + \zeta \cdot h_t + \varepsilon_t, \quad (1)$$

where  $y_t$  is the daily return constructed as the first-difference of the log market index, and  $h_t = E(\varepsilon_t^2 | t-1)$  is the conditional variance of  $\varepsilon_t$  based on the information available at time  $t-1$ .<sup>1</sup>

The conditional variance  $h_t$  follows an asymmetric GARCH process. It is well-known in the finance literature that the GARCH process describing the return variances should be modeled as asymmetric. Engle and Ng (1993) suggest that the model by Glosten, Jaganathan, and Runkle (1993) (the GJR model) be the best parametric model among a wide range of predictable

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<sup>1</sup> The lag length  $m$  is chosen by the Ljung-Box  $Q$  tests as the minimum lag that renders serially uncorrelated residuals (at the 5% significance level) up to 22 lags (approximately a month) from an OLS autoregression of  $y_t$ . To keep our model parsimonious, we remove the lags with insignificant estimated coefficients based on the OLS autoregression. The OLS autoregression is used only to determine the number of lagged returns included in the conditional mean. The coefficients in the conditional mean equation will be jointly estimated with the conditional variance equation using full information maximum-likelihood (FIML) estimation.

volatility models that they experiment with the Japanese data. Hueng and McDonald apply the GJR model and another popular asymmetric GARCH model, the EGARCH model by Nelson (1991), to the U.S. data and find no significant difference between the results from these two models. Friedmann and Sanddorf-Köhle (2002) also find that these two models perform quite similarly when applied to the daily Chinese Stock returns. However, using the weekly Chinese stock data, Wei (2002) claims that the AGARCH model included in Engle and Ng's experiments indeed outperforms the GJR model out of sample. For a robustness check, we experiment with all three popular asymmetric GARCH models.

In addition to the asymmetric variance specification, Friedmann and Sanddorf-Köhle suggest that two other factors that affect the volatilities in the Chinese markets should not be ignored: the number of non-trading days between time  $t$  and  $t-1$ , and the introduction of the 10% price change limit on December 16, 1996. The former should have a positive effect on the volatilities, while the latter should reduce the volatilities. Therefore, we include these two variables in the conditional variance equation. Finally, to control the effect of trading volume on variance, we also include past trading volume in the GARCH process.<sup>2</sup>

Our asymmetric GARCH model with the GJR specification is as follows:

$$h_t = \kappa + \alpha \cdot h_{t-1} + \beta \cdot \varepsilon_{t-1}^2 + \gamma \cdot I_{t-1}^+ \cdot \varepsilon_{t-1}^2 + \phi_1 \cdot VOL_{t-1} + \phi_2 \cdot N_t + \phi_3 \cdot D_t, \quad (2)$$

where  $I_{t-1}^+ = 1$  if  $\varepsilon_{t-1} > 0$  and  $I_{t-1}^+ = 0$  if  $\varepsilon_{t-1} < 0$ ,  $VOL_{t-1}$  is the log trading volume at  $t-1$ ,  $N_t$  is the number of non-trading days between  $t$  and  $t-1$ , and  $D_t$  is the post-12/16/1996 dummy. Similarly, our EGARCH model is specified as

$$\log(h_t) = \kappa + \alpha \cdot \log(h_{t-1}) + \beta \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \gamma \cdot I_{t-1}^+ \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \phi_1 \cdot VOL_{t-1} + \phi_2 \cdot N_t + \phi_3 D_t, \quad (2')$$

and our AGARCH model is specified as

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<sup>2</sup> We also try to include a Monday dummy as in Hueng and McDonald, but it is never statistically significant. Therefore, to keep the model parsimonious, we do not include this dummy in the final specification.

$$h_t = \kappa + \alpha \cdot h_{t-1} + \beta \cdot (\varepsilon_{t-1} - \gamma)^2 + \phi_1 \cdot VOL_{t-1} + \phi_2 \cdot N_t + \phi_3 \cdot D_t, \quad (2'')$$

where  $\gamma$  in the AGARCH model takes a positive value so that a negative  $\varepsilon_{t-1}$  has a bigger impact on  $h_t$ .

Given the conditional variance  $h_t$ , the return residuals can be expressed as  $\varepsilon_t = \sqrt{h_t} v_t$ , where  $v_t$  is a zero-mean and unit-variance random variable. The specification of the distribution of the stochastic process  $\{v_t\}$  determines the higher moments of the distribution of the returns ( $y_t$ ). In the GARCH literature, the conditional distribution of  $\{v_t\}$  is simply assumed to be independent of the information at  $t-1$ . Therefore, the only features of the conditional distribution of  $y_t$  that depend on the information at  $t-1$  are the mean and the variance. Hansen (1994) suggests the "autoregressive conditional density" (ARCD) modeling strategy to allow conditional dependence beyond the first two moments. The parameters in the conditional density function are modeled as functions of the elements of the information set so that the higher moments also depend on the conditioning information.<sup>3</sup>

To model the variation in the conditional skewness, we develop the Asymmetric Generalized-t (AGT) distribution to describe the conditional density.<sup>4</sup> The AGT distribution is an asymmetric version of McDonald and Newey's (1988) Generalized-t distribution. It is a parsimonious three-parameter distribution, but also a very flexible distribution that is able to model not only leptokurtosis and platykurtosis but also asymmetry. The derivation and the

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<sup>3</sup> An alternative modeling strategy would be the one proposed by Harvey and Siddique (1999). They specify a conditional skewness process that is similar to the conditional variance process; i.e., the conditional skewness is a function of its own past value and the cube of the past residuals.

