Forecasting Asymmetries in Aggregate Stock Market Returns:

Evidence from Conditional Skewness

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Abstract

This paper provides a time-series test for the *Differences-of-Opinion* theory proposed by Hong and Stein (2003) in the aggregate market, thus extending Chen, Hong, and Stein's (2001) cross-sectional test for this theory across individual stocks. An autoregressive conditional density model with a skewed-*t* distribution is used to estimate the effects of past trading volume on return asymmetry. Using NYSE and AMEX data from 1962 to 2000, we find that the prediction of the Hong-Stein model that negative skewness will be most pronounced under high trading volume conditions is not supported in our time-series analysis with market data.

JEL Classification: C51, G12.

Keywords: differences of opinion; asymmetry; skewed-*t* distribution; autoregressive conditional density models.

Introduction

An extensive literature documents the asymmetrical distribution of stock returns. Several economic theories have been proposed to explain the mechanism generating this asymmetry, including leverage effects, volatility feedback mechanism, and stochastic bubbles models.[1] Since the theories all focus on mechanisms in the aggregate, they could be formulated as a representative-agent model. Hong and Stein (2003), however, propose an alternative theory based on investor heterogeneity. Their model assumes that differences of opinion exist among investors and that some investors face short-sale constraints. When disagreement is high, it is more likely that bearish investors do not initially participate in the market, and their information is not fully incorporated into prices, because of their short-sale constraints. If the market receives positive news, bullish investors' information is still revealed in prices, while bearish investors' information remains hidden. On the other hand, if the market receives negative news and the previously bullish investors have a change of heart and bail out of the market, those previously bearish investors may become the marginal "support buyers" and hence reveal more of their information. Thus, accumulated hidden information tends to come out when the market is falling. That is, conditional on high investor heterogeneity, volatility is higher when the return is low, which explains why returns are negatively skewed.

A distinctive feature of the Hong-Stein model not shared by representative-agent models is that it provides testable empirical implications. Their model predicts that negative skewness will be most pronounced under high trading volume conditions. Specifically, the Hong-Stein model shows that trading volume rises with the extent of heterogeneity among investors, which is consistent with literature stating that differences of opinion drive trading volumes [Harris and Raviv (1993), Kandel and Pearson (1995), and Odean (1998)]. Therefore, when disagreement among investors is high, trading volume is high. Based on this information, we would expect the return to be from a more negatively skewed distribution.

[1] See Chen, Hong, and Stein (2001) for a summary of those mechanisms.

Motivated by the Hong-Stein model, Chen, Hong, and Stein (2001) develop a series of cross-sectional regressions in an attempt to forecast asymmetry in the daily returns for individual stocks. Their baseline measure of asymmetry is the sample skewness of market-adjusted returns over a period of six months. The key forecasting variables are the past turnover (or "relative volume" – the ratio of trading volume to the shares outstanding) and returns. The estimated coefficients from the regressions of negative skewness on past turnover and returns are always positive and strongly significant. They conclude that negative skewness is most pronounced in stocks that have experienced either an increase in turnover over the prior six months or high past returns over the prior thirty-six months.

Chen, Hong, and Stein focus mainly on the cross-sectional regressions for individual stocks and only briefly experiment with a time-series regression for the stock market as a whole, which they report would have been a more interesting practice. They argue that the statistical power issue restricts the time-series analysis. At the individual firm level, there is enough cross-sectional data, so they compute the sample skewness in daily returns measured at non-overlapping six-month intervals. However, in the time-series regressions with aggregate market data, the limited sample size would yield a loss in statistical power.

This paper uses an alternative measure of asymmetry so that Chen, Hong, and Stein's empirical work can be extended to a rigorous time-series analysis. Specifically, instead of using the sample skewness in six-month intervals, we propose a parametric model in which a parameter that measures asymmetry is changing everyday and depends on the daily available information on forecasting variables.

