Interest-Rate Risk Factor and Stock Returns: A Time-Varying Factor-Loadings Model

Peng Huang

Department of Economics
Ripon College, U.S.A.

C. James Hueng*

Department of Economics
Western Michigan University, U.S.A.

Abstract

We extend the Fama-French three-factor model to include a new risk factor that proxies for interest-rate risk faced by firms, in an attempt to reduce the pricing errors that the three-factor model cannot explain. These pricing errors are observed especially in small size and low book-to-market ratio firms, which are in general more sensitive to interest-rate risk. In addition, the factor loadings are modeled as time-varying so that the investors' learning process can be taken into account. The results show that our time-varying-loadings four-factor model significantly reduces the pricing errors.

Key words: factor model; interest-rate risk factor; time-varying factor loadings; Kalman Filter.

JEL Classifications: G11, G12

**Correspondence to:* Professor C. James Hueng
5412 Friedmann Hall
Western Michigan University
Kalamazoo, MI 49008-5330
U.S.A.
Phone: 0021-269-387-5558 Fax: 0021-269-387-5637
Email: James.Hueng@wmich.edu

I. Introduction

Modeling the cross-sectional average stock returns has been the focal point of finance for many years. The most influential asset-pricing model in the 1990s is the three-factor model proposed by Fama and French (1993). This model consists of three portfolios constructed to mimic underlying common risk factors in returns. The portfolios include the zero-investment portfolios formed on firm size (*SZ*) and book-to-market ratio (*BM*), as well as the excess return of a value-weighted market portfolio. The risk factors constructed from these portfolios are denoted as *SMB* (Small-*SZ* Minus Big-*SZ*), *HML* (High-*BM* Minus Low-*BM*), and *MKT*, respectively. Their results show that these three factors seem to explain the excess returns on a full set of *BM*- and *SZ*-sorted portfolios.

Although the three-factor model has generated impressive performance, empirical evidence indicates that it is still not able to completely capture the cross-sectional returns. In particular, statistically significant pricing errors that cannot be explained by these three factors still exist in several portfolios. For example, using monthly data on the U.S. common stocks from 1963:7 to 1990:12, Fama and French (1993) show that in the five-by-five *SZ*/*BM* double-sorted portfolios, there are three portfolios of which the pricing errors are significantly different from zero. Two of them are located in the lowest *BM* quintile. Petkova (2006) extends the sample to include the data after 1990 (up to 2001:12) and finds that six out of the 25 portfolios have pricing errors that are significantly different from zero. Four of them are located in the lowest *BM* or the smallest *SZ* quintile. Adrian and Franzoni (2005) report similar results using quarterly data from 1963 to 2004: five portfolios located in either the lowest *BM* quintile or the smallest *SZ* quintile have statistically significant pricing errors. This inefficiency of the three-factor model, in fact, is not limited to the U.S. data. Chiao and Hueng (2005) use the Japanese data and find that six out of the nine lowest *BM* and/or smallest *SZ* portfolios have pricing errors that are significantly different from zero.

Apparently, most of the pricing errors that cannot be explained by the three-factor model appear in the portfolios with either the lowest *BM* (denoted as the "growth" portfolio) or the smallest *SZ* (denoted as the "small" portfolio). Therefore, to improve the model's ability to explain the cross-sectional returns, we add a risk factor that is most likely to capture the risk faced especially by small firms and firms with a low *BM*. Studies such as Cornell (1999) and Campbell and Vuolteenaho (2004) claim that small firms and firms with a low *BM* are more sensitive to interest rate (discount rate) risk. To capture this risk, the *TERM* factor, defined as the yield spread between the ten-year and the three-month treasury rates, is incorporated into our model. As suggested by Campbell and Shiller (1991), Diebold, Rudebush, and Aruoba (2006), and Chen, Roll, and Ross (1986), this term spread represents a good proxy for risk caused by shifts in interest rates. Since the discount rate is an average of interest rates over time, a change in interest rates can lead to a change in the discount rate. Therefore, the *TERM* factor should capture information about risk related to changes in the discount rate.

In addition to the *TERM* factor, the second modification we make on the three-factor model is to model the factor loadings as time varying. Under the CAPM framework, studies such as Harvey (1989), Ferson and Harvey (1991, 1993), and Jagnnathan and Wang (1996) demonstrate that beta in the CAPM tends to be volatile through time. Franzoni (2006) shows that portfolios, especially those with small firms, exhibit considerable long-run variations in beta. Adrian and Franzoni (2004, 2005) argue that the main reason for the failure of the unconditional CAPM is that the model does not mimic investors' learning process. The unobservable time-varying beta should induce the investors' learning process. However, the OLS regression used in constant-loadings factor models cannot successfully mimic the investors' learning process, which leads to a difference between the investors' true expectation of the loadings and the constant factor loadings estimated by OLS. Indeed, Fama and French (1997) show that loadings on *MKT*, *HML*, and *SMB* of industry-sorted portfolios exhibit strong time variations. Ferson and Havey (1999) also show that loadings on the Fama-French three factors of the five-by-five *SZ*/*BM* double-sorted portfolios tend to be time-varying. Therefore, following Adrian and Franzoni's research, we use the Kalman Filter procedure to estimate the movements of the factor loadings and replicate the investors' learning process.

To evaluate the performance of our time-varying-loadings four-factor model, thereafter the "TVL4 model", we apply the model to the five-by-five *SZ*/*BM* double-sorted portfolios formed by U.S. stocks from the period 1963:7-2004:12. In addition, the time-varying-loadings three-factor model estimated by the Kalman Filter and the constant-loadings four-factor model estimated by OLS are also examined to analyze the sole contribution of the learning process and of the *TERM* factor, respectively. This paper focuses on the pricing error in each individual portfolio as well as the aggregate pricing error generated by each model.

The results show that both the time-varying-factor-loadings specification and the *TERM* factor reduce the pricing errors. The most impressive result is that the combination of these two modifications, the TVL4 model, remarkably reduces both the individual and the aggregate pricing errors generated by the Fama-French three-factor model. Those statistically significant pricing errors in the Fama-French model almost all become insignificant in the TVL4 model. The root mean squared error (RMSE) and the composite pricing error (CPE), two measures for the aggregate pricing errors, are reduced by 60% and 50%, respectively, from the Fama-French model to the TVL4 model.

To check for possible data mining problems, we further evaluate the performances of the TVL4 model in other portfolio formations. Specifically, we first experiment with the model in industry-sorted portfolios. Both the individual and the aggregate pricing errors in the Fama-French three-factor model are largely reduced in the TVL4 model. Furthermore, we use only the data prior to 1963 to form the *SZ*/*BM* double-sorted portfolios and evaluate the performances of the TVL4 model. Again, the TVL4 model outperforms the Fama-French three-factor model by greatly reducing both the individual and the aggregate pricing errors.

