

**Money Demand in an Open-Economy Shopping-Time Model: An Out-of-Sample-Prediction
Application to Canada**

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This paper contributes to the existing money demand literature by developing a shopping-time model in an open economy framework. Based on this microfoundations-of-money model, Canadian quarterly time series data for the period 1971:1-1997:2 are used to evaluate the out-of-sample prediction performance of the model. The results show that an error-correction representation of the model performs significantly better than several unrestricted and traditional open- and closed-economy models in the out-of-sample prediction of Canadian real M1 demand.

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1. INTRODUCTION

The money demand function has been extensively investigated in the literature because of its crucial importance for the formulation of monetary policies. Previous studies, however, have often been restricted to a closed economy framework [e.g., Goldfeld (1973, 1976) and Judd and Scadding (1982)]. In view of the increasing integration of world financial markets, one would intuitively expect foreign monetary developments, through their effects on foreign interest rates and/or exchange rates, to influence holding of domestic money because the portfolio choices of individuals will in this case involve not only domestic money and domestic bonds but also foreign assets. Money demand specifications that take account of foreign factors are suggested by the bulk of research on currency substitution.

Nevertheless, a criticism of many open-economy money demand studies is that they have often simply tried different measures of variables in their empirical analyses. Typically, they did not provide an explicit theoretical model to support their empirical models. For example, studies such as Brittain (1981), Bordo and Choudhri (1982), Cuddington (1983), Joines (1985), and Leventakis (1993) use levels of interest rates in their money demand regressions, while Arango and Nadiri (1981) and Ahking (1984) use logarithms of interest rates. Bahmani-Oskooee (1991) even eliminates the foreign interest rate from his regressions. As to the exchange rate, Arango and Nadiri (1981) use log nominal exchange rates, while Bahmani-Oskooee (1991) uses log real exchange rates. Brittain (1981), Bordo and Choudhri (1982), Cuddington (1983), and Ahking (1984) use the uncovered interest rate differential in their regressions instead of exchange rates.

Furthermore, Goldfeld and Sichel (1990) suggest that the disaggregation of a scale variable may be needed to appropriately reflect the nature of international transactions in an open-economy setting. However, among the studies mentioned above, only Leventakis (1993) theoretically includes a measure of foreign income in his empirical model.

This paper constructs a shopping-time model in an open economy framework to motivate the specification of the demand for money. This microfoundations-of-money model allows me to choose which

variables, and in what forms, should be used in the empirical money demand function. In addition, the model implies several long-run relations among relevant variables that can be utilized in the short-run dynamics of the money demand function.

This paper focuses on the comparisons of the out-of-sample prediction performances of different models because what really concerns policy makers may not be the goodness of fit, but the predictive power of the models. In addition, out-of-sample predictions have been used as a stability test in previous studies such as Goldfeld (1976) and Hafer and Hein (1979). I compare the performance of the error-correction representation of the open-economy shopping-time model to those from the unrestricted error-correction models often used in the literature.

Canadian national quarterly data from 1971:1 to 1997:2 are analyzed. Canada is chosen because it possesses the characteristics of a small open economy which takes foreign variables as given, as assumed in our model. The results show that our restricted error-correction model performs significantly better than several unrestricted and traditional open- and closed-economy error-correction models in the out-of-sample predictions of Canadian real M1 demand.

The rest of this paper is organized as follows: Section 2 constructs the theoretical model. The empirical analysis is set forth in Section 3. Section 4 concludes the paper.

2. THEORETICAL MODEL

Modeling money demand in terms of microeconomic principles has become popular in practice. This method of modeling allows us to specify explicitly the role of money in the economy. In this study, money is an asset held only for transactions purposes.

One way to model the role of money in the intertemporal optimization choices by an individual economic agent is to include money in the agent's utility function. The money-in-the-utility-function model has been widely used to account for the real liquidity services provided by money. Some have argued that,

however, it is the service that provides utility to agents rather than money itself. Therefore, many economists have suggested explicitly modeling the liquidity services provided by money through the agent's budget constraint.

One way to indirectly include money in the utility function is to use the cash-in-advance constraint in the model. It is assumed that the agent's consumption in any period cannot exceed the amount of money held at the beginning of that period. However, the cash-in-advance model has been criticized for two unrealistic restrictions. First, this model implies a unitary income velocity, which is not supported by many empirical studies. Guidotti (1993) addressed this problem and developed a model where the income velocity is variable. His model integrated a cash-in-advance constraint and a Baumol-Tobin type transactions technology. He showed that domestic money demand depends on consumption, domestic and foreign interest rates, and a transactions technology parameter. However, there is no empirical analysis in his paper.

The second restriction implied by the cash-in-advance model is that it places a strict upper limit on purchases during the period. To avoid this unrealistic restriction, McCallum and Goodfriend (1988) suggested the use of a more general model, the shopping-time model, which was originally introduced by Saving (1971). In this model, extra purchases are possible, but they are more expensive in terms of time. Therefore, money enters the utility function indirectly by way of a shopping-time function.

To make the derivations of the open-economy shopping-time model easier to understand, I will begin with a review of a closed-economy model. Section 2.1 derives a closed-economy model based on McCallum (1989) and McCallum and Goodfriend (1988). Section 2.2 extends this model to an open economy framework.

