The Demand for Money in an Open Economy: Some Evidence for Canada

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Abstract

This paper presents estimates of an open-economy money demand function. A cash-in-advance model is constructed to motivate the specification of the demand for money in an open economy. Special attention is paid to unit root nonstationarity in the time series data. Econometric techniques designed to accommodate integrated and cointegrated variables are used to analyze the data. Canadian quarterly data for the period 1973:2-1990:1 are used. The results suggest that changes in the foreign interest rates and exchange rates affect the demand for Canadian real Divisia money.

Resumen

Este artículo estima la función de demanda para moneda en una economía abierta a través de un modelo de efectivo-en-avance. El enfoque econometrica es el análisis de una raíz unitaria en la serie de tiempo de la base de datos y su implicación para la no-estacionaridad. El modelo está estimado usando datos Canadienses de 1973:2-1990:1 con una técnica que combina variables integrados y co-integrados. Los resultados indican que la demanda Canadiense se esta determinado por el cambio en los intereses extranjeros y cambios el tipo de cambio.
1. Introduction

Recently, the effectiveness of monetary and fiscal policies in an open economy under the floating exchange rate system has become a popular subject. Some proponents of monetary policies believe that the adjustment mechanism of the floating exchange rates can isolate economies from disturbances which originate from abroad and then provide the policymakers greater control in using stabilization policies. However, this point has been challenged by many studies regarding money demand in an open economy.

The elasticity of money demand is one of the most important determinants of the effectiveness of monetary policies. Traditional studies on money demand have often ignored the influence of foreign monetary developments. In an increasingly interdependent world, however, monetary developments in other countries may influence the domestic demand for real cash balances. If people's holdings of money change with foreign monetary developments, such as the foreign interest rate and the exchange rate, the isolation mechanism of the floating exchange rate system will break down.

A number of recent studies [e.g. Hamburger (1977), Miles (1978), Arango and Nadiri (1981), Bordo and Choudhri (1982), and Bahmani-Oskooee (1991)] have attempted to account for foreign monetary developments in addition to the traditional variables (real income and the domestic interest rate) in their money demand functions. However, they either misspecified their models or used the data of the fixed exchange rate period of pre-1973. The relative stability of the exchange rate markets under the fixed exchange rate system may reduce the effects of the foreign monetary variables on the demand for money.

The purpose of this paper is to determine whether foreign variables have significant effects on the domestic money demand during the period of the floating exchange rate system. A cash-in-advance model in an open economy framework is constructed to motivate the specification of the demand for money. The model includes both the foreign interest rate and the exchange rate in addition to the domestic interest rate and real income in the money demand function.

The national quarterly time series data of Canada for the time period of 1973:2 to 1990:1 are
analyzed, with a total of sixty-eight observations. Canada is chosen because its international trade is more than one half of its GNP and possesses the characteristics of a small open economy which takes foreign variables as given, as assumed in our model. Analysis of the Canadian data also allows us to compare results to similar studies [e.g. Arango and Nadiri (1981) and Bordo and Choudhri (1982)]. This paper gives special attention to nonstationarity in the macroeconomic time series data using econometric techniques designed to accommodate nonstationary data.

The outline of this paper is as follows: Section 2 discusses the theoretical foundation of constructing the money demand function in an open economy. The empirical results are presented in Section 3. Conclusions are set forth in Section 4.

2. Theoretical Foundation

Traditional studies on money demand only concentrated on the domestic interest rate and real income. The relation between money demand and the exchange rate was originally raised by Mundell (1963). He conjectured that demand for money could depend on the exchange rate in addition to the domestic interest rate and the level of income.

Hamburger (1977), on the other hand, analyzed different measures of foreign interest rates to see whether they should also be considered as opportunity costs of holding domestic money. He concluded that "[the domestic interest rate] and [the foreign interest rate] generally move together, . . . and when they do it is [the domestic interest rate] which determines the amount of money that is held". As a result, "if [the domestic interest rate] and [the foreign interest rate] are included in the same equation, only the coefficient of the former remains statistically significant" (p.31).

Arango and Nadiri (1981) included both the exchange rate and the foreign interest rate in addition to the traditional variables in the money demand function for Canada, Germany, the United Kingdom, and the United States. They found that "the coefficients of the level of foreign exchange rate . . . are statistically
insignificant in each regression. This suggests that . . . changes in the exchange rate do not appear to be very important in affecting demand for real cash balances in any of the countries" (pp.76-79).

There exists a flaw in both Hamburger's and Arango and Nadiri's papers. They both used the data of the fixed exchange rate period of pre-1973, while the floating exchange rate system is prevalent after 1973. The insignificance of the coefficients of the foreign monetary variables in their results may be due to the relative stability of the exchange markets under the fixed exchange rate system.

Bahmani-Oskooee (1991) eliminated this problem by using the United Kingdom data over the 1973-1987 period to estimate the money demand function. But surprisingly, he cited Hamburger's result and also excluded the foreign interest rate from his model. Bahmani-Oskooee concluded that during the period which the exchange rates are floating, the exchange rate should be included in the money demand function.