Chen, Hong, and Stein assume that the population skewness is fixed within a given six-month period, but is time-varying across six-month periods. However, the Hong-Stein model does not imply any specific time horizon during which the population skewness is fixed. The choice of six-month intervals is based on concerns of measurement errors; i.e., the horizon cannot be too short because the calculation of a higher-order sample moment would be strongly influenced by outliers in the sample. In our approach, the population skewness changes everyday. We apply an "autoregressive conditional density (ARCD) model" suggested by Hansen (1994) to estimate a conditional parameter that measures the daily conditional skewness. By specifying the distributional parameter as a function of the forecasting variables, we have sufficient statistical power to test the significance of the effects of the forecasting variables on asymmetry.

Our framework builds on a GARCH model with a flexible parametric distribution for residuals, where the parameters in the conditional density are time-varying and depend on the forecasting variables.[2] This modeling strategy permits parametric specifications for conditional dependence beyond the first two moments. To model the variation in the conditional distribution beyond the mean and variance, we use Hansen's "skewed student *t*" (ST) distribution to describe the conditional density. The ST distribution is a parsimonious two-parameter distribution, but also a flexible one, able to model not only leptokurtosis but also asymmetry.[3]

Using NYSE and AMEX daily data from July 1962 to December 2000, we find that, in contrast to the prediction of the Hong-Stein model, higher prior turnover either predicts a more *positively* skewed distribution for future market returns, or does not have a statistically significant

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[2] Numerous methods exist for estimating unknown probability distributions. These include kernel estimators, semi-nonparametric (SNP) methods, and flexible parametric forms to capture the data characteristics such as skewness, tail-thickness, and peakedness. While kernel estimators and SNP have greater flexibility in modeling an unknown fixed distribution, they do not easily lend themselves to modeling the behavior of higher order moments. Often the flexible parametric representations have sufficient flexibility to represent data characteristics and facilitate modeling dynamic behavior of the moments through their functional relationship with the distributional parameters.

[3] An alternative model to deal with skewness in the distribution of returns is the SPARCH model, which uses a mixture of two distributions. The SPARCH model, however, does not have significant advantages over our GARCH-ST model. For example, Bekaert and Harvey (1997) use a mixture of two normal distributions to model the emerging market returns. Their SPARCH model has a three-parameter distribution. Our ST distribution only has two parameters to be estimated. Besides, their SPARCH model does not offer a parameter that directly measures the asymmetry of the distribution like our model does. In other SPARCH models, the choice of a weighting scheme between the two distributions is somewhat arbitrary.

predicting power on market skewness, depending on the different measures of turnover that we use to proxy for differences of opinion. Therefore, the insignificant result shown in Chen, Hong, and Stein's time-series analysis on market data is not due to the lack of statistical power because our model provides enough statistical power. We argue that either the prediction of the Hong-Stein model is not supported in the market level, or trading volume is simply a bad proxy for opinion divergence.

On the other hand, as a byproduct of our analysis, we also find significantly negative effects of prior 36-month returns on future skewness, consistent with Chen, Hong, and Stein's empirical study. That is, when past returns have been high, we predict that the future return distribution is more negatively skewed. This result is suggested by models of stochastic bubbles [see, for example, Blanchard and Watson (1982)]: after a long period of time that the bubble has been building up, a large drop when it pops is expected.

The remainder of the paper is organized as follows. Section I introduces the ARCD-ST model. We present and discuss the empirical results in Section II. Section III concludes the paper.

I. Model

The excess return is modeled as a GARCH-in-Mean process including lagged returns in the conditional mean, in an attempt to estimate the zero-mean, serially uncorrelated residuals:

$$y_t = \mu + \sum_{i=1}^{m} \theta_i y_{t-i} + \zeta \cdot h_t + \varepsilon_t \ , \tag{1}$$

where y_t is the daily return in excess of the risk-free rate at time t, and $h_t = E(\varepsilon_t^2 \mid t-1)$ is the conditional variance of the error term ε_t based on the information available at time t-1. The lag length m is chosen by the Ljung-Box Q tests as the minimum lag that renders serially uncorrelated residuals (at the 5% significance level) up to 30 lags from an OLS autoregression of

y_t. To keep our model parsimonious, we also remove the lags with insignificant estimated coefficients based on the OLS autoregression. Note that the OLS autoregression is used only to determine the number of lagged returns included in the conditional mean. The coefficients in the conditional mean equation will be jointly estimated with the conditional variance equation using full information maximum-likelihood (FIML) estimation.