2.1 Closed-Economy Shopping-Time Model

Consider an economy inhabited by identical, infinitely-lived individuals. The representative agent maximizes the expected present value of utility over an infinite horizon:

$$U(c_t, L_t) + \sum_{j=1}^{\infty} \beta^j E_t [U(c_{t+j}, L_{t+j})], \quad (2.1.1)$$

where c_t is real consumption during period t , L_t is leisure, $\beta \in (0,1)$ is the constant discount rate, and E_t denotes the expectation conditional on information at time t . The utility function $U(\cdot)$ is assumed to be twice continuously differentiable, strictly concave, and to satisfy the Inada conditions. It is assumed that the agent knows current values of all relevant variables when making decisions.

The agent receives real income in the amount y_t and divides his wealth between money and bonds. Let M_t and B_t be the nominal money balance and nominal bonds held at the end of period t . That is, the agent begins period t with assets in the amount $M_{t-1} + B_{t-1}$. The bond is a one-period security that may be sold at discount $(1+r_t)^{-1}$, where r_t is the nominal interest rate, which pays one unit of money in $t+1$. The agent's budget constraint for period t may be written as:

$$m_t + b_t = \frac{P_{t-1}}{P_t} m_{t-1} + \frac{P_{t-1}}{P_t} (1 + r_{t-1}) b_{t-1} + y_t - c_t, \quad (2.1.2)$$

where P_t is the price level prevailing at time t , m_t is the real holding of money, and b_t is the real holding of bonds.

To acquire consumption goods, the agent must spend time in shopping, S_t . The amount of time left over for leisure is L_t . That is, $L_t = T - S_t$, where T is the total amount of time available per period. The smaller the amount of time spent in shopping, the greater the amount of time left over for leisure. For simplicity, labor is assumed to be supplied inelastically and omitted from the time constraint.

The amount of time spent in shopping depends positively on the volume of consumption. For a given volume of consumption, holding money facilitates transactions and reduces the amount of time in shopping. Therefore, the shopping-time function can be written as:

$$S_t = S(c_t, m_t)^1, \quad (2.1.3)$$

where $S_m = \frac{\partial S_t}{\partial m_t} > 0$, $S_c = \frac{\partial S_t}{\partial c_t} > 0$, $S_{cm} = \frac{\partial^2 S_t}{\partial c_t \partial m_t} > 0$, $S_{cc} = \frac{\partial^2 S_t}{\partial c_t^2} < 0$, and $S_{mm} = \frac{\partial^2 S_t}{\partial m_t^2} < 0$.

The preceding basic closed-economy setup closely follows that of McCallum (1989). It differs in that the future values of variables are known with certainty in McCallum (1989). Furthermore, he replaces the shopping-time function in (2.1.3) with a leisure function.

Given the objective function (2.1.1), subject to the constraints (2.1.2) and (2.1.3), the representative agent's utility-maximization problem is characterized by the following first-order conditions:

$$U_c - U_L S_c = \lambda_t, \quad (2.1.4)$$

$$-U_L S_m + \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = \lambda_t, \quad (2.1.5)$$

$$(1 + r_t) \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = \lambda_t, \quad (2.1.6)$$

where the Lagrange multiplier, λ_t , is the marginal utility gain of an increase in wealth.

The economic content of these first-order conditions is straightforward. Equation (2.1.4) states that the net marginal utility of consumption ($U_c - U_L S_c$) must be equal to the loss of utility necessitated by the decrease of money holdings that corresponds to one unit of consumption (λ_t). Equation (2.1.5) states that real money balances will be held to the point where their marginal benefit equals their marginal cost. The marginal benefit has two components. First, it reduces transactions time and thus frees up time for leisure ($-U_L S_m$). Second, holdings of real balances are available for consumption next period with an expected marginal benefit of $\beta \lambda_{t+1} \frac{P_t}{P_{t+1}}$. The marginal cost of holding real balances is the utility from one unit of consumption that is forgone today (λ_t). Equation (2.1.6) simply states that the expected marginal benefit of holding one unit of bond should be equal to its marginal cost.

Note that equation (2.1.6) enables us to remove the conditional expectation term $E_t \lambda_{t+1} \frac{P_t}{P_{t+1}}$. Manipulating the first-order conditions (2.1.4)-(2.1.6) yields an equation involving only three variables, c_t , m_t , and r_t :

$$\frac{U_L(c_t, T - S(c_t, m_t)) S_m(c_t, m_t)}{U_L(c_t, T - S(c_t, m_t)) S_c(c_t, m_t) - U_c(c_t, T - S(c_t, m_t))} = 1 - \frac{1}{1 + r_t}. \quad (2.1.7)$$

This equation states that the marginal rate of substitution between real cash balances and consumption equals the opportunity cost of holding money. It can also be shown that, according to our assumptions on the utility function and the shopping-time function, m_t has a positive partial derivative with respect to c_t and a negative partial with respect to r_t , as predicted by the basic economic theory.