To further check the robustness of our results, we also experiment with another time-varying-loadings four-factor model where the *TERM* factor is replaced by the momentum factor, a risk factor that has been found to be important in the literature. It turns out that the momentum factor does not help to reduce the pricing errors in the five-by-five *SZ*/*BM* double-sorted portfolios. Therefore, we conclude that the success of the TVL4 model lies on the facts that it successfully mimics the investors' evolutional learning process of loadings, and that it incorporates into the estimation process more information about risks related to changes in discount rates.

The remainder of the paper is organized as follows. The next section details our motivation for constructing the TVL4 model and the specification of the model. Section III describes the data and the methodology. Section IV shows the estimation results for the five-by-five *SZ*/*BM* double-sorted portfolios. Robustness checks are presented in Section V. Finally, Section VI concludes the paper.

II. The time-varying-loadings four-factor model

A. The *TERM* factor

Since statistically significant pricing errors are found mostly in the small/growth portfolios, it is natural for us to investigate this problem by concentrating on specific characteristics of firms in these portfolios. Previous studies have indicated that both the small and growth portfolios are more sensitive to changes in the discount rates. Based on the relationship between risk and the duration of projects, Cornell (1999) argues that the relatively higher risk of long-term projects arises from variations in the discount rate rather than variations in the cash flows. Since small and growth firms usually generate cash flows in a longer duration, their returns are likely to respond more strongly to shocks to the discount rate than the large and value (high *BM*) firms do. This is very similar to the case that long-term bonds are more sensitive than short-term bonds to shocks to the discount rates because of their longer durations.

A recent study by Campbell and Vuolteenaho (2004) provides further evidence. They decompose the market beta of a portfolio into the cash-flow beta and the discount-rate beta. Their results indicate that the discount-rate betas of the small/growth portfolios are larger than those of the large/value portfolios after 1960s. That is, the small/growth portfolios are likely to be more sensitive to the discount rate during this period. They attribute the relatively higher discount-rate betas to long durations of cash flows, future investment opportunities, and dependence on external fund raising. Like Cornell (1999), they think that small and growth firms, usually with negative current cash flows but valuable future investment opportunities, react more to the discount rate news. Furthermore, in line with Perez-Quiros and Timmermann (2000), they argue that small and young firms with little collateral rely more heavily on external financing such as bank loans. This is because these firms do not have easy access to other credit sources and therefore, are more sensitive to interest costs and external financial conditions.

Hence, we believe that the information about shocks to the discount rates plays an important role in explaining returns of the small/growth portfolios. Indeed, Cornell (1999) suggests that taking account of changes in the discount rates may improve the tests of an asset-pricing model and help to explain some anomalies in returns, especially for small or growth firms. This inspires us to add the *TERM* factor, the spread between ten-year and three-month Treasury rates, to the three-factor model.[1]

It is well-known that the term spread is a variable that contains information about future movements in interest rates [e.g., Campbell and Shiller (1991)]. When building a three-factor yield curve model, Diebold, Rudebusch and Aruoba (2006) assert that the yield spread between

---

[1] Studies that also realize the importance of the information connected to the discount rate include Elton, Gruber, and Blake (1995) and Ferson and Harvey (1999). They both suggest that the Fama-French three-factor model may leave out some important information reflected by interest-rate factors.

the ten-year bond and the three-month Treasury bill, which is highly correlated with their "slope" factor, responds significantly to innovations in Federal funds rate. Since the discount rate is an average of interest rates over time, changes in interest rates affect the value of the discount rate, and in turns affect the stock returns. Chen, Roll and Ross (1986) also claim that the discount rate changes with the term spread across different maturities.

In the factor model literature, it has been suggested that the term spread may play an important role.[2] For example, Petkova (2006) illustrates that *SMB* is positively and significantly related to the term spread, but only a very small portion (about 5%) of the surprises in the term spread can be explained by the Fama-French three factors. On the other hand, to improve the performance of their conditional CAPM, Adrian and Franzoni (2005) show that, in addition to the *MKT* and *HTM* factors, the term spread should be included as a state variable for beta. Both of these studies hint that the term spread carries information beyond the Fama-French three factors.

B. The time-varying factor-loadings model

Time-varying factor loadings have been suggested in recent asset pricing literature. For example, Franzoni (2006) finds that portfolios with small firms exhibit considerable long-run variations in market beta. Adrian and Franzoni (2004, 2005) argue that the unobservability of time-varying betas should induce the investors' learning process, and that the main reason for the failure of the unconditional CAPM is that the constant-beta model does not mimic the investors' learning process. However, the OLS regression used in the constant-loadings factor models cannot successfully mimic the investors' learning process. Therefore, they use the Kalman Filter procedure to estimate the movements of the time-varying market beta. The Kalman Filter

[2] Chen, Roll and Ross (1986), Fama and French (1993), Aretz, Bartram and Pope (2005), and Petkova (2006) use the term spread directly as a factor, while Campbell and Vuolteenaho (2004) and Adrian and Franzoni (2005) use it as a state variable for the other factors.

updates the time-varying parameters through learning from prediction errors. Their results show that the learning type of CAPM outperforms the unconditional CAPM.

Motivated by Adrian and Franzoni (2004, 2005), we implement the learning type model in the case of multiple factor loadings. Previous studies that try to document time-varying risk loadings often use conditional models. For example, Jaganathan and Wang (1996) develop a conditional CAPM in which the evolution of beta is a function of lagged state variables. Similarly, Ferson and Havery (1999) propose a conditional multi-risk-loadings model that specifies time-varying risk loadings as linear functions of lagged instrumental variables. Fama and French (1997) model a conditional three-factor model by assuming that the risk loadings on *SMB* and *HML* for a portfolio vary with the average size and book-to-market ratios of the firms in that portfolio. Different from these papers, we focus on tracking the evolution of risk loadings with an emphasis on mimicking the investors' learning process of unobservable variables, rather than using additional state variables in the model. Thus, following Adrian and Franzoni, we assume that the time-varying factor loadings are mean-reverting and that each loading has an autoregressive structure. Our time-varying-loading four-factor (TVL4) model can be represented in the following state-space form:

(1) $$r_{i,t} = \boldsymbol{x}_t{}' \boldsymbol{\beta}_{i,t} + \varepsilon_{i,t} ,$$

(2) $$\boldsymbol{\beta}_{i,t} = \boldsymbol{A}_i + \boldsymbol{F}_i \, \boldsymbol{\beta}_{i,t-1} + \boldsymbol{v}_{i,t} ,$$

where $r_{i,t}$ denotes the excess return (return minus the risk-free rate) of portfolio i at time t; $\boldsymbol{x}_t$ = (MKT_t, SMB_t, HML_t, $TERM_t$) is the vector of the common risk factors to all portfolios; $\boldsymbol{\beta}_{i,t}$ is the vector of the factor loadings; $\boldsymbol{A}_i$ is a constant vector; $\boldsymbol{F}_i$ is a diagonal matrix; and $\varepsilon_{i,t} \sim N(0, R_i)$ and $\boldsymbol{v}_{i,t} \sim N(0, \boldsymbol{Q}_i)$, where R_i is a scalar and $\boldsymbol{Q}_i$ is a four-by-four diagonal matrix. The idiosyncratic shocks to portfolio i, $\varepsilon_{i,t}$ and $\boldsymbol{v}_{i,t}$, are independent of each other and are uncorrelated with shocks to other portfolios.