<sup>4</sup> There exist many methods for modeling an asymmetric distribution, including kernel estimators, semi-nonparametric (SNP) methods, and flexible parametric methods. We adopt a parametric method in order to directly derive the effects of forecasting variables on skewness. There are also many parametric distributions that can capture the skewness of a random variable. Examples include the exponential generalized beta of the second kind (EGB2) and transformations of normally distributed variables discussed by Johnson (1949). We use the AGT distribution because it has a simple parametric form and is very flexible. Comparisons among different choices of distributions are out of the scope of this paper.

specifics of this distribution are shown in a technical appendix. Hence, the stochastic process  $\{v_t\}$  is assumed to be from the standard AGT (SAGT) distribution:

$$f_{SAGT}(v_t; p, q, r) = \begin{cases} \sigma \left\{ 1 + \left( \frac{2B\left(\frac{1}{p}, q\right) \cdot |\sigma \cdot v_t + \mu|}{p(1+r)} \right)^p \right\}^{-q-\frac{1}{p}} & \text{for } \sigma \cdot v_t + \mu \geq 0, \\ \sigma \left\{ 1 + \left( \frac{2B\left(\frac{1}{p}, q\right) \cdot |\sigma \cdot v_t + \mu|}{p(1-r)} \right)^p \right\}^{-q-\frac{1}{p}} & \text{for } \sigma \cdot v_t + \mu < 0. \end{cases} \quad (3)$$

where  $\mu = 4rp \cdot B\left(\frac{2}{p}, q - \frac{1}{p}\right) \left[ 2B\left(\frac{1}{p}, q\right) \right]^{-2}$ ,  $\sigma^2 = 2p^2 r(r^2 + 3) B\left(\frac{3}{p}, q - \frac{2}{p}\right) \left[ 2B\left(\frac{1}{p}, q\right) \right]^{-3} - \mu^2$ ,

and  $B(\cdot)$  is the Beta function. The parameters  $p$  and  $q$  control the height and tails of the density, with  $2 < p \cdot q < \infty$ . The parameter  $r$  controls the skewness of the distribution, with  $-1 < r < 1$ . Specifically, when  $r > 0$ , the distribution skews to the right, and vice-versa when  $r < 0$ .

Since  $r$  governs the skewness of the distribution, to access the effects of forecasting variables on the skewness of stock returns, we assume that  $v_t$  follows the SAGT distribution (3) with a time-varying parameter  $r_t$ :

$$r_t = -.99 + \frac{.99 - (-.99)}{1 + \exp(-\omega_t)}, \quad \omega_t = a_1 + a_2 \cdot y_{t-1} + a_3 \cdot VOL_{t-1}, \quad (4)$$

where the logistic transformation, a positive monotonic transformation, restricts  $r_t$  to be between  $-.99$  and  $.99$ , while allows  $\omega_t$  to vary over the entire real line. This specification of the law of motion for the time-varying parameter is different from those used in Hansen (1994). Hansen conjectures that since GARCH models make the conditional second moment a function of the lagged errors, it is reasonable to believe that this strategy could also work well for the other moments. Therefore, he models the distribution parameters as quadratic functions of the lagged

error terms. This paper instead uses the theoretical guidance from the Hong-Stein model to model the dynamics of the skewness. Specifically, the Hong-Stein model predicts that  $a_3$  is negative – if the trading volume proxies the differences of opinion, negative skewness is more pronounced if prior volume ( $VOL_{t-1}$ ) is higher. On the other hand, the inclusion of past return ( $y_{t-1}$ ) is to incorporate past information on returns into the determination of skewness. Chen, Hong, and Stein (2001) also argue that, based on the models of stochastic bubbles, past returns should have a positive effect on skewness.

Since, to our knowledge, there is no theoretical guidance on the determination of kurtosis, we decide to model the other two parameters  $p$  and  $q$  as constants and keep the model parsimonious.<sup>5</sup> We keep the focus of this paper on the time-series test of the Hong-Stein model, i.e., the effects of prior trading volume on stock skewness.

The conditional mean and variance equations (1)-(2) with the conditional distribution implied by (3) and (4) are jointly estimated using full information maximum-likelihood (FIML) estimation. The conditional density function (likelihood function) is obtained as:

$$g_{AGT}(\varepsilon_t | h_t, \theta_t) = f_{SAGT}(v_t | h_t, \theta_t) \times \frac{\partial v_t}{\partial \varepsilon_t} = f_{SAGT}\left(\frac{\varepsilon_t}{\sqrt{h_t}} | h_t, \theta_t\right) \times \frac{1}{\sqrt{h_t}}, \text{ where } \theta_t = \{p, q, r_t\}.$$

## II. Estimation Results

The daily data analyzed in this paper are indexes and trading volumes of the Shanghai Stock Exchange Composite (SHSEC) and Shenzhen Stock Exchange Composite (SZSEC) from

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<sup>5</sup> Cont and Bouchaud (2000) propose a theoretical model where stock kurtosis depends on the herding behaviors of the investors. However, our data do not provide information on the construction of measures of herding. Furthermore, we do experiment specifications where  $p$  and  $q$  are modeled as time-varying variables as well. But the calculation of the standard errors on this set of parameters fails, which indicates that there is not enough information in the data to identify these parameters, or these parameters cannot be identified in the specification. Including a lag in the dynamics causes the same problem. Since our focus is on the skewness, we decide to use the current specification. Adding dynamics to  $p$  and  $q$ , though, does not change the significance of the effect of prior trading volume on market skewness.