Another criticism of many open-economy money demand studies, such as those mentioned above, is that they typically did not provide a theoretical model to justify the specifications of their empirical money demand functions. Exceptions are studies by Miles (1978) and Bordo and Choudhri (1982). Miles used a CES production function for money service to derive the demand for domestic money relative to foreign money. However, as pointed out by Bordo and Choudri, his model is misspecified due to the omission of domestic interest rates and income, which are significantly different from zero and render insignificant the coefficient in his regression.

As an alternative to Miles' approach, Bordo and Choudri derived the demand for money from a money-in-the-utility-function model. However, they eliminated the effects of the exchange rate in the theoretical model by imposing the perfect interest rate arbitrage condition, which is not supported by many empirical works. In their empirical analysis, they imposed the condition again and ignored the effects of the foreign interest rates.

To motivate the specification of the demand for money in an open economy, this paper constructs a cash-in-advance model in an open economy framework. This model has several advantages. First, many
economists have suggested explicitly modelling the liquidity services provided by money through the agent's budget constraint instead of the utility function because it is the service that provides utility to agents rather than money itself.

Secondly, this model provides a more general specification which includes the foreign interest rate and the exchange rate in addition to real income and the domestic interest rate in the money demand function. The perfect interest rate arbitrage condition is not assumed in the derivation of the money demand function. In addition, it is the level, not the expected change rate, of the exchange rate that enters into the money demand function. This avoids the errors in approximating the expected changes in exchange rates.

Finally, and most importantly, this model allows us to sign the effects of the interest rates on the demand for money by doing comparative statics.

Specifically, consider a two-country world. There are two goods, two monies, and two bonds in which there is one domestic and one foreign of each. Both countries are inhabited by identical, infinitely lived individuals. The representative agent maximizes his multiperiod utility:

\[
P = u(c_t, c_t^*, L_t) + \sum_{j=1}^{\infty} \beta^j E_t[u(c_{t+j}, c_{t+j}^*, L_{t+j})],
\]

where \(c_t\) and \(c_t^*\) are home-country real consumption of domestic and foreign goods, respectively; \(L_t\) is leisure, \(\beta \in (0,1)\) is the constant discount rate, and \(E_t\) denotes the expectation conditional on information at time \(t\). The utility function \(u()\) is assumed to be twice continuously differentiable, strictly concave, and to satisfy the Inada conditions.

The representative agent holds four assets: domestic and foreign money, and domestic and foreign bonds which pay a nominal interest rate of \(r_t\) and \(r_t^*\), respectively. The agent receives real endowment in the amount \(w_t\) and divides his wealth between money and bonds. The agent's budget constraint for period \(t\) may be written as:
(2.2) \[ M_t + B_t + e_t M_t^* + e_t B_t^* = M_{t-1} + (1 + r_t) B_{t-1} + e_t M_{t-1}^* + e_t (1 + r_t^*) B_{t-1}^* + P_t (w_t - e_t) - e_t P_t^* c_t, \]

where \( M_t \) and \( M_t^* \) are holdings of domestic and foreign money, respectively; \( B_t \) and \( B_t^* \) are holdings of domestic and foreign bonds, respectively; \( r_t \) and \( r_t^* \) are domestic and foreign nominal interest rates, respectively; \( P_t \) and \( P_t^* \) are domestic and foreign price levels, respectively; and \( e_t \) is the nominal exchange rate defined as units of domestic currency per unit of foreign currency.

Divided by \( P_t \), (2.2) can be written as:

(2.3) \[ m_t + b_t + q_t m_t^* + q_t b_t^* = (1 + \pi_{t-1})^{-1} m_{t-1} + (1 + \pi_{t-1})^{-1} (1 + r_t) B_{t-1} + q_t (1 + \pi_{t-1}^*)^{-1} m_{t-1}^* + q_t (1 + \pi_{t-1}^*)^{-1} (1 + r_t^*) b_{t-1}^* + w_t - e_t - q_t e_t^*, \]

where \( m_t \) and \( m_t^* \) are real holdings of domestic and foreign money, respectively; \( b_t \) and \( b_t^* \) are real holdings of domestic and foreign bonds, respectively; \( \pi_t \) and \( \pi_t^* \) are domestic and foreign inflation rates, respectively; and \( q_t = e_t P_t^*/P_t \) is the real exchange rate.

Following Stockman (1980), Lucas (1982), and Guidotti (1989), it is assumed that domestic and foreign goods have to be purchased with domestic and foreign currencies, respectively. Given this assumption, the agent is subject to two cash-in-advance constraints:

(2.4) \[ c_t = N_t m_t \quad \text{and} \quad c_t^* = N_t^* m_t^*. \]

where \( N_t \) and \( N_t^* \) denote the number of times the agent withdraws domestic and foreign currencies, respectively. Leisure is forgone in withdrawing both domestic and foreign currencies according to the following technology:

(2.5) \[ 1 - L_t = f(N_t) + f^*(N_t^*), \]

where \( f' > 0, f^*' > 0, f'' < 0 \), and \( f'''' < 0 \).
Using (2.4) and (2.5), (2.1) can be rewritten as the following indirect utility function:

\[
W = u(c_t, c_t^*, 1 - f^*(\frac{c_t}{m_t}) - f^*(\frac{c_t^*}{m_t^*})), \quad \sum_{j=1}^\infty \beta^j E_t \{ u(c_{t-j}, c_{t-j}^*, 1 - f^*(\frac{c_{t-j}}{m_{t-j}}) - f^*(\frac{c_{t-j}^*}{m_{t-j}^*}) \} \\
= U(c_t, c_t^*, m_t, m_t^*) + \sum_{j=1}^\infty \beta^j E_t \{ U(c_{t-j}, c_{t-j}^*, m_{t-j}, m_{t-j}^*) \},
\]

where \( U(c_t, c_t^*, m_t, m_t^*) = u(c_t) - f(c_t, m_t) - u(c_t^*) + f(c_t^*, m_t^*) \).