The conditional variance h_t follows a GARCH process. It is well documented in the finance literature that stock returns have asymmetric effects on predictable volatilities; see among others, Glosten, Jaganathan, and Runkle (1993) and Bekaert and Wu (2000). Therefore, we use an asymmetric GARCH model proposed by Glosten, Jaganathan, and Runkle (the GJR model), which is claimed to be the best parametric model among a wide range of predictable volatility models experimented by Engle and Ng (1993). For a robustness check, we also experiment with another popular asymmetric GARCH model, the EGARCH model, proposed by Nelson (1991).

In addition to the asymmetric variance specification, we also include a Monday dummy in the GARCH process, as suggested by Harvey and Siddique (1999), who argue that Mondays are characterized by substantially greater volatility than the other days of the week. Finally, to control the effect of turnover on variance, we also include past turnover in the GARCH process. The role of turnover in forecasting volatilities is suggested by the mixture hypothesis in the finance literature [see Lamoureux and Lastrapes (1990) and Laux and Ng (1993)], which states that the GARCH effect in daily stock returns reflects time dependence in the process generating information flow to the market, and turnover proxies for information arrival time.

Our asymmetric GARCH model with the GJR specification is as follows:

$$h_t = \kappa + \alpha \cdot h_{t-1} + \beta \cdot \varepsilon_{t-1}^2 + \gamma \cdot I_{t-1}^{+} \cdot \varepsilon_{t-1}^2 + \delta \cdot TO_t + \phi \cdot MON_t, \tag{2}$$

where $I_{t-1}^{+} = 1$ if $\varepsilon_{t-1}>0$ and $I_{t-1}^{+} = 0$ if $\varepsilon_{t-1}<0$, TO_t is a measure of past turnover (to be defined later), and MON_t is the Monday dummy. Similarly, our EGARCH model is specified as

(2') $$\log(h_t) = \kappa + \alpha \cdot \log(h_{t-1}) + \beta \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \gamma \cdot I_{t-1}^{+} \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \delta \cdot TO_t + \phi \cdot MON_t .$$

The relationship between the return residuals and its conditional variance can be specified as $\varepsilon_t = \sqrt{h_t} v_t$, where v_t is a zero-mean and unit-variance random variable. The specification of the distribution of the stochastic process $\{v_t\}$ determines the distribution of y_t. In response to the levels of kurtosis found in stock return data, Bollerslev (1987) combined a *t*-distribution and a GARCH (1,1) model. However, the *t*-GARCH model is still symmetric and unable to model the skewness in stock returns. Hansen (1994) proposes a skewed student's *t* (ST) distribution for v_t.[4] The density function of the ST distribution is written as

(3) $$g_{ST}(v_t \mid \eta, \lambda) = \begin{cases} \sigma \cdot c \cdot \left[1 + \frac{1}{\eta - 2} \left(\frac{\sigma \cdot v_t + \mu}{1 + \lambda} \right)^2 \right]^{-\frac{\eta+1}{2}} & \text{for} \quad v_t \geq -\frac{\mu}{\sigma}, \\ \sigma \cdot c \cdot \left[1 + \frac{1}{\eta - 2} \left(\frac{\sigma \cdot v_t + \mu}{1 - \lambda} \right)^2 \right]^{-\frac{\eta+1}{2}} & \text{for} \quad v_t < -\frac{\mu}{\sigma}, \end{cases}$$

where $2 < \eta < \infty$, $-1 < \lambda < 1$, $\mu = 4 \cdot \lambda \cdot c \cdot \frac{\eta - 2}{\eta - 1}$, $\sigma = \sqrt{1 + 3 \cdot \lambda^2 - \mu^2}$, and $c = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\Gamma\left(\frac{\eta}{2}\right)}$.

For details on the ST density, see the appendix in Hansen (1994). The ST distribution is able to model not only leptokurtosis but also asymmetry. The parameter η controls the tails and the peak of the density and λ controls the rate of descent of the density around $v_t = 0$. Specifically, when $\lambda > 0$, the mode of the density is to the left of zero and the distribution skews to the right, and vice-versa when $\lambda < 0$. When $\lambda = 0$, the distribution is symmetric.