June 1995 to August 2001. There are a total of 1519 observations in the SHSEC and 1517 observations in the SZSEC. We start the sample in June 1995 to avoid the abnormal 60% return swing between 05/18/1995 and 05/23/1995 caused by the crash of the Chinese government bond futures market. Figures 1-4 plot the returns (measured as the first-difference of the log indexes) and the log trading volumes for these two markets.

Table 1 shows the estimation results from three different asymmetric GARCH specifications: the GJR model, the EGARCH model, and the AGARCH model. Panel (A) shows the results from the SHSEC and Panel (B) shows those from the SZSEC. For the conditional mean equation, the results are very similar across different models and markets. The only difference is the GARCH-in-Mean effect: there exists a significant risk-return tradeoff in the SZSEC, but the tradeoff is not statistically significant in the SHSEC. The positive sign on this variable indicates that an asset with a higher perceived risk would pay a higher return on average.

In the conditional variance equation, as expected, all results show high persistence in the GARCH process. The estimated signs on the asymmetry parameter  $\gamma$  in different models are consistent with the findings by Harvey and Siddique (1999) and Hueng and McDonald (2005), who use both the GJR model and the EGARCH model, and the findings by Wei (2002), who uses the GJR model and the AGARCH model. However, the asymmetric GARCH effect is not statistically significant in the GJR specifications.

Trading volumes do not provide statistically significant information in predicting volatilities after controlling for the other variables. The number of non-trading days before the market opens, as argued by Friedmann and Sanddorf-Köhle (2002), has a significantly positive effect on the volatilities in the Chinese stock markets. The impact is highly significant across different markets and models. On the other hand, the price regulation regime also changes the volatilities in the Chinese stock markets. The introduction of a 10% price change limit on December 16, 1996 reduces the market volatilities. This effect is statistically significant in the SHSEC, but not in the SZSEC except for the EGARCH specification.

In the conditional density function, the estimates of  $p$  and  $q$  show the expected leptokurtosis of the distribution in the Chinese stock returns. For forecasting the skewness, consistent with what Chen, Hong, and Stein and Hueng and McDonald find in the U.S. data, the prior returns have a positive effect on future skewness. This predicting power is statistically significant in the SHSEC, but not in the SZSEC.

Most importantly, we find the evidence that supports the Hong-Stein model: higher trading volume today predicts a more negatively skewed distribution on the next trading day. The predicting power is statistically significant in all three models and in both markets.

### **III. Conclusion**

Hong and Stein's (2003) Differences-of-Opinion model is not supported by Hueng and McDonald's (2005) time-series test in the U.S. aggregate market. Since the Hong-Stein model is based on the assumption that some investors face a short-sales constraint, one possible explanation for Hueng and McDonald's finding is that, given the liquid derivative markets in the United States, the short-sale constraint may not be binding at the aggregate market level.

This paper uses an alternative sample that fits the Hong-Stein model's assumption perfectly to test the model in a time-series setting. The Chinese stock markets are young and lack of stock lending mechanisms. The index derivatives are still at the early stage of development. Therefore, it is very difficult to short-sell the Chinese stocks. Using daily data of the Shanghai and Shenzhen composite indexes from June 1995 to August 2001, we find that higher trading volume today predicts a more negatively skewed distribution for tomorrow's stock return. Thus, the Hong-Stein model is supported in the Chinese stock markets.

The goal of this paper is to see whether past trading volume has a significantly negative effect on return skewness, as suggested by the Hong-Stein model, in a time-series analysis for aggregate markets. The selection of the predicting variables is by no means the complete list of factors that influence the volatility and asymmetry in the Chinese stock markets. We seek

theoretical supports from the literature but at the same time try to keep our model parsimonious. Due to the limitation of the data availability, a more rigorous specification of the models is not feasible. Testing the Hong-Stein model in other markets that have short-sale constraints and more data would be an interesting extension for future studies.

## Appendix: Specifics of the Asymmetric Generalized t Distribution.

The derivation of the AGT distribution is similar to Hansen's (1994) derivation of his asymmetric Student's t distribution.

The Generalized-t (GT) distribution in McDonald and Newey (1988) has the following pdf:

$$f_{GT}(x_t; \omega, p, q) = \frac{p}{2\omega q^{\frac{1}{p}} B\left(\frac{1}{p}, q\right) \left(1 + \frac{|x_t|^p}{q\omega^p}\right)^{q + \frac{1}{p}}},$$

where  $B(\cdot)$  is the beta function. The parameters  $p$  and  $q$  are positive and  $p \times q > n$  is required for the  $n^{\text{th}}$ -order moment to exist. This distribution can accommodate both leptokurtosis (thicker tailed than the normal distribution) and platykurtosis (thinner tailed than the normal distribution). Special cases of the GT include the t ( $p=2$ ) and the normal ( $p=2$  and  $q \rightarrow \infty$ ) distributions.