This model implies a relation that is similar to those normally described in the literature as "money demand functions." McCallum and Goodfriend (1988) refer to this type of expression as a portfolio-balance relationship. To provide an example, McCallum (1989) assumes that the utility function takes a Cobb-Douglas form: $U(c_t, L_t) = c_t^\alpha L_t^{1-\alpha}$, where $0 < \alpha < 1$, and leisure is a function of the consumption velocity: $L_t = T - S_t = \left(\frac{m_t}{c_t}\right)^a$, where $0 < a < 1$. Substituting these specific functional forms into the model yields:

$$m_t = \frac{a - a\alpha}{\alpha - (a - a\alpha)} c_t \left(1 + \frac{1}{r_t}\right). \quad (2.1.8)$$

Taking logs on both sides of (2.1.8) yields a linear closed-economy money demand function:

$$\ln m_t = \phi_0 + \ln c_t - \ln i_t, \quad (2.1.9)$$

where $i_t = \frac{r_t}{1 + r_t}$ and N_0 is a constant. This model implies unitary consumption and interest rate elasticities.

2.2 Open-Economy Shopping-Time Model

Now we are ready to construct a shopping-time model in an open economy framework. Specifically, consider a small open economy which takes foreign variables as given. There are two goods, two monies, and two bonds in which there is one domestic and one foreign of each. The representative agent maximizes his multiperiod utility:

$$U(c_t, c_t^*, L_t) + \sum_{j=1}^{\infty} \beta^j E_t [U(c_{t+j}, c_{t+j}^*, L_{t+j})], \quad (2.2.1)$$

where c_t and c_t^* are home-country real consumption of domestic and foreign goods, respectively. The derivatives $U_i > 0$, $U_{ii} < 0$, and $U_{ik} > 0$ ($I, k = c, c^*, L; I..k$). The scale variable is disaggregated into two parts: consumption of domestically produced goods and consumption of imports. This intertemporal structural consumption model has been used in Clarida (1994) and Guidotti (1993).

The representative agent holds four assets: domestic and foreign money, and domestic and foreign bonds which pay a nominal interest rate of r_t and r_t^* , respectively. The agent's budget constraint for period t may be written as:

$$m_t + b_t + q_t m_t^* + q_t b_t^* = \frac{P_{t-1}}{P_t} m_{t-1} + \frac{P_{t-1}}{P_t} (1 + r_{t-1}) b_{t-1} + q_t \frac{P_{t-1}^*}{P_t^*} m_{t-1}^* + q_t \frac{P_{t-1}^*}{P_t^*} (1 + r_{t-1}^*) b_{t-1}^* + y_t - c_t - q_t c_t^*, \quad (2.2.2)$$

where m_t and m_t^* are real holdings of domestic and foreign money, respectively; b_t and b_t^* are real holdings of domestic and foreign bonds, respectively; and P_t and P_t^* are domestic and foreign price levels, respectively. The real exchange rate is $q_t = e_t P_t^* / P_t$, where e_t is the nominal exchange rate defined as units of domestic currency per unit of foreign currency.

As to the transactions technology, following Stockman (1980), Lucas (1982), and Guidotti (1993), it is assumed that domestic and foreign goods have to be purchased with domestic and foreign currencies, respectively. Therefore, the time spent in purchasing domestic (foreign) goods only depends on consumption of domestic (foreign) goods and holdings of domestic (foreign) money. That is,

$$S_t = S(c_t, c_t^*, m_t, m_t^*) = S^D(c_t, m_t) + S^F(c_t^*, m_t^*), \quad (2.2.3)$$

where $S_c, S_{c^*} > 0$; $S_m, S_{m^*} \neq 0$; $S_{ij} > 0$ and $S_{ij} \neq 0$ for $I, j = c, c^*, m, m^*, I..j$. S^D is the time spent in purchasing domestic goods and S^F in purchasing foreign goods.

One may argue that the average consumer simply uses domestic currency to buy foreign goods and spends no extra time in acquiring foreign goods. However, the economy as a whole does use foreign currencies and spends time in acquiring foreign currencies to buy imported goods. The importers are those who hold foreign currency accounts and spend time in acquiring foreign goods. Therefore, we may consider

that the importers are representatives for consumers in buying foreign goods in the first stage, whereas in the second stage the consumers use domestic currency to buy foreign goods from the importers.

Given the objective function (2.2.1), subject to the constraints (2.2.2) and (2.2.3), the first-order conditions necessary for optimality of the agent's choices imply²:

$$\frac{U_L S_m}{U_L S_c - U_c} = 1 - \frac{1}{1 + r_t}, \quad (2.2.4)$$

$$\frac{U_L S_{m^*}}{U_L S_{c^*} - U_{c^*}} = 1 - \frac{1}{1 + r_t^*}, \quad (2.2.5)$$

$$\frac{U_c - U_L S_c}{U_{c^*} - U_L S_{c^*}} = \frac{1}{q_t}. \quad (2.2.6)$$

Equations (2.2.4) and (2.2.5) are necessary conditions analogous to (2.1.7) in the closed-economy model. Equation (2.2.6) states that the marginal rate of substitution between domestic and foreign goods equals their relative price.