Given the objective function (2.6), subject to the budget constraint (2.3), the first-order conditions necessary for optimality of the agent's choices imply:

\[
\frac{U_t^*(c_t, c_t^*, m_t, m_t^*)}{U_t(c_t, c_t^*, m_t, m_t^*)} = 1 - \frac{1}{1 + r_t},
\]

\[
\frac{U_m^*(c_t, c_t^*, m_t, m_t^*)}{U_m^*(c_t, c_t^*, m_t, m_t^*)} = 1 - \frac{1}{1 + r_t^*},
\]

\[
\frac{U_c^*(c_t, c_t^*, m_t, m_t^*)}{U_c^*(c_t, c_t^*, m_t, m_t^*)} = \frac{1}{q_t}.
\]

Equations (2.7) and (2.8) state that the marginal rate of substitution between real cash balances and consumption equals the opportunity cost of holding money. Equation (2.9) says that the marginal rate of substitution between domestic and foreign goods equals their relative price.

Since both the indirect utility function and the budget constraint are twice continuously differentiable and the optimal solutions exist, according to the implicit function theorem, this model implies a relation that is similar to those normally described as money demand functions. It can be shown that \( m_t \) and \( m_t^* \) can be expressed as functions of \( c_t, c_t^*, r_t, r_t^*, \) and \( q_t \). In particular, the domestic money demand function in an open economy takes the form:
Denoted by $y_t$ and $y_t^*$ are the domestic and foreign real output. In the symmetric world, equilibrium in the goods market implies: $c_t = y_t/2$ and $c_t^* = y_t^*/2$. Therefore (2.9) can be rewritten as:

\[ \frac{M_t}{P_t} = F(c_t, c_t^*, r_t, r_t^*, q_t). \]

Another important implication from the model is that the effects of an increase in the interest rates on the demand for domestic money can be signed by doing comparative statics on (2.7)-(2.9). Specifically, $\frac{M_m}{M_r} < 0$ and $\frac{M_m}{M_r^*} > 0$. That is, an increase in the domestic interest rate raises the opportunity cost of holding domestic money and then lowers the holding of money. On the other hand, since domestic and foreign moneys are imperfect substitutes precisely through the fact that withdrawals of both currencies require time, when the opportunity cost of holding foreign currency increases, it is optimal to increase the number of foreign currency withdrawals and raise the holding of domestic money.

However, the sign of the effect of the real exchange rate on the demand for domestic money can not be determined. It can be positive or negative. A depreciation of the domestic currency decreases the demand for domestic money. However, it also reduces the rate of return to foreign residents of domestic bonds and raises their demand for domestic money.

3. Empirical Analysis

3.1 Model and Data

As in many open-economy money demand studies, the foreign real income $y^*$ is suppressed from the empirical model. Assuming loglinearity, the model of money demand including real income, the domestic interest rate, the foreign interest rate, and the exchange rate is specified as
\[
\ln \frac{M_t}{P_t} = \$_0 + \$_1 \ln y_t + \$_2 \ln r_t + \$_3 \ln r_t^* + \$_4 \ln q_t + u_t,
\]

\[u_t = D u_{t-1} + e_t,\]

where u's are error terms and e's are iid. The coefficients $\$_i (i=1,\ldots,4)$ are the elasticities of money demand with respect to those explanatory variables, and $D$ is the population first autocorrelation coefficient of the disturbances $u_t$ and also of the dependent variable $\ln \frac{M_t}{P_t}$.

Canadian quarterly data for five variables are used in the following analysis for the period from 1973:2 to 1990:1 with a total of sixty-eight observations. All data, except for the nominal money stock, are taken from *International Financial Statistics* (IFS) CD ROM database of International Monetary Fund (IMF).

Previous studies on money demand often used the narrowly defined money stock (M1) as the desired nominal money balance. Recent deregulation of the financial market has led to the use of a broader aggregate, M2. However, these official monetary aggregates constructed by simple-sum aggregation have been criticized for assuming that all monetary components are perfect substitutes. Most of the short-term instruments included in these measures are actually poor substitutes for each other. Therefore, methods of weighting different components of money by the degree of moneyness have been widely used.

The most well-known index of monetary aggregate is the Divisia money. This index weights the growth of each asset using a formula based upon its user cost, which is designed to reflect its liquidity in making transactions. This paper uses Canadian Divisia M2, which is a broader measure and usually used for policy purposes, provided by Fisher (1996). The real money balance is then defined as Divisia M2 divided by the Consumer Price Index (CPI, 1990=100), where the CPI is from IFS line 64.

Real income (Y) is defined as real GDP at 1990 prices and is from IFS line 99b. The domestic nominal interest rate (r) is defined as the short-term interest rate and the three-month treasury-bill rate (IFS line 60c) is used.