To transform this symmetric distribution to an asymmetric one, define

$$h(z_t) = \begin{cases} f_{GT}\left(\frac{|z_t|}{1+r}\right) & \text{for } z_t \geq 0, \\ f_{GT}\left(\frac{|z_t|}{1-r}\right) & \text{for } z_t < 0, \end{cases}$$

where  $-1 < r < 1$ . This specification is a proper pdf and allows different rates of descent for  $z_t > 0$  and  $z_t < 0$ , thus allowing for skewness. Specifically, when  $r > 0$ , the mode of the density is to the left of zero and the distribution skews to the right, and vice-versa when  $r < 0$ . When  $r = 0$ , the distribution is symmetric.

Next, we scale  $z_t$  by defining  $x_t = \frac{z_t}{A}$ , where  $A$  is a positive scaling constant for simplifying the notation. The density function of the AGT distribution can be written as

$$f_{AGT}(x_t) = h(z_t) \cdot \frac{\partial z_t}{\partial x_t} = h(A \cdot x_t) \cdot A = \begin{cases} f_{GT}\left(\frac{A|x_t|}{1+r}\right) \cdot A & \text{for } x_t \geq 0, \\ f_{GT}\left(\frac{A|x_t|}{1-r}\right) \cdot A & \text{for } x_t < 0. \end{cases}$$

Specifically, let  $A = \frac{2\omega q^{\frac{1}{p}} B\left(\frac{1}{p}, q\right)}{p}$ , and the pdf becomes

$$f_{AGT}(x_t; p, q, r) = \begin{cases} \left\{ 1 + \left( \frac{2B\left(\frac{1}{p}, q\right) \cdot |x_t|^p}{p(1+r)} \right)^p \right\}^{-q-\frac{1}{p}} & \text{for } x_t \geq 0, \\ \left\{ 1 + \left( \frac{2B\left(\frac{1}{p}, q\right) \cdot |x_t|^p}{p(1-r)} \right)^p \right\}^{-q-\frac{1}{p}} & \text{for } x_t < 0. \end{cases}$$

This distribution nests several popular distributions. In particular, when  $r = 0$ , AGT = GT; when  $r = 0$  and  $p = 2$ , AGT = Student's t; and when  $r = 0$ ,  $p = 2$ , and  $q \rightarrow \infty$ , AGT converges to a normal distribution. A nonzero  $r$  yields asymmetric versions of corresponding distributions. The following Table shows the skewness and kurtosis for some combinations of  $p$ ,  $q$ , and  $r$ . Note that the respective negative of  $r$  will generate identical tables for skewness and kurtosis except that the skewness will have a negative sign.

Skewness						Kurtosis					
$r=0$	$q=0.5$	1	10	50	100	$r=0$	$q=0.5$	1	10	50	100
$P=0.5$	0	0	0	0	0	$P=0.5$	.	.	635.040	35.835	29.798
1	0	0	0	0	0	1	.	.	10.286	6.527	6.251
10	0	0	0	0	0	10	3.550	2.070	1.892	1.886	1.885
50	0	0	0	0	0	50	1.821	1.809	1.805	1.804	1.804
100	0	0	0	0	0	100	1.805	1.802	1.801	1.801	1.801
$r=0.5$						$r=0.5$					
$P=0.5$	.	.	12.296	4.720	4.352	$P=0.5$	.	.	958.926	53.539	44.199
1	.	.	2.360	1.756	1.699	1	.	.	15.339	8.809	8.342
10	0.631	0.173	0.067	0.062	0.062	10	5.653	2.187	1.910	1.901	1.900
50	0.015	0.007	0.003	0.003	0.003	50	1.827	1.812	1.806	1.805	1.805
100	0.004	0.002	0.001	0.001	0.001	100	1.806	1.803	1.801	1.801	1.801
$r=0.9$						$r=0.9$					
$P=0.5$	.	.	12.757	5.054	4.677	$P=0.5$	.	.	935.237	54.721	45.348
1	.	.	2.801	2.118	2.053	1	.	.	17.760	10.022	9.469
10	1.111	0.302	0.115	0.107	0.106	10	9.773	2.429	1.948	1.935	1.933
50	0.027	0.012	0.006	0.006	0.006	50	1.842	1.817	1.808	1.808	1.808
100	0.006	0.003	0.002	0.001	0.001	100	1.809	1.804	1.802	1.802	1.802

The  $n^{\text{th}}$  raw moment of  $x_t$  is

$$M_n = \int_{-\infty}^{\infty} x^n f_{AGT}(x) dx = \int_0^{\infty} x^n f_1(x) dx + \int_{-\infty}^0 x^n f_2(x) dx = \int_0^{\infty} x^n f_1(x) dx + (-1)^n \int_0^{\infty} x^n f_2(x) dx.$$