[4] Other candidates would include the exponential generalized beta of the second kind (EGB2) and transformations of normally distributed variables discussed by Johnson (1949). Wang et al. (2001) apply the EGB2 to GARCH models. Hansen, McDonald, and Theodossiou (2002) compare the performance of several asymmetric distributions with GARCH models and find similar performance. Thus the ST appears to be an excellent choice for attempting to model dynamic behavior of higher-order moments.

In the GARCH literature, the conditional distribution of the stochastic process $\{v_t\}$ is simply assumed to be independent of the conditioning information at t-1. The only features of the conditional distribution of y_t that depend on the information at t-1 are the mean and the variance. Hansen (1994) suggests that conditional dependence should be allowed beyond the first two moments. His "autoregressive conditional density" (ARCD) modeling strategy is to model the parameters in the conditional density function as functions of the elements of the information set so that the higher moments also depend on the conditioning information.

We build our model based on Hansen's setup, but our model differs from Hansen's in the specification of the laws of motion for the time-varying parameters.[5] Hansen conjectures that since GARCH models make the conditional second moment a function of the lagged errors, it is reasonable to believe that this strategy could also work well for the other moments. Therefore, he models the distribution parameters as quadratic functions of the lagged error terms. This paper utilizes the theoretical guidance from the Hong-Stein model and the empirical evidence from Chen, Hong, and Stein (2001) to specify the laws of motion for the time-varying parameters. That is, we assume that $v_t (= \varepsilon_t / \sqrt{h_t})$ follows the ST distribution (3) with time-varying parameters:

(4) $$\eta_t = 2.01 + \frac{30 - 2.01}{1 + \exp(-\omega_{1,t})}, \qquad \omega_{1,t} = a_1 + b_1 \cdot TO_t + c_1 \cdot RET_t + d_1 \cdot \sqrt{h_{t-1}} + e_1 \cdot \omega_{1,t-1},$$

(5) $$\lambda_t = -.99 + \frac{.99 - (-.99)}{1 + \exp(-\omega_{2,t})}, \qquad \omega_{2t} = a_2 + b_2 \cdot TO_t + c_2 \cdot RET_t + d_2 \cdot \sqrt{h_{t-1}} + e_2 \cdot \omega_{2,t-1},$$

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5 Harvey and Siddique (1999) use an alternative method to estimate conditional skewness. Instead of modeling the conditional skewness through the distributional parameter as Hansen (1994) and we do, they specify a conditional skewness process that is similar to the conditional variance process; i.e., the conditional skewness is a function of its own past value and the cube of the past residuals. Their methodology, however, is more computationally difficult than ours in that they first obtain the estimated conditional moments and then use them to compute the distributional parameters to be used in the likelihood function. By using the ST distribution, they need to solve two equations for two unknowns in each iteration of the MLE. Therefore, our approach has a computational advantage over the Harvey-Siddique model.

where the logistic transformation $\theta = L + \frac{(U-L)}{1+\exp(-\omega)}$ is used to set constraints on the parameters. With this transformation, even if ω is allowed to vary over the entire real line, θ will be constrained to lie in the region [L, U].

The key variable in the specifications of the time-varying parameters (4) and (5) is the past turnover TO_t. According to the Hong-Stein model, when differences of opinion are large, those bearish investors who are subject to the short-sales constraint will sit out of the market and their information is not revealed. This accumulated hidden information tends to come out during market declines. Therefore, the Hong-Stein model predicts that negative skewness in returns will be most pronounced after periods of heavy trading, because trading volume (and therefore turnover) proxies for the differences of opinion. However, the length of this information accumulation period is a subject of debate. A short information accumulation period may make TO_t a noisy proxy for differences of opinion. Chen, Hong, and Stein (2001) suggest the use of the average over the past six-month period. Therefore, we define TO_t as the average of turnover ratios from date t-125 to date t-1.

The other variables in (4) and (5) are included as control variables in an attempt to isolate the effects of turnover. In particular, Chen, Hong, and Stein (2001) find that when past returns have been high, skewness is forecasted to be more negative. They claim that returns as far back as 36 months still have some ability to predict negative skewness. Therefore, we define RET_t as the average of the returns from date t-750 to date t-1.