Equations (1) and (2) are estimated by the Kalman Filter. The Kalman Filter is a dynamic procedure that updates unobservable factor loadings by learning through prediction errors in returns that contain the most updated information. Since the factor loadings are affected by idiosyncratic shocks and change over time, the Kalman Filter is a better methodology than the OLS in capturing the dynamics of the factor loadings.

The Kalman Filter consists of two steps: prediction and updating. Since the factor loadings are not observable, investors need to form their expectations based on available information. In the prediction step, the one-period ahead forecast of $\boldsymbol{\beta}_i$ at time t-1 can be expressed as:

(3) $$\boldsymbol{\beta}_{i,t|t-1} = \boldsymbol{A}_i + \boldsymbol{F}_i\, \boldsymbol{\beta}_{i,t-1|t-1} .$$

Based on the expectations of the risk loadings, the expected excess return of portfolio *i* is:

(4) $$r_{i,t|t-1} = \boldsymbol{x}_t' \boldsymbol{\beta}_{i,t|t-1} .$$

After $r_{i,t}$ is realized at time t, the prediction error can be expressed as:

(5) $$\eta_{i,t|t-1} = r_{i,t} - r_{i,t|t-1} ,$$

where $\eta_{i,t|t-1}$ contains new information about $\boldsymbol{\beta}_{i,t}$ beyond $\boldsymbol{\beta}_{i,t|t-1}$.

In the updating step, based on the prediction error, the inference of risk loading $\boldsymbol{\beta}_{i,t}$, denoted as $\boldsymbol{\beta}_{i,t|t}$, can be updated with information up to time t:

(6) $$\boldsymbol{\beta}_{i,t|t} = \boldsymbol{\beta}_{i,t|t-1} + \boldsymbol{K}_{i,t} \times \eta_{i,t|t-1} ,$$

where $\boldsymbol{K}_{i,t}$ is the Kalman gain, which determines how much weigh to be assigned to the prediction error $\eta_{i,t|t-1}$.[3] In practice, investors continue to adjust their inference about risk loadings through

[3] The Kalman gain $\boldsymbol{K}_{i,t} = \boldsymbol{P}_{i,t|t-1} \boldsymbol{X}_t f^{-1}_{i,t|t-1}$, where $\boldsymbol{P}_{i,t|t-1} = E[(\boldsymbol{\beta}_{i,t} - \boldsymbol{\beta}_{i,t|t-1})(\boldsymbol{\beta}_{i,t} - \boldsymbol{\beta}_{i,t|t-1})']$ and $f_{i,t|t-1} = \boldsymbol{X}_t' \boldsymbol{P}_{i,t|t-1} \boldsymbol{X}_t + R_i$.

learning new information. The dynamic process in equation (6) mimics the investors' learning process on the unobservable risk loadings. After incorporating the new information from the prediction error $\eta_{i,t|t-1}$, the updated risk loadings $\boldsymbol{\beta}_{i,t|t}$ can be used to form the expectation of the risk loadings at time t+1, $\boldsymbol{\beta}_{i,t+1|t}$. Here we only briefly introduce the idea of how the Kalman Filter mimics the investors' learning process of time-varying risk loadings. For details on the Kalman Filter methodology, see Hamilton (1994) and Kim and Nelson (2001).

III. Data and methodology

A. Data

The data used in this paper are kindly provided by Professor Kenneth R. French on his website at Dartmouth. They are monthly returns of 25 portfolios double-sorted by size (*SZ*) and book-to-market ratio (*BM*) covering the period from July 1963 to December 2004. A portfolio's *BM* in year *t* is the sum of book equities of the firms in the portfolio for the fiscal year ending in year *t*-1 divided by the sum of their *SZ* in December of year *t*-1. A portfolio's *SZ* is the average of the logarithms of market equities of the firms in the portfolio at the end of June in year *t*. On the formation dates (the beginning of July from 1963 to 2004), all NYSE, AMEX, and NASDAQ stocks are sorted into five groups based on their *SZ*. The breakpoints are the NYSE market equity quintiles at the end of June in each year. These stocks are then divided independently into five equal *BM* groups. Twenty-five portfolios are constructed from the intersections of the five *SZ* and five *BM* groups. The average monthly return is calculated over the period from July in year *t* to June in year *t*+1. All stocks in a portfolio are value weighted on the formation date. The five-by-five *SZ* and *BM* double-sorted portfolios are a standard set for testing asset pricing models. Previous empirical results show that the returns of the small (the smallest *SZ*) and growth (the lowest *BM*) portfolios cannot be adequately explained by the Fama-French three-factor model.

The Fama-French three-factor model consists of three portfolios constructed to mimic underlying risk factors in returns. The portfolios include the zero-investment portfolios formed on *SZ* (denoted as *SMB*) and *BM* (denoted as *HML*), as well as the excess return of a value-weighted market portfolio (denoted as *MKT*). The *TERM* factor we propose is defined as the yield spread between the ten-year government bond and the three-month Treasury bill.[4] The data of these two variables are taken from the Federal Reserve Bank of St. Louis website.

Table 1 reports the time-series means and the standard deviations of the monthly returns of the five-by-five *SZ*/*BM* double-sorted portfolios, and Table 2 reports those of the four factors.[5] Table 1 exhibits a familiar pattern: Within a specific *BM* (*SZ*) group, the average return increases as *SZ* falls (*BM* rises). The only exception is at the lowest *BM* quintile. This exception shows the first signal that Fama-French's risk factors constructed by *SZ* (*SMB*) and *BM* (*HML*) may not be able to explain the cross-sectional returns in these growth portfolios (the portfolios in the lowest *BM* quintile).

The other signal that may hint on the possible failure of the Fama-French model is that the standard deviations of the small portfolios (the portfolios in the smallest *SZ* quintile) and the growth portfolios are larger compared to the other portfolios. This suggests that the returns of the small/growth portfolios are more volatile than those of the other portfolios. Since the values of the risk factors are the same across different portfolios, relatively higher volatilities in the returns

---

[4] Studies using yield spread between a long-term bond and a short-term treasury bill as a proxy for interest rate risk include Chen, Roll, and Ross (1986), Fama and French (1993), Adrian and Franzoni (2005), and Petkova (2006).