Using the formula:  $\int_0^{\infty} x^n (1+k \cdot x^p)^{-m} dx = B\left(\frac{n+1}{p}, m - \frac{n+1}{p}\right) \left(p \cdot k^{\frac{n+1}{p}}\right)^{-1}$  and letting

$$k_1 = \left(\frac{2B\left(\frac{1}{p}, q\right)}{p(1+r)}\right)^p, \quad k_2 = \left(\frac{2B\left(\frac{1}{p}, q\right)}{p(1-r)}\right)^p, \quad \text{and } m = q + \frac{1}{p}, \quad \text{the } n^{\text{th}} \text{ raw moment becomes}$$

$$\begin{aligned} M_n &= B\left(\frac{n+1}{p}, m - \frac{n+1}{p}\right) \left(p \cdot k_1^{\frac{n+1}{p}}\right)^{-1} + (-1)^n B\left(\frac{n+1}{p}, m - \frac{n+1}{p}\right) \left(p \cdot k_2^{\frac{n+1}{p}}\right)^{-1} \\ &= [(-1)^n (1-r)^{n+1} + (1+r)^{n+1}] p^n B\left(\frac{n+1}{p}, q - \frac{n}{p}\right) \left[2B\left(\frac{1}{p}, q\right)\right]^{-n-1}. \end{aligned}$$

Therefore, the mean is  $\mu \equiv M_1 = 4rp \cdot B\left(\frac{2}{p}, q - \frac{1}{p}\right) \left[2B\left(\frac{1}{p}, q\right)\right]^{-2}$ ; the variance is  $\sigma^2 \equiv M_2 - \mu^2$ ;

the skewness is  $\frac{E(x - \mu)^3}{\sigma^3} = \frac{M_3 - 3\mu\sigma^2 - \mu^3}{\sigma^3}$ ; and the kurtosis is

$$\frac{E(x - \mu)^4}{\sigma^4} = \frac{M_4 - 4\mu \cdot SK \cdot \sigma^3 - 6\mu^2\sigma^2 - \mu^4}{\sigma^4}.$$

Note that  $\Pr(x \geq 0) = \int_0^{\infty} f_1(x) dx = \int_0^{\infty} x^0 f_1(x) dx = (1+r)B\left(\frac{1}{p}, q\right) \left[2B\left(\frac{1}{p}, q\right)\right]^{-1} = \frac{1+r}{2}$  and,

similarly,  $\Pr(x < 0) = \int_{-\infty}^0 f_2(x) dx = \frac{1-r}{2}$ , which shows the asymmetry of the distribution when  $r \neq$

0. Furthermore,  $\Pr(x \geq 0) + \Pr(x < 0) = 1$  confirms that  $f_{AGT}$  is a proper pdf.

In a different framework, Theodossiou (1998) also develops a skewed version of the GT distribution. His Skewed-GT distribution has four parameters. Our pdf is less complicated than his and easier to be implemented in the ARCD model.

## Reference:

Chen, J., H. Hong, and J. C. Stein, 2001, "Forecasting crashes: trading volume, past returns and conditional skewness in stock prices." *Journal of Financial Economics*, 61, 345-381.

Cont, R. and J. P. Bouchaud, 2001, "Herd behavior and aggregate fluctuations in financial markets." *Macroeconomic Dynamics*, 4(2), 170-96.

Engle, R. F. and V. K. Ng, 1993, "Measuring and Testing the Impact of News on Volatility." *Journal of Finance*, 48, 1749-1778.

Figlewski, S. and G. P. Webb, 1993, "Options, Short Sales, and Market Completeness." *Journal of Finance*, 48, 761-777.

Friendmann, R. and W. G. Sanddorf-Köhle, 2002, "Options, Short Sales, and Market Completeness." *Journal of Finance*, 48, 761-777.

Glosten, L. R., R. Jagannathan, and D. E. Runkle, 1993, "On the Relation between the Expected Value and the Volatility of the Normal Excess Return on Stocks." *Journal of Finance*, 48, 1779-1801.

Hansen, B. E., 1994, "Autoregressive conditional density estimation." *International Economic Review*, 35, 705-730.

Harvey C. R., and A. Siddique, 1999, "Autoregressive Conditional Skewness." *Journal of Financial and Quantitative Analysis*, 34, 465-487.

Hong, H., and J. C. Stein, 2003, "Differences of Opinion, Short-Sales Constraints and Market Crashes." *Review of Financial Studies*, 16, 487-525.

Hueng, C. J., and J. B. McDonald, 2005, "Forecasting Asymmetries in Aggregate Stock Market Returns: Evidence from Conditional Skewness." *Journal of Empirical Finance*, forthcoming.

Johnson, N. L., 1949, "Systems of Frequency Curves Generated by Methods of Translation." *Biometrika*, 36, 149-176.

McDonald, J. B., and W. K. Newey, 1988, "Partially adaptive estimation of regression models via the generalized t distribution." *Econometric Theory*, 4, 428-457.

Nelson, D. B., 1991, "Conditional Heteroskedasticity in Asset Return: A New Approach." *Econometrica*, 59, 347-370.

Theodossiou, P., 1998, "Financial Data and the Skewed Generalized t Distribution." *Management Science*, 44, 1650-1661.

Wei, W., 2002, "Forecasting stock market volatility with non-linear GARCH models: a case for China." *Applied Economics Letters*, 9, 163-166.