Since over two-thirds of Canada's international trade is conducted with the United States, the U.S.
T-Bill rates and the Canadian dollar/U.S. dollar exchange rates (IFS line ae) are used as the foreign interest rate ($r^*$) and the nominal exchange rate, respectively. The real exchange rate ($q$) is defined as the nominal exchange rate multiplied by the U.S. CPI and divided by the Canadian CPI.

Log levels of all variables are plotted in Figures 1 to 5 and their means and standard deviations are shown in the top half of Table I. The bottom half of Table I reports the means and standard deviations of log differences of the variables.

3.2 Unit Root Tests

Before analyzing the models, I identify the order of integration maintained by each of the time series in this paper by using Phillips-Perron $Z_D$ and $Z_t$ tests and the augmented Dickey-Fuller (D-F) $D$ tests. The advantage of these tests is that they allow for serially correlated observations. The details of these tests are described in Hamilton (1994, ch.17).

Different regressions apply to different time series. Real money and real income would be expected to exhibit a persistent upward trend due to a growing population and technological improvements. However, we do not know whether this trend arises from the positive drift term of a random walk or from a stationary process with a deterministic time trend. Therefore the null hypothesis is:

\[
X = \ln y, \ln m
\]

(Phillips-Perron Tests) \[ X_t = \mu + X_{t-1} + u_t \]

(Augmented D-F Test) \[ X_t = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots + \varepsilon_p + X_{t-p} + \mu + X_{t-1} + u_t. \]

The alternative hypothesis is:

(Phillips-Perron Tests) \[ X_t = \mu + DX_{t-1} + \mu_t + u_t \]

$D \neq 1$,
(Augmented D-F Test) \( X_t = \epsilon_1 X_{t-1} + \epsilon_2 X_{t-2} + \ldots + \epsilon_p X_{t-k} + D X_{t-1} + *t + u_t \).

On the other hand, there is no economic theory which suggests that the interest rates and the exchange rates should exhibit a deterministic time trend. Therefore, a natural null hypothesis is that the true model is a random walk without trend:

\[
X = \ln r, \ln r^*, \ln q
\]

(Phillips-Perron Tests) \( X_t = X_{t-1} + u_t \),

(Augmented D-F Test) \( X_t = \epsilon_1 X_{t-1} + \epsilon_2 X_{t-2} + \ldots + \epsilon_p X_{t-k} + X_{t-1} + u_t \).

Since these series obviously should have a positive mean, a plausible alternative hypothesis would be a stationary process including a constant term:

(Phillips-Perron Tests) \( X_t = " + DX_{t-1} + u_t \),

(Augmented D-F Test) \( X_t = \epsilon_1 X_{t-1} + \epsilon_2 X_{t-2} + \ldots + \epsilon_p X_{t-k} + " + DX_{t-1} + u_t \).

In the Phillips-Perron \( Z_D \) and \( Z_t \) tests, the "lag truncation parameter" or "bandwidth" in calculating weighted sums of estimated autocovariances of cross products of residuals is calculated from the formula proposed by Newey and West (1994). For the choice of the order of autoregression \( k \) in the augmented Dickey-Fuller tests, Ng and Perron (1995) have found that the lag length has important size and power implications. They show that a data-dependent rule that takes sample information into account is superior to a deterministic rule. In addition, they use simulations to show the distinctive behaviors of three data-dependent rules in finite samples: the Akaike Information Criterion (AIC), Schwartz's (1978) criterion, and methods using sequential testing for the significance of coefficients on additional lags. For small samples, they find that information-based rules such as the AIC and the Schwartz criterion are associated with larger size distortions. The sequential testing methods, on the other hand, are associated with power loss.

Therefore, to show that the criteria for choosing lag length do not affect inferences, this paper uses the AIC, the Schwartz criterion, and a sequence of 5% \( t \) tests to choose \( k \) for each series. The lag lengths are assumed to have an upper bound 4 and a lower bound 0.
The results of the unit root tests are shown in Table II.a and Table II.b. Table II.a shows the test statistics of the log level of each series and Table II.b shows those of the log difference of each series. All the statistics in Table II.a are insignificant at the 5% level and almost all the statistics in Table II.b are significant at the 5% level. The results indicate that all the variables are better modeled as I(1) than as I(0).

<< Insert Tables II.a and II.b about here >>

3.3 Methodology

When constructing a model where all the variables are integrated of order one and no lags of the dependent variable are included, one should be concerned about the possibility of a spurious regression. A spurious regression arises if the error term in the regression is I(1), that is, if $D = 1$ in equation (3.1). In this case, the OLS estimates in a spurious regression will not be consistent and arbitrarily different large samples will have randomly differing estimates.

The classical approach to dealing with the spurious regression is to difference the nonstationary variables before estimating regression. Once all series have been transformed to stationarity, the usual regression models may be specified and the standard asymptotic results apply. However, if the error term in a regression containing unit root processes is I(0), that is, if the I(1) variables are cointegrated, the dynamic relation among these variables will be misspecified if one simply differences the variables.