[5] Since estimating the time-varying parameter model via the Kalman Filter needs wild guess for initial values of parameters, the first five years of estimates (1963:7 to 1968:12) are eliminated to offset the effect of the wild guess. Thus, Tables 1 and 2 report the statistics for the data from 1968:7 to 2004:12.

of these portfolios imply that these factors may not be able to explain the cross-sectional returns in these portfolios.

B. Measurement of the pricing errors

In the constant-loadings models, the pricing error is defined as the excess portfolio return that cannot be explained by the proposed risk factors:

$$\alpha_{i,t} = r_{i,t} - \boldsymbol{x_t}'\boldsymbol{\beta_i}\,, \tag{7}$$

where $\boldsymbol{x_t}$ is the vector of the common risk factors and $\boldsymbol{\beta_i}$ is the vector of the corresponding factor loadings. Jensen (1968) suggests a time series regression to estimate the pricing error. That is, the excess return is regressed on a constant and the risk factors by OLS. The estimated intercept is the estimated pricing error ($\hat{\alpha}_i$). This estimated intercept should not be statistically different from zero if the model can well explain the returns of portfolio *i*. A *t*-test can be used to check the statistical significance.

In our time-varying-loadings setup, following Adrian and Franzoni (2005), the pricing error at time *t* of portfolio *i* is estimated as:

$$\hat{\alpha}_{i,t} = r_{i,t} - \boldsymbol{x_t}'\boldsymbol{\hat{\beta}_{i,t|t-1}}\,, \tag{8}$$

where $\boldsymbol{\hat{\beta}_{i,t|t-1}}$ is estimated by the Kalman Filter [see equations (1)-(6) in Section II]. Then the estimated pricing error of portfolio *i* ($\hat{\alpha}_i$) is the time-series mean of $\hat{\alpha}_{i,t}$:

$$\hat{\alpha}_i = \frac{1}{T-1}\sum_{t=2}^{T}\hat{\alpha}_{i,t}\,. \tag{9}$$

The standard deviation of $\hat{\alpha}_i$ is:

$$\hat{\sigma}_i = \sqrt{\frac{1}{T-2}\sum_{t=2}^{T}(\hat{\alpha}_{i,t} - \hat{\alpha}_i)^2}\,. \tag{10}$$

In a well-specified asset-pricing model, the pricing error $\hat{\alpha}_i$ should be insignificantly different from zero. The standard *t*-statistics can be used to check the statistical significance.

There are two measures of the overall performance of the model across all portfolios. The first one is the root mean squared error (RMSE), which gives equal weight to the pricing errors of individual portfolios:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i^2}\,, \tag{12}$$

where N is the number of portfolios. The second measure is the composite pricing error (CPE) suggested by Campbell and Vuolteenaho (2004):

$$\text{CPE} = \sqrt{\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{\Omega}}^{-1}\hat{\boldsymbol{\alpha}}} \tag{13}$$

where $\hat{\boldsymbol{\alpha}}$ is the $N\times 1$ vector consists of the individual estimated pricing errors and $\hat{\boldsymbol{\Omega}}$ is a $N\times N$ diagonal matrix with $\hat{\sigma}_i^2$'s on its diagonal. The weighting matrix $\hat{\boldsymbol{\Omega}}^{-1}$ places less weight on more volatile portfolios.[6]

IV. Results

Table 3 reports the individual and aggregate pricing errors from the four models considered in this paper, (1) the constant-loadings three-factor model (the Fama-French model), (2) the constant-loadings four-factor model, (3) the time-varying-loadings three-factor model, and (4) the time-varying-loadings four-factor model (the TVL4 model), in Panels (A)-(D), respectively. The constant-loadings models are estimated by OLS and the time-varying-loadings models are estimated by the Kalman Filter (with Maximum Likelihood Estimations).

[6] The composite pricing error suggested by Campbell and Vuolteenaho (2004) is $\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{\Omega}}^{-1}\hat{\boldsymbol{\alpha}}$. We take a squared root of the measure to express the pricing errors in levels.

Panel (A) shows that, among the 25 portfolios, seven portfolios have a pricing error that is significantly different from zero at the 5% level. Five of these seven portfolios are located either in the small (smallest *SZ*) portfolios quintile or in the growth (lowest *BM*) portfolios quintile. The findings are in line with previous studies [e.g., Petkova (2006) and Adrian and Franzoni (2005)] in that the returns of the small and the growth portfolios cannot be successfully explained by the Fama-French three-factor model.

Incorporating the *TERM* factor into the conventional Fama-French model, Panel (B) shows that the number of significant individual pricing errors is decreased from seven to two, one of which is located in the lowest *BM* quintile. For the aggregate pricing errors, the RMSE and the CPE are reduced from 0.153 % and 0.423% in Panel A to 0.115% and 0.373% in Panel (B), respectively. Since the *TERM* factor represents a proxy for the risk related to the innovations in the discount rates, the results confirm our conjectures that the returns of the small or growth portfolios are sensitive to such risk, and that the *TERM* factor documents the information about risk related to changes in the discount rates that the Fama-French factors cannot fully capture. Adding the *TERM* factor into the conventional Fama-French model can help to reduce the abnormal returns in the small and growth portfolios.

Panel (C) reports the pricing errors for the three-factor model with time-varying factor-loadings. Compared with Panel (A), even though those small/growth portfolios with significant pricing errors in Panel (A) still show significant pricing errors in Panel (C), those two portfolios with significant pricing error in the other quintiles do not show significant pricing errors in Panel (C). The overall performance of the time-varying loadings model is better than that of the constant-loadings model: both the RMSE and the CPE are reduced by about 10%. The better performance of the time-varying loadings model implies that the learning process mimicked by the Kalman Filter improves the accuracy of forecasts for risk loadings. This is consistent with the findings of Adrian and Franzoni (2005) in their time-varying beta model.

Comparing Panel (C) and Panel (D), which reports the pricing errors for the TVL4 model, we can see the importance of the *TERM* factor again. The performance of the TVL4 model is very impressive. The null hypothesis that the pricing error of individual portfolio is equal to zero can only be rejected at the 5% level in one growth portfolio. For the overall performance, the RMSE and the CPE are reduced by 57% and 50%, respectively, when the *TERM* factor is added to the time-varying loadings model.

Similarly, comparing Panel (D) and Panel (B), we also see an improvement by modeling the factor loadings as time-varying. Although the additional *TERM* factor conveys information beyond the Fama-French factors, the OLS regression is still not able to track the time-varying factor-loadings, which leads to imprecise estimates of the loadings. Under the framework of a learning process, the accuracy of the estimates of risk loadings improves as the Kalman Filter captures the dynamics of the risk loadings.

Finally, the improvement of the performances from the Fama-French model to the TVL4 model can be seen by comparing Panel (A) and Panel (D). The number of portfolios with significant pricing errors drops from seven to one. The TVL4 model reduces the aggregate pricing error in the Fama-French model by more than 50%.