**Table 1: Estimation Results**

The parameters are shown in equations (1)--(4):

$$(1) \quad y_t = \mu + \sum_{i=1}^m \theta_i y_{t-i} + \zeta \cdot h_t + \varepsilon_t, \quad \varepsilon_t = \sqrt{h_t} v_t,$$

$$(2) \quad \text{GJR} \quad h_t = \kappa + \alpha \cdot h_{t-1} + \beta \cdot \varepsilon_{t-1}^2 + \gamma \cdot I_{t-1}^+ \cdot \varepsilon_{t-1}^2 + \phi_1 \cdot VOL_{t-1} + \phi_2 \cdot N_t + \phi_3 \cdot D_t,$$

$$(2') \quad \text{EGARCH}$$

$$\log(h_t) = \kappa + \alpha \cdot \log(h_{t-1}) + \beta \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \gamma \cdot I_{t-1}^+ \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \phi_1 \cdot VOL_{t-1} + \phi_2 \cdot N_t + \phi_3 D_t,$$

$$(2'') \quad \text{AGARCH} \quad h_t = \kappa + \alpha \cdot h_{t-1} + \beta \cdot (\varepsilon_{t-1} - \gamma)^2 + \phi_1 \cdot VOL_{t-1} + \phi_2 \cdot N_t + \phi_3 \cdot D_t,$$

$$(3) \quad f_{SAGT}(v_t; p, q, r) = \begin{cases} \sigma \left\{ 1 + \frac{\left( 2B\left(\frac{1}{p}, q\right) \cdot |\sigma \cdot v_t + \mu| \right)^p}{p(1+r)} \right\}^{-q-\frac{1}{p}} & \text{for } \sigma \cdot v_t + \mu \geq 0, \\ \sigma \left\{ 1 + \frac{\left( 2B\left(\frac{1}{p}, q\right) \cdot |\sigma \cdot v_t + \mu| \right)^p}{p(1-r)} \right\}^{-q-\frac{1}{p}} & \text{for } \sigma \cdot v_t + \mu < 0. \end{cases}$$

$$(4) \quad r_t = -.99 + \frac{.99 - (-.99)}{1 + \exp(-\omega_t)}, \quad \omega_t = a_1 + a_2 \cdot y_{t-1} + a_3 \cdot VOL_{t-1},$$

Market		(A) SHSEC			(B) SZSEC		
Model		GJR	EGARCH	AGARCH	GJR	EGARCH	AGARCH
<b>Mean Equation</b>	$\mu$ (constant)	0.014 (0.749)	0.001 (0.978)	0.009 (0.833)	-0.021 (0.628)	-0.063 (0.176)	-0.027 (0.527)
	$\theta_l$ (AR3)	<b>0.105</b> <b>(0.000)</b>	<b>0.105</b> <b>(0.000)</b>	<b>0.102</b> <b>(0.000)</b>	<b>0.097</b> <b>(0.000)</b>	<b>0.098</b> <b>(0.000)</b>	<b>0.100</b> <b>(0.000)</b>
		---	---	---	0.006 (0.806)	-0.001 (0.980)	0.009 (0.720)
	$\theta_6$ (AR6)	<b>-0.054</b> <b>(0.016)</b>	<b>-0.054</b> <b>(0.012)</b>	<b>-0.054</b> <b>(0.018)</b>	<b>-0.056</b> <b>(0.019)</b>	<b>-0.065</b> <b>(0.005)</b>	<b>-0.054</b> <b>(0.024)</b>
		$\zeta$ (GARCH-in-Mean)	0.017 (0.264)	0.023 (0.185)	0.017 (0.276)	<b>0.028</b> <b>(0.046)</b>	<b>0.040</b> <b>(0.015)</b>

The numbers in parentheses are P-values. A value of 0.000 indicates that the true value is smaller than 0.0005.