To express the money demand function explicitly, assume that the utility function takes a Cobb-Douglas form and that the utilities from consuming domestically produced goods and from consuming imports are the same:

$$U(c_t, c_t^*, L_t) = c_t^{\frac{\alpha}{2}} c_t^{*\frac{\alpha}{2}} L_t^{1-\alpha}, \quad (2.2.7)$$

where $0 < \alpha < 1$. In addition, assume that shopping time is constrained by a technical relationship that reflects the transaction-facilitating properties of money. This technical relationship is assumed to be characterized by a Baumol-Tobin type technology in which cash withdrawals involve time instead of monetary transaction costs as in Baumol (1952). Specifically, assume that $S_t^D = \frac{c_t}{m_t}$ and $S_t^F = \rho \frac{c_t^*}{m_t^*}$, where $D > 0$ is a transactions technology parameter. The open-economy shopping-time function can then be expressed as:

$$S_t = \frac{c_t}{m_t} + \rho \frac{c_t^*}{m_t^*}. \quad (2.2.8)$$

Since the flow of consumption to be financed occurs continuously, cash withdrawals are spread evenly throughout the time period. Therefore, $\frac{c_t}{m_t}$ and $\frac{c_t^*}{m_t^*}$ can be interpreted as frequencies of withdrawals in domestic and foreign currencies, respectively. The cost of using domestic currency in units of time is set to one, and D reflects how much more time it takes proportionally to obtain foreign currency instead of domestic currency.

Substituting (2.2.7) and (2.2.8) into (2.2.4)-(2.2.6) yields:

$$m_t = \frac{q_t c_t^* - c_t}{i_t - \sqrt{\rho i_t i_t^* q_t \frac{c_t^*}{c_t}}}, \quad (2.2.9)$$

where $i_t = \frac{r_t}{1 + r_t}$, and $i_t^* = \frac{r_t^*}{1 + r_t^*}$. For the real money balance in (2.2.9) to be positive, the following

inequality has to hold:

$$(q_t c_t^* - c_t)(i_t - \sqrt{\rho i_t i_t^* q_t c_t^* / c_t}) > 0.$$

This implies,

$$\begin{aligned} \left(1 - \frac{c_t}{q_t c_t^*}\right) \left(1 - \sqrt{\rho \frac{c_t^*}{c_t} \frac{i_t^* q_t}{i_t}}\right) &= \left(1 - \frac{c_t}{q_t c_t^*}\right) \left(1 - \sqrt{\rho \frac{c_t^*}{c_t} \frac{S_m^*}{S_m}}\right) = \left(1 - \frac{c_t}{q_t c_t^*}\right) \left(1 - \frac{\rho c_t^* / m_t^*}{c_t / m_t}\right) \\ &= \left(1 - \frac{c_t}{q_t c_t^*}\right) \left(1 - \frac{S_t^F}{S_t^D}\right) > 0. \end{aligned}$$

The first equality is from (2.2.4)-(2.2.6). Therefore, either $\frac{S_t^D}{S_t^F} < 1 < \frac{c_t}{q_t c_t^*}$ or $\frac{S_t^D}{S_t^F} > 1 > \frac{c_t}{q_t c_t^*}$ has to hold.

In words, if the expenditure on the domestically produced goods is larger (smaller) than that on imports, the

agent must spend less (more) time in shopping for domestically produced goods than in shopping for imports.

Since c_t and c_t^* yield the same utility to the agent, these restrictions seem reasonable enough.

Equation (2.2.9) is neither linear nor loglinear in relevant variables. To express it as a linear equation like those in many money demand studies, rewrite (2.2.9) as:

$$\begin{aligned} \ln m_t - \ln c_t + \ln i_t = & -(\ln c_t - \ln c_t^* - \ln q_t) + \ln[1 - \exp(\ln c_t - \ln c_t^* - \ln q_t)] \\ & - \ln\left\{1 - \exp\left[\frac{1}{2}\ln\rho - \frac{1}{2}(\ln c_t - \ln c_t^* - \ln q_t - \ln i_t^* + \ln i_t)\right]\right\}. \end{aligned} \quad (2.2.10)$$

Next consider the following first-order Taylor series expansion:

$$\ln[1 - \exp(d_t)] \approx \ln[1 - \exp(\bar{d})] - \frac{\exp(\bar{d})}{1 - \exp(\bar{d})} (d_t - \bar{d}), \quad (2.2.11)$$

where $\bar{\div}$ is the steady state value of d_t . Let

$$\begin{aligned} d_{1t} &= \ln c_t - \ln c_t^* - \ln q_t, \text{ and} \\ d_{2t} &= \frac{1}{2}\ln\rho - \frac{1}{2}(\ln c_t - \ln c_t^* - \ln q_t - \ln i_t^* + \ln i_t). \end{aligned}$$

Equation (2.2.10) can be expressed as:

$$\ln m_t - \ln c_t + \ln i_t = \theta_0 + \theta_1(\ln c_t - \ln c_t^* - \ln q_t) + \theta_2(\ln i_t^* - \ln i_t) + \epsilon_t. \quad (2.2.12)$$

Let $k_1 = [1 - e^{\bar{d}_1}]^{-1} - 1$, $k_2 = [1 - e^{\bar{d}_2}]^{-1} - 1$, and $\bar{\div}_1$ and $\bar{\div}_2$ be the steady state values of d_{1t} and d_{2t} , respectively. Then: $\theta_0 = \ln(k_2 + 1) + k_1(\bar{d}_1) - \ln(k_1 + 1) - k_2\bar{d}_2 + \frac{1}{2}k_2\ln\rho$, $\theta_1 = -1 - k_1 - \frac{1}{2}k_2$, and $\theta_2 = \frac{1}{2}k_2$. The higher-order terms of the Taylor series expansion are included in the residual ϵ_t . Equation (2.2.12) states that the domestic consumption velocity adjusted for the domestic interest rate is a function of the ratio of consumer's expenditure on domestically produced goods to that on imported goods in terms of domestic currency, and the ratio of the return on foreign bonds to that on domestic bonds

Note that in order to use the first-order Taylor series expansion, d_{1t} and d_{2t} must be stationary for their steady state values to exist. That is, the model implies that $(\ln c_t - \ln c_t^* - \ln q_t)$, $(\ln i_t^* - \ln i_t)$, and $(\ln m_t - \ln c_t + \ln i_t)$ are all stationary. Therefore, (2.2.12) can be interpreted as the long-run money demand function implied by our model.