V. Robustness Checks

The above results confirm our hypotheses that (1) the *TERM* factor contains information related to innovations to discount rates for which the Fama-French three factors cannot fully account; and that (2) the Kalman filter improves the accuracy of the estimation of factor-loadings since the learning process mimicked by the filter captures the dynamics of the factor-loadings that the common OLS estimation cannot.

However, we have only focused on the five-by-five *SZ*/*BM* double-sorted portfolios because these are where our motivation is derived from. These portfolios are also the most commonly used ones in the literature. Whether or not our TVP4 model derived from this particular portfolio design can still perform well in other samples, that is, the data mining problem, may be an issue of concerns. Therefore, it is necessary to investigate the performance of our model in other samples.

A. An alternative portfolio formation

Daniel and Titman (1997) argue that it could be wrongful to examine asset pricing models only within portfolios sorted by characteristics known to be related to average returns. Therefore, to check the importance of the two modifications we make on the three-factor model in general, we further evaluate the performances of the models in industry-sorted portfolios. Fama and French (1997) find that the risk loadings of the industry-sorted portfolios exhibit great time variation and are difficult to be estimated precisely. Therefore, it would be interesting to see whether our TVL4 model can solve this problem. We use the ten industry-sorted portfolios provided by Professor Kenneth R. French on his website. The industries are categorized by the firms' four-digit SIC codes at the time of the portfolio formation. Panel (A) of Table 4 shows the means and the standard deviations of these portfolios.

Panel (B) of Table 4 reports the pricing errors from those four models considered in this paper. In the constant-loadings three-factor model, statistically significant pricing errors are shown in the Consumer Durables and the Healthcare industries. Adding the *TERM* factor into the model does not improve the model like it does in the *SZ*/*BM* double-sorted portfolios. This is not too surprising because firms that are more sensitive to the interest rate risk do not particularly reside in any specific industry. However, the time-varying-loadings specification does improve the performance of the model. Especially, when combined with the addition of the *TERM* factor, our TVL4 model does not generate any statistically significant pricing error for any individual industry. The model's overall performance is also much better than the traditional Fama-French model. Both the RMSE and the CPE are reduced by more than 40%.[7]

B. An alternative sample period

Previous studies have intensively explored the post-1963:7 sample for the five by five *SZ/BM* double-sorted portfolios. This is because first of all, the book values for firms are not generally available in the pre-1963 COMPUSTAT dataset. Secondly, the COMPUSTAT has a serious selection bias prior to 1963, which are tilted toward big historically successful firms [see Fama and French (1992)]. Finally, the CAPM is usually found to fail in explaining cross-sectional returns, especially the book-to-market anomaly, in the post-1963:7 sample [see Campbell and Vuolteenaho (2004) and Adrian and Franzoni (2005)].

Since many researchers investigate the asset pricing model with the same dataset, data mining has become a potential problem. Ferson and Harvey (1999) suggest that out-of-sample studies may reduce the risk of data mining. A good example is Davis, Fama, and French (1998), who extend the analysis of the U.S. data back to 1929. Campbell and Vuolteenaho (2004) argue that pre-1963 sample provides an opportunity for an out-of-sample test because this sample is relatively untouched in comparison with the well mined post-1963 sample. Therefore, in this subsection, we experiment with the pre-1963:7 data of the five-by-five *SZ/BM* double-sorted

[7] We also experiment with 30 and 48 industry-sorted portfolios provided by Professor Kenneth R. French on his website. The results, not shown here but available upon requests, confirm the superiority of our TVL4 model. When the constant three-factor model is used, six of the 30 industry-sorted portfolios, and nine of the 48 industry-sorted portfolios, have a statistically significant pricing error. When the TVL4 model is used, there is no statistically significant pricing error existing in either set of portfolios. Both the RMSE and the CPE are reduced by more than 30%.

portfolios. Due to the availability of the bond return data, we can only extend our experiment back to 1953.

Table 5 shows the results of the five-by-five *SZ/BM* double-sorted portfolios for the period 1953:4-1963:6. Panel (A) shows the pricing errors from the Fama-French constant-loadings three-factor model and Panel (B) shows those from the TVL4 model. There are five individual pricing errors that are significantly different from zero in Panel (A). Note that these significant pricing errors are not observed particularly in the lowest *BM* or the smallest *SZ* portfolios. Although these observations are not where our original motivation is derived from, they can be explained by the selection bias (toward big historically successful firms) prior to 1963.[8] Nonetheless, our TVL4 model still eliminates all these pricing errors. The model does not produce any significant individual pricing errors for all 25 portfolios. Both the RMSE and the CPE are reduced by more than 80% from the Fama-French three-factor model to our TVL4 model.

C. An alternative factor

As a robustness check, we further compare the effect of the *TERM* factor to that of a popular factor that has been shown to explain abnormal stock returns in the literature. The momentum factor is the average of the returns on high prior-return portfolios minus the average of the returns on low prior-return portfolios. This factor is to pick up the short-run delayed price reactions to firm-specific information. Jegadeesh and Titman (1993) argue that the moment

[8] Small-size and low-book-to-market-ratio portfolios prior to 1963 likely include those successful firms independent of external financial conditions (interest rates). Furthermore, Campbell and Vuolteenaho (2004) suggest that the NYSE has very strict profitability requirements for a firm to be listed on the exchange in earlier years, and therefore growth portfolios tend to be consistently profitable and insensitive to external financing.

effect is distinct from the value effect captured by *BM*. Fama and French (1996) show that their three factors cannot successfully explain the returns of portfolios formed on short-term (two to 12 months) past returns. Carhart (1997) is the first to add the momentum factor into the Fama-French three-factor model. He finds that, compared to the Fama-French three-factor model, his four-factor model noticeably reduces the average pricing errors of portfolios sorted by past returns. Here we evaluate the performance of the momentum factor in the five-by-five *SZ*/*BM* double-sorted portfolios and compare it to the performance of the *TERM* factor in such portfolios.

Data on the momentum factor and the method of constructing it are provided by Professor Kenneth R. French on his website. Panel (A) of Table 6 shows the pricing errors from the constant-loadings four-factor model and Panel (B) shows those from the time-varying-loadings four-factor model. Both use the momentum factor as the fourth factor. Comparing Panel (A) of Table 6 and Panel (A) of Table 3, we can see that the addition of the momentum factor only reduces the number of pricing errors from seven to five; while Panel (B) of Table 3 shows that the addition of the *TERM* factor reduces the number of pricing errors from seven to two. Incorporating the time-varying-loadings specification, Panel (B) of Table 6 shows that there are still four portfolios with significant pricing errors when the momentum factor is used. Recall that in Panel (D) of Table 3, there is only one portfolio with a significant pricing error when the *TERM* factor is used. Comparing the RMSEs and the CPEs, the models with the *TERM* factor have smaller aggregate pricing errors than the models with the momentum factor. Therefore, the *TERM* factor outperforms the momentum factor in explaining the abnormal returns in the *SZ*/*BM* sorted portfolios.