**Table 1: Estimation Results (continued)**

Market		(A) SHSEC			(B) SZSEC		
Model		GJR	EGARCH	AGARCH	GJR	EGARCH	AGARCH
<b>Variance Equation:</b>	$\kappa$ (constant)	2.53e-5	<b>-0.807</b>	2.53e-5	2.53e-5	<b>-0.962</b>	2.53e-5
		.	<b>(0.013)</b>	.	(0.969)	<b>(0.000)</b>	(0.972)
	$\alpha$ (GARCH)	<b>0.698</b>	<b>0.883</b>	<b>0.701</b>	<b>0.707</b>	<b>0.906</b>	<b>0.706</b>
		<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>
	$\beta$ (ARCH)	<b>0.275</b>	<b>-0.488</b>	<b>0.228</b>	<b>0.285</b>	<b>-0.469</b>	<b>0.279</b>
		<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>
	$\gamma$ (asymmetry)	-0.099	<b>0.825</b>	<b>0.384</b>	-0.018	<b>0.848</b>	0.152
		(0.097)	<b>(0.000)</b>	<b>(0.003)</b>	(0.754)	<b>(0.000)</b>	(0.125)
$\phi_1$ (volume)	0.001	0.034	0.002	-0.008	<b>0.045</b>	-0.005	
	(0.945)	(0.115)	(0.858)	(0.500)	<b>(0.007)</b>	(0.682)	
$\phi_2$ (closing days)	<b>0.391</b>	<b>0.130</b>	<b>0.362</b>	<b>0.302</b>	<b>0.103</b>	<b>0.293</b>	
	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.001)</b>	<b>(0.001)</b>	<b>(0.001)</b>	
$\phi_3$ (12/16/96 dummy)	<b>-0.355</b>	<b>-0.141</b>	<b>-0.367</b>	-0.154	<b>-0.133</b>	-0.191	
	<b>(0.016)</b>	<b>(0.002)</b>	<b>(0.012)</b>	(0.161)	<b>(0.001)</b>	(0.103)	
<b>Distribution Parameters:</b>	$p$	<b>1.775</b>	<b>1.720</b>	<b>1.845</b>	<b>2.288</b>	<b>2.251</b>	<b>2.330</b>
		<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>
	$q$	<b>2.812</b>	<b>3.212</b>	<b>2.518</b>	<b>1.845</b>	<b>1.901</b>	<b>1.774</b>
		<b>(0.013)</b>	<b>(0.024)</b>	<b>(0.010)</b>	<b>(0.004)</b>	<b>(0.004)</b>	<b>(0.004)</b>
	$a_1$ (constant)	<b>3.194</b>	<b>3.232</b>	<b>3.412</b>	<b>4.360</b>	<b>4.111</b>	<b>4.499</b>
		<b>(0.005)</b>	<b>(0.003)</b>	<b>(0.007)</b>	<b>(0.001)</b>	<b>(0.001)</b>	<b>(0.001)</b>
	$a_2$ (return)	<b>0.068</b>	<b>0.062</b>	<b>0.064</b>	0.017	0.024	0.018
		<b>(0.026)</b>	<b>(0.025)</b>	<b>(0.045)</b>	(0.644)	(0.461)	(0.647)
	$a_3$ (turnover)	<b>-0.215</b>	<b>-0.218</b>	<b>-0.229</b>	<b>-0.302</b>	<b>-0.289</b>	<b>-0.311</b>
		<b>(0.003)</b>	<b>(0.002)</b>	<b>(0.005)</b>	<b>(0.000)</b>	<b>(0.000)</b>	<b>(0.000)</b>

The numbers in parentheses are P-values. A value of 0.000 indicates that the true value is smaller than 0.0005.

Figure 1: SHSEC Returns (%)  
[Mean: 0.063 Standard Deviation: 1.870]

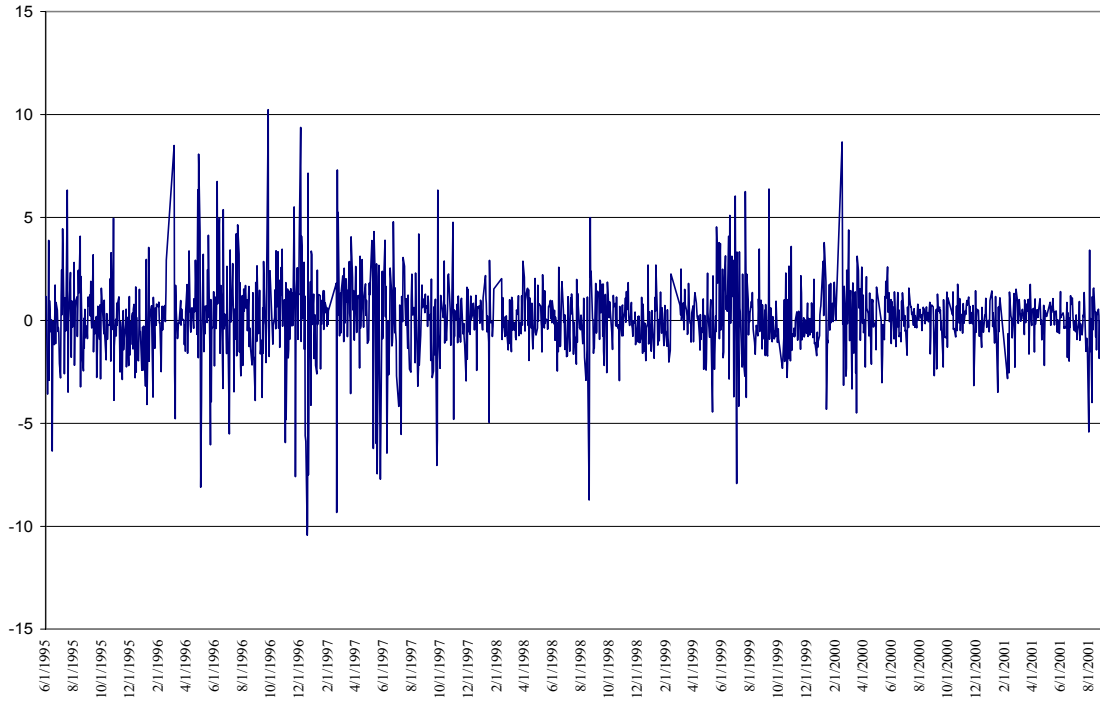


Figure 2: SHSEC log trading volume  
[Mean: 15.357 Standard Deviation: 0.763]