The models in this section describe the behavior of the household sector. There exist, of course, other economic units, such as firms. To construct a model for a firm that is analogous to our model, one would post the maximization of the present value of profits or the minimization of costs. The shopping-time function would be replaced by a relationship depicting resources used in conducting transactions. But the general aspects of the analysis would be similar. Therefore, we proceed under the presumption that the crucial points are well represented in our models, which recognize only the household sector.

3. OUT-OF-SAMPLE PREDICTIONS

Prediction is of fundamental importance in economic decision making. Whether a model exhibits sufficient stability to be useful for extrapolation is what really concerns the policy makers. Since predictions are often conducted in a quarterly horizon, a model with short-run dynamics should be used for predictions. This paper uses error-correction models to make predictions. The error-correction model is widely used in the literature [e.g., Baum and Furno (1990), Hueng (1998), Miller (1991), and Mehra (1993)] because it not only shows the short-run dynamics of the variables but also takes into account the long-run relations among the variables.

Performances from several error-correction models are compared. In the first model, the long-run relations implied by the model in Section 2 are imposed. The second model uses an unrestricted long-run relation in an open-economy framework. The third one is an unrestricted closed-economy model. Finally, it is also interesting to see how the traditional money demand functions, which use income instead of consumption as the scale variable, perform.

3.1 The Data

This paper uses Canadian quarterly data for the period 1971:1 to 1997:2, with a total of one hundred and six observations. The sample starts in 1971 because of data availability. All data are taken from the CANSIM

CD-ROM data base published by Statistics Canada. Variables are seasonally adjusted.

Since a representative-agent model is used to derive the money demand function, money and consumption are measured in per capita terms. That is, data on money and consumption are divided by population before estimation.

For the nominal money stock (M_t), theories based on a transactions approach often lead to a narrow definition of money, M1. This aggregate allows for more direct comparison to previous studies.

As to the domestic interest rate (r_t), the three-month treasury bill rate is used. The foreign interest rate (r_t^*) is approximated by the U.S. three-month treasury bill rate in Canada, which is equal to the Canadian treasury bill rate minus the sum of the “USA dollar in Canada 90-day forward differential” and the “covered differential Canada/USA three-month treasury bill rate”³³.

Since representative consumer models generally exclude durable goods from the scale variables, the variable c_t^* is defined as real imports of non- and semi-durable goods and services at 1986 prices. This variable is constructed by the nominal imports of non- and semi-durable goods and services in units of Canadian dollars and the corresponding price indices.

The real consumption of domestically produced goods (c_t) is then defined as the difference between total consumer expenditures on non- and semi-durable goods and services at 1986 prices and the imported non- and semi-durable goods and services at 1986 prices. Dividing the corresponding price index of c_t^* ($e_t P_t^*$) by that of c_t (P_t) yields the real exchange rate (q_t).

3.2 The Models

Assume that the short-run dynamics of our model can be identified by a VAR system. Specifically, assume that the (6×1) vector $y_t = (\ln m_t \ \ln c_t \ \ln c_t^* \ \ln q_t \ \ln i_t \ \ln i_t^*)'$ can be characterized by a VAR(2) in level. If the variables in y are cointegrated, the following Error-Correction representation applies [see Hamilton (1994, pp. 579-581)]:

$$\Delta \mathbf{y}_t = \boldsymbol{\alpha} + \boldsymbol{\beta} \Delta \mathbf{y}_{t-1} - \mathbf{B} \mathbf{A}' \mathbf{y}_{t-1} + \mathbf{e}_t, \quad (3.2.1)$$

where $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and \mathbf{B} are constant vectors, and \mathbf{A} is the $(h \times 6)$ matrix composed of h cointegrating vectors. The coefficients in the vector \mathbf{B} can be interpreted as the effects of the short-run deviations from the long-run equilibria. The coefficients in $\boldsymbol{\beta}$ are the short-run effects of the explanatory variables on the money demand, given that the long-run relationships are in equilibrium.

Recall that the open-economy shopping-time model implies three cointegrating vectors: $\ln c_t - \ln c_t^* - \ln q_t$, $\ln i_t^* - \ln i_t$, and $\ln m_t - \ln c_t + \ln i_t$. By imposing these cointegrating relations on (3.2.1), the first equation of the error-correction models can be expressed as:

Model 1:

$$\begin{aligned} \Delta \ln \frac{M_t}{P_t} = & \beta_{10} + \beta_{11} \Delta \ln m_{t-1} + \beta_{12} \Delta \ln c_{t-1} + \beta_{13} \Delta \ln c_{t-1}^* + \beta_{14} \Delta \ln q_{t-1} + \beta_{15} \Delta \ln i_{t-1} + \beta_{16} \Delta \ln i_{t-1}^* \\ & + b_{11} (\ln m_{t-1} - \ln c_{t-1} + \ln i_{t-1}) + b_{12} (\ln c_{t-1} - \ln c_{t-1}^* - \ln q_{t-1}) + b_{13} (\ln i_{t-1}^* - \ln i_{t-1}) + e_{1t}. \end{aligned} \quad (3.2.2)$$

Given the well-known unit root property of the macroeconomic time series and the long-run relations implied by the model, each term in (3.2.2) is $I(0)$. In addition, all terms on the right hand side are predetermined. Therefore, OLS applies.