The popular ways to test for cointegration are the residual-based cointegration tests suggested by Phillips and Ouliaris (1990) and Hansen (1992). These tests, however, are based on the assumption that there is a unique cointegrating relation in which the dependent variable is included and its coefficient is normalized to be one. An alternative approach, which can be used to test the numbers of cointegrating relations without the above assumption, is the full-information maximum likelihood (FIML) ratio test proposed by Johansen (1988).
The key statistic in this approach is the likelihood ratio: 

\[- T \ln(1 - 8_i) \]

where \( T \) is the number of observations and \( 8_i \) is the \( i \)th largest normalized eigenvalue of the matrix \( E - v_1 v \) \( E \) \( v_1 u \) \( E \) \( u v \). The variable \( E x z \) denotes the estimated variance-covariance matrix of \( x \) and \( z \). Assume that the (5 x 1) vector \( y_t = (\ln m, \ln y_t, \ln r_t, \ln r_t, \ln q_t) \)' can be characterized by a VAR(P) in levels. The variable \( v \) is the residual from the regression of \( y_{t,1} \) on \( (1) y_{t,1} y_{t,2} y_{t,3} \ldots y_{t,p+1} \), and \( u \) is from the regression of \( y_t \) on \( (1) y_{t,1} y_{t,2} y_{t,3} \ldots y_{t,p+1} \). Johansen and Juselius (1990) demonstrated that statistical tests based on cointegration vectors may depend on whether any of the I(1) variables exhibit deterministic time trends. I apply the approach that allows for possible deterministic time trends. For more detail about the FIML tests, see Hamilton (1994, ch.20).

Another advantage of using the FIML tests is that it allows us to obtain both the long-run and the short-run effects of the explanatory variables on the dependent variable by way of an error-correction representation. Specifically, assume that the variables in \( y \) are cointegrated. The following Error-Correction Model (ECM) applies [see Hamilton (1994, pp. 579-581)]:

\[
\Delta y_t = \alpha + \zeta_1 y_{t-1} + \zeta_2 \Delta y_{t-2} + \ldots + \zeta_{p-1} \Delta y_{t-p+1} - B A' y_{t-1} + \epsilon_t,
\]

where \( \epsilon, \ldots, \epsilon \) and \( B \) are constant vectors, and \( A \) is the (h x 5) matrix composed of h cointegrating vectors. Note that \( y_{t,i} \) \((i=0, \ldots, p-1)\) and \( A' y_{t,1} \) are all I(0).

The first equation of the error-correction model (3.2) can be expressed as:

\[
\begin{align*}
\Delta \ln m_t &= \alpha_1 + \zeta_{111} \Delta \ln m_{t-1} + \zeta_{122} \Delta \ln y_{t-1} + \zeta_{113} \Delta \ln r_{t-1} + \zeta_{114} \Delta \ln r_{t-1} + \zeta_{115} \Delta \ln q_{t-1} + \\
&+ \zeta_{121} \Delta \ln m_{t-2} + \zeta_{122} \Delta \ln y_{t-2} + \zeta_{123} \Delta \ln r_{t-2} + \zeta_{124} \Delta \ln r_{t-2} + \zeta_{125} \Delta \ln q_{t-2} + \\
&+ \ldots + \\
&+ \zeta_{1(p-1)} \Delta \ln m_{t-p+1} + \zeta_{1(p-2)} \Delta \ln y_{t-p+1} + \zeta_{1(p-1)} \Delta \ln r_{t-p+1} + \zeta_{1(p-2)} \Delta \ln r_{t-p+1} + \zeta_{1(p-1)} \Delta \ln q_{t-p+1} + \\
&- B A' y_{t-1} + \epsilon_t,
\end{align*}
\]

where \( \alpha, \ldots, \alpha \) and \( B \) are constant vectors, and \( A \) is the (h x 5) matrix composed of h cointegrating vectors.
The linear combination in the error-correction term \((A'y_{t-1})\) may be interpreted as a long-run equilibrium demand for the real money balance. Therefore \(B\) in equation (3.3) can be interpreted as the effect of the short-run deviation from the money demand equilibrium, and the coefficients \(\cdot_{ij}\) are the short-run effects of \(y_{t-1}\) on money demand given that the money demand is in equilibrium.

3.4 Empirical Results

In this section, Johansen's FIML tests are used to test the number of cointegrating relations and the cointegrating vectors. Two tests for determining the number of cointegrating vectors, \(h\), are presented in Table III. The first, denoted as the Trace test, examines the null hypothesis of at most \(q\) cointegrating vectors against the general alternative that \(h=5\). The second, denoted as the Maximum Value test, tests the null hypothesis \(h=q\) against the alternative that \(h=q+1\).

The null hypothesis that there is no cointegration is rejected at the 5% significance level in both tests. Therefore, there exists long-run relationships among these five variables and there is no spurious regression.

Next, the null hypothesis that there is exactly one cointegration vector against the alternative that there are two (Maximum Value Test) or five (Trace Test) cointegration vectors is tested. The null is accepted in both tests at the 5% level. Therefore, I proceed with one cointegration vector in the long-run Canadian Divisia M2 demand.

The cointegration vector can be derived from the eigenvector of the matrix \(E_{\nu v}^{-1}E_{\nu u}^{-1}E_{\nu v}\) associated with the largest eigenvalue \(\lambda_1\). Taking the first element to be unity, the cointegration vector from the FIML test is given by \((1.000 \quad -3.432 \quad 0.191 \quad -0.212 \quad 0.890\) ), as shown in the top half of Table IV. That is, the resulting cointegrating relation is:
The second issue considered here is whether all the variables should be included in the cointegrating relation. I test this by following the procedure for testing restrictions on the cointegrating vectors suggested by Johansen (1989). The null hypothesis of this test is that one of the variables does not appear in the cointegrating relation. The test statistics have a $\mathcal{P}(1)$ distribution and are listed under the cointegrating vectors in Table IV. The null is rejected at the 5% significance level for all five variables. Therefore, all the variables proposed by our model should be included in the cointegrating relation.