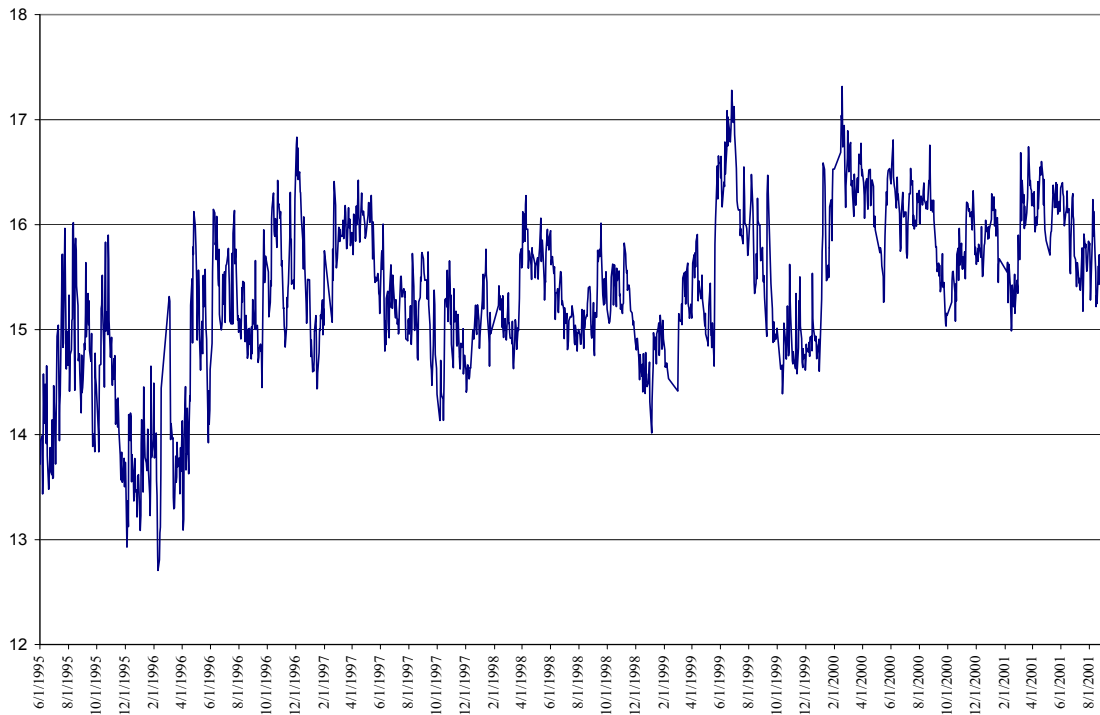


Figure 3: SZSEC Returns (%)  
[Mean: 0.092 Standard Deviation: 2.016]

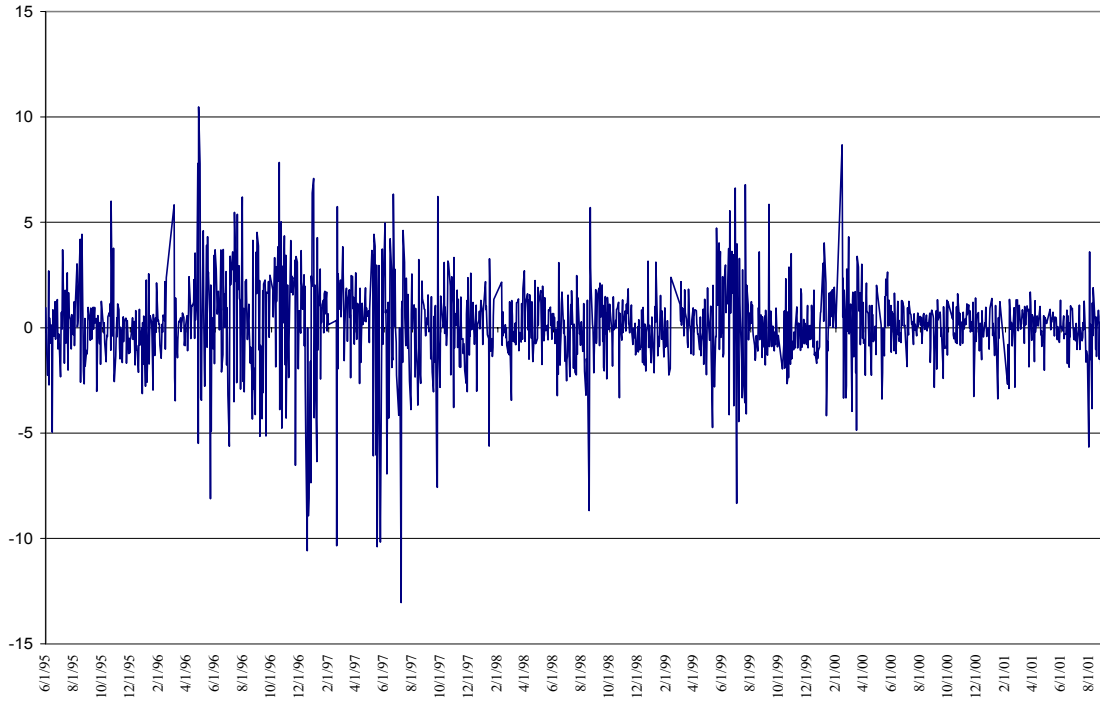


Figure 4: SZSEC log trading volume  
[Mean: 15.200 Standard Deviation: 0.993]