The selection of the variables that affect the distribution parameters is based on Chen, Hong, and Stein (2001). This is, however, by no means the complete list of factors that influence the asymmetry. Our goal is to see whether past turnover ratio has a significantly negative effect on asymmetry, as suggested by the Hong-Stein model, in the time-series setup. Some omitted variables that have impacts on asymmetry and are correlated with turnover may exaggerate the impacts of turnover. However, due to the lack of theoretical justifications in the literature on possible impacts of other variables on asymmetry, and to keep our model parsimonious, we only include variables similar to those used in Chen, Hong, and Stein (2001).

Recall that λ governs the asymmetry of the distribution. When $\lambda > 0$, the distribution is positively skewed. When $\lambda < 0$, the distribution is negatively skewed. Therefore, the parameter of particular interest is b_2, the effect of the average of prior turnover ratios on λ_t. The Hong-Stein model predicts that b_2 is negative – if the prior turnover proxies the differences of opinion, negative skewness is more pronounced if prior turnover is higher. On the other hand, Chen, Hong, and Stein's (2001) empirical evidence also emphasizes that c_2 is negative: when past returns have been high, skewness is forecasted to become more negative.

II. Estimation Results

A. Data and Specification Test

We collect daily data on stock prices, returns, trading volumes, and shares outstanding for all NYSE and AMEX firms from the CRSP databank for the period from July 1962 to December 2000. The market returns and turnover ratios are value-weighted based on the market values of individual stocks. The risk-free rate is defined as the three-month T-bill rates at daily rates taken from the FRED database published by the Federal Reserve Bank of St. Louis. The historical market excess returns and turnover ratios are plotted in Figures 1 and 2. As expected, all the values around October 19, 1987 are very volatile. As is common practice in the literature, to address the concern that the enormous daily movements during October 1987 may dominate our inferences, we also show the regression results omitting October 1987.

Since the specification of the conditional density plays an important role in our paper, before estimating the model we conduct an out-of-sample conditional density forecasts evaluation proposed by Diebold, Gunther, and Tay (1998). Their method evaluates the "probability integral transform series." This series is obtained as follows. First, the sample is split into in-sample and out-of-sample periods for model estimation and density forecast evaluation. The data up to 1990 are used as the in-sample observations and the data from the last ten years of our sample (1991-2000) are used for out-of-sample density forecasts. We use the in-sample observations to estimate the model and then freeze the estimated model. Based on the estimated model and the available observations of the variables, the forecast of the next day's conditional density is formed. After the next day's return is realized, we calculate the implied CDF value of this return in the forecasted distribution. The time-series of CDFs generated by this process in the out-of-sample period is the "probability integral transform series." This series should follow an i.i.d. $U(0,1)$ distribution.

Denote the integral transform series as z_t. In addition to the standard tests such as the Kolmogorov-Smirnov test of i.i.d. $U(0,1)$, Diebold, Gunther, and Tay (1998) suggest the use of histogram of z_t and the correlograms of $(z_t-\overline{z_t})$, $(z_t-\overline{z_t})^2$, $(z_t-\overline{z_t})^3$, and $(z_t-\overline{z_t})^4$ to evaluate the conditional density specification. Figures 3a-d plots the histograms of z_t with 20 bins for each of the four models we consider: the GJR specification and the EGARCH specification with and without October 1987 observations. Figure 4a-d shows the correlograms. The dashed lines superimposed on the charts are approximate 95% confidence intervals under the null that z_t is i.i.d. $U(0,1)$.[6] In general, the histograms are very close to uniform and no "butterfly" pattern is revealed, which shows the ability of the ST distribution in modeling fat tails and asymmetry of the distribution. On the other hand, the correlograms also do not reveal strong serial correlations. Most of the autocorrelations are within the 95% confidence intervals. For those statistically significant observations, the autocorrelations are all below 0.1. Finally, the Kolmogorov-Smirnov test of i.i.d. $U(0,1)$ fails to reject the null at the 5% level for two of the four models. The test statistics from the other two models are very close to the 5% critical value.

[6] The 95% confidence intervals for the bin heights in the histograms are obtained from 10,000 Monte Carlo simulations. The 95% confidence intervals for the correlograms are obtained by using Bartlett's formula.