Next we consider unrestricted models which do not impose any predetermined cointegration relations. Let $y_t = \ln m_t$ and \mathbf{x}_t be the vector consist of the variables other than $\ln m_t$. If y_t and \mathbf{x}_t can be characterized by a VAR(2) in level and are cointegrated with the cointegrating vector $(1 \ \boldsymbol{\gamma})$, the following error-correction representation results:

$$y_t = \boldsymbol{\alpha} + \boldsymbol{\beta} (y_{t-1} \ \mathbf{x}_{t-1})' + \boldsymbol{\gamma} (y_{t-1} - \boldsymbol{\gamma} \mathbf{x}_{t-1}) + e_t. \quad (3.2.3)$$

This is simply an alternative expression of (3.2.1). Unlike (3.2.2), where the long-run relations implied by our models are imposed, the cointegrating vector $(1 \ \boldsymbol{\gamma})$ in (3.2.3) is estimated directly without imposing any prior restriction.

One way to estimate equation (3.2.3), which also makes the out-of-sample predictions easier, is the two-step method proposed by Engle and Granger (1987). The first step is to regress y_t on x_t , including a constant term. This will yield a super-consistent estimate of β . The second step is to replace β by this estimate and then regress equation (3.2.3) using OLS. Engle and Granger (1987) show that the estimates of β , λ , and γ are consistent and asymptotically normal.

Let $x_t = (\ln c_t \ \ln c_t^* \ \ln q_t \ \ln i_t \ \ln i_t^*)'$, (3.2.3) yields the unrestricted open-economy model,

Model 2:

$$\begin{aligned} \Delta \ln m_t = & \beta_{20} + \beta_{21} \Delta \ln m_{t-1} + \beta_{22} \Delta \ln c_{t-1} + \beta_{23} \Delta \ln c_{t-1}^* + \beta_{24} \Delta \ln q_{t-1} + \beta_{25} \Delta \ln i_{t-1} + \beta_{26} \Delta \ln i_{t-1}^* \\ & + b_2 (\ln m_{t-1} - \lambda_{21} \ln c_{t-1} - \lambda_{22} \ln c_{t-1}^* - \lambda_{23} \ln q_{t-1} - \lambda_{24} \ln i_{t-1} - \lambda_{25} \ln i_{t-1}^*) + e_{2t}; \end{aligned} \quad (3.2.4)$$

and $x_t = (\ln c_t \ \ln i_t)'$ gives the unrestricted closed-economy model,

Model 3:

$$\Delta \ln m_t = \beta_{30} + \beta_{31} \Delta \ln m_{t-1} + \beta_{32} \Delta \ln c_{t-1} + b_3 (\ln m_{t-1} - \lambda_{31} \ln c_{t-1} - \lambda_{32} \ln i_{t-1}) + e_{3t}. \quad (3.2.5)$$

The next two models use the variables often seen in the literature. The traditional closed-economy money demand function [e.g., Goldfeld (1973, 1976) and Judd and Scadding (1982)] and the open-economy money demand function [e.g., Arango and Nadiri (1981) and Hueng (1998)] use real GDP, instead of consumption, as the scale variable and measure money and GDP at aggregate levels. Incorporating error correction, we have the traditional open-economy model,

Model 4:

$$\begin{aligned} \Delta \ln m'_t = & \beta_{40} + \beta_{41} \Delta \ln m'_{t-1} + \beta_{42} \Delta \ln y_{t-1} + \beta_{43} \Delta \ln q_{t-1} + \beta_{44} \Delta \ln i_{t-1} + \beta_{45} \Delta \ln i_{t-1}^* \\ & + b_4 (\ln m'_{t-1} - \lambda_{41} \ln y_{t-1} - \lambda_{42} \ln q_{t-1} - \lambda_{43} \ln i_{t-1} - \lambda_{44} \ln i_{t-1}^*) + e_{4t}; \end{aligned} \quad (3.2.6)$$

and the traditional closed-economy model,

Model 5:

$$\Delta \ln m'_t = \beta_{50} + \beta_{51} \Delta \ln m'_{t-1} + \beta_{52} \Delta \ln y_{t-1} + \beta_{53} \Delta \ln i_{t-1} + b_5 (\ln m'_{t-1} - \lambda_{51} \ln y_{t-1} + \lambda_{52} \ln i_{t-1}) + e_{5t}, \quad (3.2.7)$$

where y_t is aggregate real GDP at 1986 prices, m'_t is aggregate real M1, and β 's are the normalized

cointegrating coefficients, which are to be estimated by the Engle-Granger two-step estimations. The closed-economy error-correction model has been used in Baum and Furno (1990), Miller (1991), and Mehra (1993). The open-economy model has been used in Hueng (1998). To compare the prediction performances of these two models to that of our open-economy shopping-time model, the per-capita terms in (3.2.2) are replaced by aggregate levels.