V. Conclusion

Fama and French's three-factor model has produced an influential impact on the development of asset pricing models. However, empirical studies show that this model cannot fully capture the cross-sectional average stock returns in the *SZ*/*BM* double-sorted portfolios, especially in the small and the growth portfolios. The three-factor model generates unexplained pricing errors in several of these 25 portfolios. A special characteristic of small and growth firms is that they are more sensitive to interest rate risk. Therefore, we are motivated to add into the model the *TERM* factor, which proxies for risk caused by shifts in interest rates. In addition, since it has been shown in the literature that factor loadings exhibit time variations, we use the Kalman Filter procedure to estimate the movements of our factor loadings and to replicate investors' learning processes.

Using U.S. stock market data, we show that both modifications are essential to improving the performance of the three-factor model in the *SZ*/*BM* double-sorted portfolios. Especially, our time-varying-loadings four-factor (TVL4) model explains the cross-sectional returns without any statistically significant pricing error in 24 out of the 25 *SZ*/*BM* double-sorted portfolios. Our model also reduces the aggregate pricing errors generated by the three-factor model by more than 50%. These results confirm our hypotheses that (1) the *TERM* factor contains information related to innovations to discount rates that the Fama-French three factors cannot fully account for; and that (2) the Kalman filter improves the accuracy of the estimation of the loading because the learning process mimicked by the filter captures the dynamics of the factor loadings that the common OLS estimation cannot.

To check the robustness of our results, we further experiment with an alternative portfolio formation, with a different sample period, and with an alternative risk factor. We first evaluate the performance of the TVL4 model in industry-sorted portfolios. It is shown that no statistically significant pricing error is observed when the TVL4 model is used. The aggregate pricing error is also reduced significantly. The second experiment is an out-of-sample test suggested by Ferson and Harvey (1999). We check the performances of the TVL4 model on the five-by-five *SZ*/*BM* double-sorted portfolios for the sample period 1953:4-1963:6. The results show that the TVL4 model remarkably reduces both the individual and aggregate pricing errors in comparison with the Fama-French three-factor model. Finally, we replace the *TERM* factor with the momentum factor and evaluate the models in the *SZ*/*BM* double-sorted portfolios. The results show that, in contrast to the outstanding performance of the TVL4 model with the *TERM* factor, the TVL4 model with the momentum factor only moderately improves the Fama-French three-factor model.

The focus of this paper is to test whether the *TERM* factor and the time-varying-loadings specification are able to eliminate the pricing errors generated by the Fama-French three-factor model, especially in the small and the growth portfolios. We successfully prove that these two modifications improve the three-factor model in these portfolios. However, it is not proper to claim a full victory of our TVL4 model because it needs to be tested in other types of portfolio formations and/or in other samples. Due to the limitation on publicly available data, we only experiment with several industry-sorted portfolios and a different sample period. Further robustness checks, such as investigations of data from other countries, should be a topic of interest for future studies. Nonetheless, this paper provides a good starting point and shows that future studies should not ignore the effects of the interest-rate risk and the investors' learning process.

References:

Adrian, T. and F. Franzoni, 2004. "Learning about beta: a new look at CAPM tests." *Staff reports of Federal Reserve Bank of New York*, No.193.

Adrian, T. and F. Franzoni, 2005. "Learning about beta: time-varying factor loadings, expected returns, and the conditional CAPM." Working paper, HEC School of Management, Paris.

Aretz, K., S. Bartram, and P. Pope, 2005. "Macroeconomic risks and the Fama and French model." Working paper, Lancaster University, Department of Accounting and Finance.

Campbell, J. and R. Shiller, 1991. "Yield spreads and interest rate movements: a bird eye view." *Review of Economic Studies*, 58, 495-514.

Campbell, J. and T. Vuolteenaho, 2004. "Bad beta, Good beta." *American Economic Review*, Vol. 94, No. 5.

Carhart, M., 1997. "On persistence in mutual fund performance." *Journal of Finance*, 52, 57-82.

Cornell, B., 1999. "Risk, duration, and capital budgeting: new evidence on some old questions." *Journal of Business*, 72, 183-200.

Chen, N., R. Roll, and S. Ross, 1986. "Economic forces and the stock market." *Journal of Business*, 59, 383-403.

Chiao, C. and C. J. Hueng, 2005. "Overreaction effects independent of risk and characteristics: Evidence from the Japanese stock market." *Japan and the World Economy*, 17, 431-455.

Daniel, K. and S. Titman, 1997. "Evidence on the characteristics of cross sectional variation in stock returns." *Journal of Finance*, 52, 1-33.

Davis J., E. Fama, and K. French, 2000. "Characteristics, Covariances, and Average Returns: 1929 to 1997." *Journal of Finance*, 55, 389-406.

Diebold, F., G. Rudebusch, and B. Aruoba, 2006. "The macroeconomy and the yield curve: a dynamic latent factor approach." *Journal of Econometrics*, 131, 309-338.

Elton, E., M. Gruber, and C. Blake, 1995. "Fundamental Economic Variables, Expected Returns, and Bond Fund Performance." *Journal of Finance*, 50, 1229-1256.

Fama, E. and K. French, 1992. "The cross-section of expected stock returns." *Journal of Finance*, 47, 427-465.

Fama, E. and K. French, 1993. "Common risk factors in the returns on stocks and bonds." *Journal of Financial Economics*, 33, 3-56.

Fama, E. and K. French, 1996. "Multifactor explanations of asset pricing anomalies." *Journal of Finance*, 51, 55-84.

Fama, E. and K. French, 1997. "Industry costs of equity." *Journal of Financial Economics*, 43, 153-193.

Ferson, W. and C. Harvey, 1991. "The variation of economic risk premiums." *Journal of Political Economy*, 99, 385-415.

Ferson, W. and C. Harvey, 1993. "The risk of predictability of international equity returns." *Review of Financial Studies*, 6, 527-566.

Ferson, W. and C. Harvey, 1999. "Conditioning Variables and the Cross Section of Stock Returns." *Journal of Finance*, 54, 1325-1360.

Franzoni, F., 2006. "Where is beta going? The riskiness of value and small stocks." Working paper, HEC School of Management, Paris.

Hamilton, J., 1994. *Time Series Analysis*. New Jersey: Prentice Hall.

Harvey, C., 1989. "Time-varying conditional covariance in tests of asset pricing models." *Journal of Financial Economics,* 24, 289-317.

Jagannathan, R. and Z. Wang, 1996. "The conditional CAPM and the cross-section of expected returns." *Journal of Finance,* 51, 3-54.

Jegadeesh, N. and S. Titman, 1993. "Returns to buying winners and selling losers: implications for stock market efficiency." *Journal of Finance*, 48, 65-91.

Jensen, M., 1968. "The performance of mutual funds in the period of 1945-1954." *Journal of Finance*, 23, 389-416.