Several features of the results in Table IV are of particular interest. First of all, the positive effect of the real income on money demand is consistent with the basic economic theory: an increase in the volume of income causes higher consumption and raises the holding of money. On the other hand, however, the estimated income elasticity of money demand runs counter to the hypothesis of economies of scale in money holding predicted by the transactions and precautionary theories. This may be due to the fact that a broader definition of money is used here. Several assets which were viewed as savings balances are included in our measure because of the liquidity service they provide after recent deregulation of the financial market. As stated in Laidler (1993), “Broader definitions of money . . . produce higher estimates of the income or wealth elasticity of the demand for money than narrower ones” (P. 169). Furthermore, Fried (1973) also has an argument that supports our finding: as the level of wages, the opportunity cost of time, increases due to the increase in income, one would expect the volume of money holding associated with any planned volume of transactions to rise.

Another interesting result in Table IV is that the signs of the estimated coefficients on the two interest rates are consistent with the comparative statics from our model. An increase in the domestic interest

\[(3.4) \quad \frac{\ln M_t}{P_t} = 3.432 \ln y_t - 0.191 \ln r_t + 0.212 \ln r_t^* - 0.890 \ln q_t.\]
rate raises the opportunity cost of holding domestic money and then lowers the holding of money. On the other hand, the positive effect of the foreign interest rate indicates that, since domestic and foreign moneys are imperfect substitutes precisely through the fact that withdraws of both currencies require time, it is optimal to increase the holding of domestic money and the number of foreign currency withdraws when the opportunity cost of holding foreign currency increases.

In addition, the results in Table IV also reveal an interesting phenomenon: the absolute value of the foreign interest rate elasticity is higher than that of the domestic interest rate elasticity. That is, the long-run demand for Canadian Divisia M2 responds more to the U.S. interest rate than to the domestic interest rate. This result is similar to the argument of Barro (1990) that the U.S. economy may have a large, perhaps dominant, influence on the Canadian economy.

Finally, the effect of the real exchange rate on the demand for domestic money is negative. This indicates that a depreciation of the domestic currency decreases the domestic demand for domestic money, which overcomes the positive effect caused by the increase of foreign demand of domestic money as mentioned in Section 2.

Next we compare our results with those of the other studies. First of all, the effects of the domestic and foreign interest rates on the money demand have opposite signs. This contradicts Hamburger's (1977) conclusion that, because "[the domestic interest rate] and [the foreign interest rate] generally move together, if [the domestic interest rate] and [the foreign interest rate] are included in the same equation, only the coefficient of the former remains statistically significant". We find that the foreign interest rate is also a significant factor in the money demand function. Therefore, Bahmani-Oskooee (1991), who cited Hamburger's results, also misspecified his model because the foreign interest rate was eliminated from his model.

Secondly, the model used by Bordo and Choudhri (1982) is incorrectly specified. According to our model and empirical results, both the foreign interest rate and the exchange rate should be included in the
Canadian money demand function. There is no need to assume perfect interest rate arbitrage and to leave out either the foreign interest rate or the exchange rate.

Finally, our results contradicts Arango and Nadiri's (1981) finding that changes in the exchange rate do not appear to be very important in affecting demand for real cash balances in Canada. This contrast could be due to the different data periods we used. They have pooled data from the fixed and floating exchange rate periods. Under the fixed exchange rate system, the relative stability of the exchange markets may not give people as many opportunities to take advantage of the variations in exchange rates by adjusting their holdings of money.

Now we turn to the short-run dynamic of the money demand function and estimate an error-correction model. The lag length in the autoregression was chosen as the minimum length for which there is no significant autocorrelation in the estimated residuals. It turns out that one lag on the right hand side of the ECM is sufficient. The results are:

\[
\begin{align*}
\hat{\ln} m_t &= -0.058 - 0.004 (\ln m_{t-1} - 3.432 \ln y_{t-1} + 0.191 \ln r_{t-1} - 0.212 \ln r^*_{t-1} + 0.890 \ln q_{t-1}) \\
&\quad + 0.270 \hat{\ln} m_{t-1} + 0.122 \hat{\ln} y_{t-1} - 0.042 \hat{\ln} r_{t-1} + 0.001 \hat{\ln} r^*_{t-1} - 0.125 \hat{\ln} q_{t-1} \\
\end{align*}
\]

\[
R^2 = 0.337 \quad \text{Durbin-Watson Statistics} = 2.069.
\]

The standard errors are in parentheses. The numbers under the standard errors are P-Values.

The error-correction model (3.6) shows that, first, the effect of deviations from the long-run money demand equilibrium is marginally significant at the 9% level and the adjustment is on the right direction. Second, the signs of all explanatory variables in the short run are also consistent with a priori specifications, which again supports the validity of our model.

In addition, the coefficient of the lagged dependent variable implies that, after we take into account the arbitrage opportunities in the foreign exchange market, there is a very short adjustment period for real
Divisia M2. Finally, even though changes in the U.S. interest rates do not have significant effects on Canadian Divisia M2 demand, the exchange rate still plays an important role in the short-run dynamics.