3.3 The Results

Before proceeding to the out-of-sample prediction, I list the in-sample regression results for each model in Table 1. In general, the long-run relations among the relevant variables, which are shown inside the error-correction terms, are significant and have the expected signs⁴. On the other hand, the short-run effects of the explanatory variables on money demand are not significant, except for the foreign exchange rate in the open-economy models. In addition, the estimated coefficients on the error-correction terms show that the adjustments from the deviations are significant and on the right direction, except for the interest rate differential in Model 1. The estimated coefficient of this term has the wrong sign. However, it is insignificant.

Now let's proceed with the out-of-sample predictions. The sample is separated into two parts. The first decade (1971:1-1980:4) of the sample period is the smallest sample used for regression estimation⁵. The sample size of the regression grows as one makes successive predictions. This regression method is called the "recursive scheme" in West (1996). The prediction period is from 1981:1 to 1997:2.

The Mean Square Prediction Errors (MSPE) from the one-quarter ahead out-of-sample predictions are used to evaluate the performances. In comparing the MSPEs, the prediction inference techniques described in West (1996) are used to test whether the prediction errors from competing models are different. The null hypothesis of the test statistic is that the difference between the MSPEs from competing models is zero.

The prediction performances of the models are shown in Table 2. To express the MSPEs in economically interpretable terms, I report the root MSPEs in percentage terms. For example, 2.529 denotes the root mean square prediction error of real money growth rate to be 2.529%.

The first row in Panel A shows that the open-economy shopping-time model (Model 1) predicts better than the unrestricted open-economy model (Model 2). The P-value of the test statistic from the West Test shows that the difference is marginally significant at the 7.7% significance levels. The second row shows that the open-economy shopping-time model performs significantly better than the unrestricted closed-economy model (Model 3) at the traditional significance level.

Panel B shows the results from comparing the traditional models with the open-economy shopping-time model using aggregate money stock and scale variables. The results show that the traditional money demand functions, either in an open-economy (Model 4) or a closed-economy (Model 5) setting, predict worse than the model proposed by this paper. The differences are significantly different from zero at the traditional significance level. The use of consumption as the scale variable is justified. This corresponds to Mankiw and Summers (1986), who argue for the superiority of consumption to GNP as the scale variable in the money demand function. In addition, it also shows that, as suggested by Goldfeld and Sichel (1990), Clarida (1994), and Guidotti(1993), the disaggregation of the scale variable to reflect the nature of international transactions in an open economy is essential.

4. CONCLUSION

Traditional studies on money demand have only concentrated on the domestic interest rate and real income. Some previous attempts to take account of the foreign monetary developments in the money demand function have tried different measures of variables in their empirical analysis with little theoretical justification. Conversely, this paper develops a shopping-time model in an open economy framework to motivate the specification of the money demand function.

Canadian national quarterly time series data for the period 1971:1-1997:2 are used to evaluate the out-of-sample prediction performance of the model. The results show that first, the open-economy shopping-time model performs better than the unrestricted models in the out-of-sample prediction of Canadian real M1 demand. In addition, our model predicts better than the money demand functions similar to ones used in the literature.

In regards to policy implications, the results of this paper raise questions about the usefulness of the traditional closed-economy money demand specification for monetary targeting. Policy makers need to account for the influence of foreign monetary developments on the domestic environment rather than simply relying on a money demand function with only domestic variables for setting monetary targets.

The availability of the real imports consumption data restrict the sample size of this study and therefore our ability to test the long-run relations among the variables. Instead, I impose the long-run relations suggested by the theory on the empirical model without testing them. I do this just as others have imposed a unitary price elasticity without testing it in the money demand functions as can be seen in the literature. Extensions for future research include searching for other measures which can provide a larger data set for long-run analyses.

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Table 1 In-Sample Regression Results

Model 1:

$$\left. \begin{aligned} \ln m_t = & -0.197 - 0.185 \ln m_{t-1} - 0.280 \ln c_{t-1} + 0.174 \ln c_{t-1}^* - 1.492 \ln q_{t-1} + 0.027 \ln i_{t-1} - 0.020 \\ \ln i_{t-1}^* & (0.063) (0.130) (0.481) (0.353) (0.544) (0.023) (0.027) \\ & - 0.042 (\ln m_{t-1} - \ln c_{t-1} + \ln i_{t-1}) + 0.125 (\ln c_{t-1} - \ln c_{t-1}^* - \ln q_{t-1}) - 0.014 (\ln i_{t-1}^* - \ln i_{t-1}) \\ & (0.011) (0.070) (0.011) \end{aligned} \right\}$$

Durbin Watson Statistic = 1.975. $R^2 = 0.201$.Model 2:

$$\left. \begin{aligned} \ln m_t = & 0.365 + 0.088 \ln m_{t-1} - 0.139 \ln c_{t-1} + 0.064 \ln c_{t-1}^* - 1.085 \ln q_{t-1} + 0.013 \ln i_{t-1} - 0.008 \\ \ln i_{t-1}^* & (0.202) (0.132) (0.510) (0.377) (0.546) (0.024) (0.026) \\ & - 0.119 \{ \ln m_{t-1} - 0.094 \ln c_{t-1} - 0.594 \ln c_{t-1}^* - 1.255 \ln q_{t-1} + 0.096 \ln i_{t-1} - 0.099 \ln i_{t-1}^* \} \\ & (0.066) (0.258) (0.180) (0.280) (0.022) (0.021) \end{aligned} \right\}$$