Kim, C. and C. Nelson, 1999. *State-space Models with Regime Switching: Classical and Gibbs-sampling Approaches with Applications*. Cambridge, Mass: MIT Press.

Perez-Quiros, G. and A. Timmermann, 2000. "Firm size and cyclical variations in stock returns." *Journal of Finance*, 55, 1229-1262.

Petkova, R., 2006. "Do the Fama-French factors proxy for innovations in predictive variables?" *Journal of Finance*, 61, 581-612.

Table 1

Means and standard deviations of the returns in the five-by-five *SZ*/*BM* double-sorted portfolios

Mean					
	Low (growth)	2	*BM*	4	High (value)
Small	0.490	1.166	1.253	1.458	1.530
2	0.762	1.075	1.282	1.377	1.471
SZ	0.832	1.160	1.130	1.267	1.456
4	0.986	0.971	1.208	1.273	1.314
Big	0.873	1.037	1.010	1.051	1.080
Standard deviation					
	Low (growth)	2	*BM*	4	High (value)
Small	8.407	7.161	6.091	5.620	5.956
2	7.656	6.200	5.384	5.196	5.794
SZ	7.067	5.606	5.038	4.800	5.436
4	6.304	5.376	4.987	4.723	5.440
Big	4.968	4.749	4.493	4.344	4.909

The monthly returns (in percentage) cover the period from July 1968 to December 2004. A portfolio's *BM* in year *t* is the sum of book equities of the firms in the portfolio for the fiscal year ending in year *t*-1 divided by the sum of their *SZ* in December of year *t*-1. A portfolio's *SZ* is the average of the logarithms of market equities of the firms in the portfolio at the end of June in year *t*. On the formation dates, all NYSE, AMEX, and NASDAQ stocks are sorted into five groups based on their *SZ*. The breakpoints are the NYSE market equity quintiles at the end of June in each year. These stocks are then divided independently into five equal *BM* groups. Twenty-five portfolios are constructed from the intersections of the five *SZ* and five *BM* groups. The average monthly return is calculated over the period from July in year *t* to June in year *t*+1. All stocks in a portfolio are value weighted on the formation date.

Table 2

Means and standard deviations of the factors (in percentage)

	Mean	Standard deviation
MKT	0.440	4.649
SMB	0.040	3.331
HML	0.520	3.097
TERM	1.610	1.319

MKT is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus one-month Treasury bill rate. *SMB* is the average return of the small-size portfolios minus the average return of the big-size portfolios. *HML* is the average return of the high book-to-market ratio portfolios minus the average return of the low book-to-market ratio portfolios. *TERM* is the yield spread between 10-year government bond and three-month Treasury bill.

Table 3

The pricing errors (in percentage) in the five-by-five *SZ*/*BM* double-sorted portfolios

Panel (A): Constant-loadings three-factor model					
	Low (growth)	2	*BM*	4	High (value)
Small	**-0.506*** **(-4.348)**	0.039 (0.465)	0.061 (0.911)	**0.220*** **(3.310)**	0.137 (1.930)
2	**-0.171*** **(-2.108)**	-0.088 (-1.176)	0.049 (0.717)	0.077 (1.166)	0.017 (0.250)
SZ	-0.028 (-0.363)	0.011 (0.132)	-0.123 (-1.542)	-0.043 (-0.577)	0.003 (0.030)
4	**0.180*** **(2.350)**	**-0.183*** **(-2.083)**	-0.025 (-0.285)	0.003 (0.036)	-0.123 (-1.244)
Big	**0.184*** **(2.978)**	0.031 (0.418)	-0.037 (-0.431)	-0.144 (-1.932)	**-0.232*** **(-2.085)**
	RMSE	0.153		CPE	0.423
Panel (B): Constant-loadings four-factor model					
	Low (growth)	2	*BM*	4	High (value)
Small	-0.081 (-0.456)	0.195 (1.517)	0.075 (0.726)	**0.213*** **(2.070)**	-0.008 (-0.073)
2	0.092 (0.746)	0.083 (0.724)	-0.020 (-0.195)	0.076 (0.749)	0.128 (1.221)
SZ	-0.002 (-0.016)	-0.002 (-0.018)	-0.111 (-0.903)	-0.065 (-0.566)	0.184 (1.386)
4	**0.264*** **(2.244)**	-0.117 (-0.861)	0.108 (0.811)	0.069 (0.572)	-0.027 (-0.174)
Big	0.141 (1.477)	0.093 (0.819)	0.052 (0.397)	0.090 (0.790)	-0.087 (-0.505)
	RMSE	0.115		CPE	0.373

The numbers in parentheses are *t*-statistics. The asterisk indicates the statistical significance at the 5% level. RMSE is the root-mean-squared-error and CPE is the composite pricing error. Both are measures of the aggregate pricing errors and are shown in equations (12) and (13), respectively.

Table 3 (continued)

Panel (C): Time-varying-loadings three-factor model					
	Low (growth)	2	*BM*	4	High (value)
Small	**-0.475*** **(-4.234)**	0.032 (0.399)	0.060 (0.953)	**0.216*** **(3.446)**	**0.138*** **(2.000)**
2	-0.113 (-1.545)	-0.023 (-0.358)	0.061 (1.020)	0.077 (1.274)	0.024 (0.381)
SZ	-0.006 (-0.084)	0.050 (0.664)	-0.070 (-1.035)	-0.014 (-0.213)	0.042 (0.528)
4	**0.208*** **(2.825)**	-0.086 (-1.174)	-0.001 (-0.011)	0.012 (0.160)	-0.104 (-0.119)
Big	**0.156*** **(2.690)**	0.062 (0.969)	-0.012 (-0.152)	-0.102 (-1.446)	-0.197 (-1.847)
	RMSE	0.137		CPE	0.390
Panel (D): Time-varying-loading four-factor model					
	Low (growth)	2	*BM*	4	High (value)
Small	-0.017 (-0.151)	0.092 (1.160)	0.057 (0.904)	0.096 (1.505)	0.019 (0.280)
2	0.035 (0.484)	0.007 (0.106)	-0.069 (-1.170)	-0.015 (-0.256)	0.070 (1.102)
SZ	0.003 (0.038)	-0.055 (-0.732)	-0.084 (-1.242)	-0.062 (-0.946)	0.062 (0.791)
4	**0.162*** **(2.193)**	-0.038 (-0.523)	-0.012 (-0.166)	-0.006 (-0.077)	-0.054 (-0.585)
Big	0.002 (0.034)	0.033 (0.508)	0.026 (0.323)	0.041 (0.589)	-0.025 (-0.237)
	RMSE	0.058		CPE	0.197

The numbers in parentheses are t-statistics. The asterisk indicates the statistical significance at the 5% level. RMSE is the root-mean-squared-error and CPE is the composite pricing error. Both are measures of the aggregate pricing errors and are shown in equations (12) and (13), respectively.