4. Conclusion

The traditional money demand function is not complete in an open economy framework. Many previous attempts to take account of the foreign monetary developments in the money demand function have mistakenly excluded either the exchange rate or the foreign interest rate by inappropriately using a misspecified model or data from the fixed exchange rate period.

This paper constructs an open-economy cash-in-advance model to motivate the specification of the money demand function. Canadian quarterly data for the period 1973:2 - 1990:1 are used to estimate the money demand function which includes both a foreign interest rate and an exchange rate. Under the floating exchange rate system, changes in the foreign variables should affect people's incentives to hold money. By carefully examining the nonstationarity of the time series processes, I found that the empirical results in this paper support this argument. The Canadian Divisia M2 demand depends on the foreign interest rate and exchange rate in addition to real income and a domestic interest rate. Thus, ignoring the effects of the foreign interest rate and/or the exchange rate will lead to the misspecification of the demand for money.

The conventional argument that monetary policy is relatively more effective under flexible exchange rates than under fixed exchange rates is challenged by the results in this paper. The effectiveness of monetary policies is affected by people's decisions to hold money and people's money demand is affected by foreign monetary developments. Therefore, this paper concludes that policymakers need to account for the influence of foreign monetary developments on the domestic environment rather than simply depending on the adjustment mechanism of the floating exchange rate system.

Acknowledgment
I am grateful to Kenneth D. West, Arthur S. Goldberger, Donald D. Hester, James M. Johannes and two anonymous referees for helpful comments. I also thank Douglas Fisher for providing the Canadian Divisia money data and the Chiang Ching-Kuo Foundation for its financial support. Naturally, all remaining errors are mine.

Note

1. Assume that $u_c - f_c u_L > 0$ and $u_{c^*} - f_{c^*} u_{L} > 0$. That is, the direct effects of changes in consumption on utility are stronger than the indirect effects working through the transaction technology’s impact on leisure, which seems reasonable enough. It can be shown that the model allows us to sign all the cross partial derivatives of the indirect utility function, with the exception of $U_{cc^*}$. Specifically, $U_{cc} < 0$, $U_{cc^*} < 0$, $U_{mm} < 0$, $U_{m^*m^*} < 0$, $U_{cm} > 0$, $U_{cm^*} > 0$, $U_{c^*m} > 0$, $U_{c^*m^*} > 0$, and $U_{mm^*} < 0$.

Recall equations (2.7) and (2.8). Let $i = 1 - 1/(1+r) = U_{m}/U_{c}/T(m, m^*)$, and $i^* = 1 - 1/(1+r^*) = U_{m^*}/U_{c^*}/X(m, m^*)$. Then $T_m$, $T_{m^*}$, $X_m$, $X_{m^*} < 0$ according to the results of the cross partial derivatives. The comparative statics show that $M_n/M = X_m/M < 0$ and $M_n/M^* = -T_m/M > 0$, where $M = T_m X_m$, $-T_m X_m > 0$. Therefore, $M_n/M < 0$ and $M_n/M^* > 0$. 


REFERENCES


Johansen, S. and K. Juselius. 1990. "Maximum Likelihood Estimation and Inference on Cointegration-


### Table I Means and Standard Deviations of the Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>(\ln m_t)</th>
<th>(\ln y_t)</th>
<th>(\ln r_t)</th>
<th>(\ln r_t^*)</th>
<th>(\ln q_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>7.060</td>
<td>6.246</td>
<td>-3.715</td>
<td>-3.947</td>
<td>0.217</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>(0.066)</td>
<td>(0.161)</td>
<td>(0.272)</td>
<td>(0.297)</td>
<td>(0.080)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable (Multiplied by 100):</th>
<th>(\ln m_t)</th>
<th>(\ln y_t)</th>
<th>(\ln r_t)</th>
<th>(\ln r_t^*)</th>
<th>(\ln q_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.411</td>
<td>0.840</td>
<td>1.361</td>
<td>0.239</td>
<td>0.047</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>(1.225)</td>
<td>(0.900)</td>
<td>(11.57)</td>
<td>(11.64)</td>
<td>(2.134)</td>
</tr>
</tbody>
</table>

*Sources: International Financial Statistics; Fisher (1996).*

The sample period is 1973:2-1990:1. Variable definitions are as follows: \(m_t\) is Divisia M2 divided by CPI; \(y_t\) is real GDP at 1990 prices; \(r_t\) is domestic nominal interest rate; \(r_t^*\) is foreign nominal interest rate; and \(q_t\) is real exchange rate.
Table II.a  Unit Root Test Statistics for the Log Levels of the Variables

<table>
<thead>
<tr>
<th></th>
<th>Phillips-Perron Tests</th>
<th>Augmented Dickey-Fuller Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m^g</td>
<td>Z_D</td>
</tr>
<tr>
<td>ln m_t</td>
<td>5</td>
<td>0.976</td>
</tr>
<tr>
<td>ln q_t</td>
<td>4</td>
<td>-3.572</td>
</tr>
</tbody>
</table>

The sample period is 1973:2-1990:1. The variables are defined in the note to Table I.