Durbin Watson Statistic = 1.963. $R^2 = 0.082$.Model 3:

$$\left. \begin{aligned} \ln m_t = & 0.306 + 0.110 \ln m_{t-1} - 0.030 \ln c_{t-1} - 0.111 \{ \ln m_{t-1} - 0.636 \ln c_{t-1} + 0.079 \ln i_{t-1} \} \\ & (0.135) (0.107) (0.110) (0.049) (0.070) (0.017) \end{aligned} \right\}$$

Durbin Watson Statistic = 2.008. $R^2 = 0.056$.Model 4:

$$\left. \begin{aligned} \ln m_t^! = & 0.306 + 0.032 \ln m_{t-1}^! - 0.219 \ln y_{t-1} - 0.849 \ln q_{t-1} + 0.009 \ln i_{t-1} + 0.001 \ln i_{t-1}^* \\ & (0.167) (0.118) (0.316) (0.486) (0.023) (0.027) \\ & - 0.072 \{ \ln m_{t-1}^! - 0.438 \ln y_{t-1} + 0.958 \ln q_{t-1} + 0.227 \ln i_{t-1} - 0.060 \ln i_{t-1}^* \} \\ & (0.040) (0.047) (0.308) (0.028) (0.035) \end{aligned} \right\}$$

Durbin Watson Statistic = 1.949. $R^2 = 0.079$.Model 5:

$$\left. \begin{aligned} \ln m_t^! = & 0.482 + 0.099 \ln m_{t-1}^! - 0.356 \ln y_{t-1} + 0.010 \ln i_{t-1} - 0.087 \{ \ln m_{t-1}^! - 0.330 \ln y_{t-1} + 0.206 \ln i_{t-1} \} \\ & (0.206) (0.111) (0.287) (0.021) (0.038) (0.032) (0.017) \end{aligned} \right\}$$

Durbin Watson Statistic = 2.030. $R^2 = 0.066$.

The numbers in parentheses are standard errors. The sample period is 1971:1-1997:2. The variables are defined as follows. m_t and $m_t^!$ are per capita and aggregate real holdings of domestic money, respectively; c_t and c_t^* are per capita real consumption of domestically-produced goods and imports, respectively; i_t and i_t^* are domestic and foreign interest rates, respectively; y_t is aggregate real GDP; and q_t is the real exchange

rate defined as units of domestic currency per unit of foreign currency.

Table 2 Root Mean Square Prediction Errors of Real Money Growth Rates (in Percentage)^a

Restricted Open-Economy Shopping-Time Model P-Value ^b	Unrestricted Model		P-Value ^b
	Closed-Economy	Open-Economy	
A) Consumption as the scale variable:			
2.529	-----	2.860	0.077
2.529	2.851	-----	0.046
B) GDP as the scale variable in the Unrestricted Models ^c :			
2.512	-----	3.365	0.002
2.512	3.030	-----	0.009

- a. The smallest sample used for regression estimation is 1971:1-1980:4. The sample size of the regression grows as one makes successive predictions. The prediction period is 1981:1-1997:2, and the number of predictions is 66. The Root Mean Square Prediction Errors are in percentage terms. That is, for example, 2.529 denotes that the root mean square prediction error of real money growth rate is 2.529%.
- b. These P-Values are the levels at which the observed test statistics would be just significantly different from zero. The null hypothesis of the test statistics is that the difference between the MSPEs from different models is zero.
- c. In this section, money and scale variables are measured in aggregate levels.

Notes:

1. As noted in McCallum (1989), it might be that the real money held at the start of period t , rather than at the end, is the relevant magnitude. In actuality, however, the real balances held at each instant of time during the period are relevant. Therefore, for simplicity, the current specification is used.

2. The first order conditions with respect to domestic and foreign consumption, money, and bonds are:

(1) $U_c - U_L S_c = \delta_b$, (2) $U_{c^*} - U_L S_{c^*} = \delta_{q_b}$, (3) $-U_L S_m + E_t \delta_{t+1} P_t / P_{t+1} = \delta_b$, (4) $-U_L S_{m^*} + E_t \delta_{t+1} q_{t+1} P_t^* / P_{t+1}^* = \delta_{q_b}$, (5) $\$(1+r_t)E_t \delta_{t+1} P_t / P_{t+1} = \delta_b$, and (6) $\$(1+r_t^*)E_t \delta_{t+1} q_{t+1} P_t^* / P_{t+1}^* = \delta_{q_b}$. Combining (1) and (2) yields (2.2.6). Manipulating (1), (3), and (5) yields (2.2.4). Likewise, manipulating (2), (4), and (6) yields (2.2.5).

3. The data for the U.S. three-month T-Bill rate *in Canada* are not available. This variable is equal to the Canadian T-Bill rate minus the “Canada/USA T-Bill rates differential *in Canada*”. The latter can be constructed by two available series: “the covered differential Canada/USA three-month treasury bill rate” and “the USA dollars in Canada 90-day forward differential.” The second variable is used to remove the “covered exchange rates risk” from the first variable.

4. The sign on the effect of the real exchange rate on the demand for domestic money can be positive or negative. A depreciation of the domestic currency decreases the demand for domestic money. However, it also reduces the rate of return to foreign residents of domestic bonds and raises their demand for domestic money.

5. The first two data points are used as initial conditions. Therefore, the sample size in the first regression is 38.