Table 4

The results for the ten industry-sorted portfolios

Industry	Sample Statistics		Estimated Pricing Errors			
	Mean	Standard deviation	Constant-loadings three-factor model	Constant-loadings four-factor model	Time-varying-loadings three-factor model	Time-varying-loadings four-factor model
(1)	1.102	4.681	0.088 (0.687)	0.088 (0.446)	0.105 (0.966)	-0.045 (-0.415)
(2)	0.909	5.857	**-0.373*** **(-2.270)**	**-0.613*** **(-2.420)**	-0.288 (-1.802)	-0.188 (-1.175)
(3)	0.929	4.998	-0.143 (-1.522)	-0.250 (-1.728)	-0.094 (-1.132)	-0.099 (-1.198)
(4)	1.115	5.438	0.107 (0.546)	0.201 (0.666)	0.122 (0.656)	0.148 (0.796)
(5)	0.920	7.070	0.258 (1.640)	0.028 (0.115)	0.128 (0.892)	-0.082 (-0.575)
(6)	0.960	4.848	0.065 (0.435)	0.166 (0.726)	-0.008 (-0.056)	0.016 (0.113)
(7)	1.021	5.543	-0.047 (-0.354)	0.033 (0.159)	-0.010 (-0.088)	-0.082 (-0.692)
(8)	1.100	5.203	**0.450*** **(2.899)**	**0.850*** **(3.574)**	**0.405*** **(2.996)**	0.249 (1.836)
(9)	0.900	4.245	-0.184 (-1.274)	-0.167 (-0.749)	-0.134 (-0.963)	-0.039 (-0.288)
(10)	1.063	5.245	-0.084 (-1.042)	-0.025 (-0.197)	-0.016 (-0.224)	0.012 (0.170)
RMSE			0.223	0.356	0.178	0.121
CPE			0.232	0.369	0.194	0.134

The industries are: (1) Consumer Nondurables; (2) Consumer Durables; (3) Manufacturing; (4) Energy; (5) High Technology; (6) Telecommunication; (7) Wholesale and Retail; (8) Healthcare; (9) Utilities; and (10) Other Industries (firms that are not included in the first nine industries). The numbers in parentheses are t-statistics. The asterisk indicates the statistical significance at the 5% level. RMSE is the root-mean-squared-error and CPE is the composite pricing error. Both are measures of the aggregate pricing errors and are shown in equations (12) and (13), respectively.

Table 5

The Pricing Errors (in percentage) in the Five-by-five *SZ*/*BM* Double-Sorted Portfolios

(April 1953 to June 1963)

Panel (A): Constant-loadings three-factor model					
	Low (growth)	2	*BM*	4	High (value)
Small	-0.217 (0.529)	-0.213 (0.728)	-0.088 (0.424)	**0.217*** **(2.009)**	0.159 (1.424)
2	-0.333 (-1.603)	0.167 (0.967)	-0.001 (-0.005)	0.022 (0.200)	**0.238*** **(2.007)**
SZ	0.082 (0.604)	0.091 (0.714)	-0.008 (-0.071)	0.105 (0.897)	-0.246 (-1.658)
4	-0.074 (-0.635)	0.131 (1.284)	**0.255*** **(2.167)**	-0.121 (-0.902)	-0.384 (-1.872)
Big	-0.092 (-1.147)	-0.034 (-0.286)	**0.434*** **(2.946)**	**-0.391*** **(-2.747)**	-0.310 (-1.735)
	RMSE	0.362		CPE	1.193
Panel (B): Time-varying-loading four-factor model					
	Low (growth)	2	*BM*	4	High (value)
Small	-0.025 (-0.065)	-0.132 (-0.472)	-0.011 (-0.059)	0.058 (0.575)	0.103 (0.954)
2	-0.083 (-0.417)	0.035 (0.210)	0.030 (0.281)	0.062 (0.601)	0.022 (0.194)
SZ	0.027 (0.211)	-0.090 (-0.750)	-0.007 (-0.063)	-0.029 (-0.261)	0.072 (0.505)
4	-0.017 (-0.157)	0.044 (0.448)	-0.011 (-0.099)	0.027 (0.207)	-0.037 (-0.192)
Big	-0.009 (-0.116)	0.007 (0.059)	0.116 (0.810)	-0.050 (-0.380)	-0.128 (-0.744)
	RMSE	0.062		CPE	0.221

The numbers in parentheses are t-statistics. The asterisk indicates the statistical significance at the 5% level. RMSE is the root-mean-squared-error and CPE is the composite pricing error. Both are measures of the aggregate pricing errors and are shown in equations (12) and (13), respectively.

Table 6

The pricing errors in the four-factor models with the momentum factor as the fourth factor

Panel (A): Constant-loadings four-factor model (including the momentum factor)					
	Low (growth)	2	*BM*	4	High (value)
Small	**-0.447*** **(-3.751)**	0.045 (0.521)	0.058 (0.846)	**0.218*** **(3.177)**	**0.180*** **(2.497)**
2	-0.109 (-1.326)	-0.023 (-0.297)	0.085 (1.224)	0.092 (1.364)	0.025 (0.362)
SZ	0.016 (0.202)	0.050 (0.569)	-0.075 (-0.914)	-0.010 (-0.129)	0.077 (0.876)
4	**0.170*** **(2.161)**	-0.133 (-1.485)	0.036 (0.405)	0.053 (0.670)	-0.051 (-0.507)
Big	**0.201*** **(3.176)**	0.038 (0.503)	-0.058 (-0.666)	-0.087 (-1.148)	-0.173 (-1.516)
	RMSE	0.136		CPE	0.395
Panel (B): Time-varying-loadings four-factor model (including the momentum factor)					
	Low (growth)	2	*BM*	4	High (value)
Small	**-0.436*** **(-4.044)**	0.046 (0.591)	0.055 (0.888)	**0.171*** **(2.734)**	0.127 (1.864)
2	-0.067 (-0.917)	0.022 (0.349)	0.062 (1.046)	0.065 (1.080)	0.000 (-0.001)
SZ	0.014 (0.191)	0.063 (0.845)	-0.026 (-0.382)	0.016 (0.255)	0.011 (0.141)
4	**0.188*** **(2.607)**	0.026 (0.362)	0.095 (1.294)	0.048 (0.673)	-0.082 (-0.891)
Big	**0.118*** **(2.145)**	0.096 (1.508)	-0.004 (-0.045)	-0.049 (-0.689)	-0.129 (-1.223)
	RMSE	0.119		CPE	0.342

These four-factor models include the Fama-French factors (MKT, SMB, and HML) as well as the momentum factor. The numbers in parentheses are t-statistics. The asterisk indicates the statistical significance at the 5% level. RMSE is the root-mean-squared-error and CPE is the composite pricing error. Both are measures of the aggregate pricing errors and are shown in equations (12) and (13), respectively.