The upper half reports the lag lengths and the test statistics of the Phillips-Perron Tests and the augmented Dickey-Fuller Tests for the following null hypotheses:

(Phillips-Perron Tests) \( X_t = \theta + X_{t-1} + u_t \)

(Augmented D-F Test) \( X_t = \epsilon_1 X_{t-1} + \epsilon_2 X_{t-2} + \ldots + \epsilon_p X_{t-k} + \theta D_{X_{t-1}} + \theta^* t + u_t \)

The alternative hypotheses are:

(Phillips-Perron Tests) \( X_t = \theta + D X_{t-1} + \theta^* t + u_t \)

(Augmented D-F Test) \( X_t = \epsilon_1 X_{t-1} + \epsilon_2 X_{t-2} + \ldots + \epsilon_p X_{t-k} + \theta D X_{t-1} + \theta^* t + u_t \)

The lower part reports those of the following null hypotheses:

(Phillips-Perron Tests) \( X_t = X_{t-1} + u_t \)

(Augmented D-F Test) \( X_t = \epsilon_1 X_{t-1} + \epsilon_2 X_{t-2} + \ldots + \epsilon_p X_{t-k} + X_{t-1} + u_t \)

The alternative hypotheses are:

(Phillips-Perron Tests) \( X_t = \theta + D X_{t-1} + u_t \)

(Augmented D-F Test) \( X_t = \epsilon_1 X_{t-1} + \epsilon_2 X_{t-2} + \ldots + \epsilon_p X_{t-k} + \theta D X_{t-1} + u_t \)

The variable \( m \) is the "lag truncation parameter" or "bandwidth" in calculating weighted sums of estimated autocovariances of cross products of residuals. The value of \( m \) is derived from the formula proposed by Newey and West (1994).
Table II.b    Unit Root Test Statistics for the Log Differences of the Variables

<table>
<thead>
<tr>
<th></th>
<th>Phillips-Perron Tests</th>
<th>Augmented Dicky-Fuller Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>AIC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Schwartz's Criterion</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sequential 5% t tests</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>Z_D</td>
</tr>
<tr>
<td>ln m_t</td>
<td>4</td>
<td>-56.37</td>
</tr>
<tr>
<td>ln y_t</td>
<td>2</td>
<td>-43.52</td>
</tr>
<tr>
<td>ln r_t</td>
<td>5</td>
<td>-46.27</td>
</tr>
<tr>
<td>ln r_t*</td>
<td>4</td>
<td>-50.42</td>
</tr>
<tr>
<td>ln q_t</td>
<td>4</td>
<td>-63.65</td>
</tr>
</tbody>
</table>

See the notes to Table II.a.

In this regression, the estimated coefficient is 0.598 and the standard error is 0.202.
### Table III  Johansen's FIML Test Statistics of the Cointegration Rank

<table>
<thead>
<tr>
<th>Number of cointegrating vectors (h)</th>
<th>Trace Test Statistics</th>
<th>Maximum Value Statistics</th>
<th>5% Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h=0</td>
<td>86.89* 38.46*</td>
<td>77.74</td>
<td>Trace Test 36.41</td>
</tr>
<tr>
<td>h=#1</td>
<td>48.42 20.34</td>
<td></td>
<td>Maximum Value Test 30.33</td>
</tr>
<tr>
<td>h=#2</td>
<td>28.09 13.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h=#3</td>
<td>14.17 8.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h=#4</td>
<td>5.20 5.20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table presents tests of the number of cointegrating vectors in the system (ln m_t, ln y_t, ln r_t, ln r_t^*, ln q_t), where the variables are defined in the note to Table I. The autoregression includes a constant term, a time trend, and four lags. An asterisk denotes significance at the 5% level. The Trace test examines the null hypothesis of at most q cointegrating vectors against the general alternative that h=5. The Maximum Value test tests the null hypothesis h=q against the alternative that h=q+1. The 5% critical values are taken from Osterwald-Lenum (1992).
Table IV  Johansen's FIML Tests for Cointegrating Vector

<table>
<thead>
<tr>
<th></th>
<th>ln m_t</th>
<th>ln y_t</th>
<th>ln r_t</th>
<th>ln r_t*</th>
<th>ln q_t*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized Coint. Vectors:</td>
<td>1</td>
<td>-3.432</td>
<td>0.191</td>
<td>-0.212</td>
<td>0.890</td>
</tr>
<tr>
<td>Restriction Test Statistics</td>
<td>4.070</td>
<td>15.700</td>
<td>5.570</td>
<td>11.980</td>
<td>9.061</td>
</tr>
<tr>
<td>P-Value</td>
<td>0.044</td>
<td>0.000</td>
<td>0.018</td>
<td>0.001</td>
<td>0.003</td>
</tr>
</tbody>
</table>

The upper part of this table presents the cointegrating vector with the coefficient on ln m_t normalized to one. The lower part shows the test statistics and the P-values of the null hypothesis that the corresponding variable is not in the cointegration relation. The variables are defined in the note to Table I.
Figure 1: $\ln m_t$ (Real Divisia M2)

Figure 2: $\ln y_t$ (Real GDP)

Figure 3: $\ln r_t$ (Treasury Bill Rates)
Figure 4: \( \ln r^*_t \) (U.S. Treasury Bill Rates)

Figure 5: \( \ln q_t \) (Canadian Dollar/U.S. Dollar Exchange Rates)