Overall, both the GJR and EGARCH specifications are able to pick up the dynamics of the moments, and whether or not to include the October 1987 observations does not change the results of the specification tests. These findings justify our specification of the conditional density.

B. Estimation Results

Table 1 shows the estimation results for the four models mentioned above using the whole sample. First of all, the conditional mean and variance equations fit very well for all four models. All the coefficients have the expected signs and are almost all statistically significant at the 5% level. In the conditional mean equation, the conditional variance has a positive effect on returns, which is consistent with the finance theory that an asset with a higher perceived risk would pay a higher return on average.

In the conditional variance equation, the signs on the estimated coefficients of the GARCH process and Monday dummy are consistent with the findings by Harvey and Siddique (1999), who also use a GJR model and an EGARCH model (without the turnover variable) for value-weighted daily returns on the S&P500 index from 1969 to 1997. First, the GARCH process is highly persistent. Second, the return shocks have asymmetric effects on predictable variance. Negative shocks tend to cause more volatilities than positive shocks do. Finally, the conditional variance tends to be higher on Mondays and when prior turnover is high.

Now turn to the law of motions of the distribution parameters. The estimated coefficients on the lagged parameters are statistically significant and close to one, indicating that the dynamic processes of the parameters, and therefore the higher moments, are very persistent. The conditional variance has a positive and significant effect on λ in the GJR model including October 1987 observations, and on η in the EGARCH model excluding October 1987 observations.

Our focus is on the law of motion for λ. Especially, the effects of prior turnover ratios on λ (b_2) are all positive, a result that contradicts the prediction of the Hong-Stein model. The positive effects are only marginally significant though, with P-values equal to 0.091, 0.120, 0.050, and 0.097, respectively in these four models. On the other hand, consistent with the findings of Chen, Hong, and Stein (2001), prior 36-month returns predict a more negatively skewed return: the estimated c_2 are all negative and statistically significant. This result can be explained by models of stochastic bubbles. That is, higher past returns imply that the bubble has been building up for a while and a larger drop is expected when it pops.

C. Estimation Results from Alternative Measures of Trading Volume

As mentioned in the previous section, turnover ratios are a proxy for the differences of opinion, the variable that actually predicts negative skewness in the Hong-Stein model. The average of prior six-month turnover ratio we use to obtain the above results may be a noisy and crude proxy for opinion divergence and therefore, we may not provide a fair test for the Hong-Stein model. To address this concern, we experiment with several measures of turnover ratios to check the robustness of our results.

First, the Hong-Stein model predicts that negative skewness in returns will be most pronounced after periods of heavy trading. However, the length of the periods is a subject of debate. The choice of the six-month period used by us and Chen, Hong, and Stein (2001) is arbitrary. Therefore, in Table 2, we try different information accumulation periods for TO_t and rerun the estimations using the GJR specification for the whole sample. In particular, we experiment with prior one-day (daily), five-day (weekly), 21-day (monthly), 250-day (annual), and 750-day (36-month) average turnover ratios. As can be seen from Table 2, the results are very consistent across different definitions of TO_t. An important difference is that the effects of prior turnover ratios on asymmetry are not only positive, but also significantly different from zero at the 5% level when the prior daily, weekly, monthly, and 36-month average turnover ratios are used. This finding strengthens our argument that Hong and Stein's prediction is not supported in the aggregate market.

The second test for robustness we perform is suggested by the empirical trading volume literature [e.g., Campbell, Grossman, and Wang (1993) and Lee and Swaminathan (2000)]. Turnover is used as the volume measure in most previous studies because it reduces the low-frequency variation in volume. However, as can be seen from Figure 2, it does not eliminate it completely. Turnover still has an upward trend in the late 1960's, in the period between the elimination of fixed commissions in 1975 and the crash of 1987, and during the market boom in the 1990's. These trends in the level of turnover may be due in part by technology innovation or increased stock market participation. Lee and Swaminathan (2000) suggest that the information content of turnover ratios be due to intertemporal variations in normal trading activity. Therefore, they compute the *change* in turnover ratios as a measure of the abnormal trading activity. Following their methodology, we define the change in turnover ratios as the average daily turnover ratio over the past six months minus the average daily turnover ratio from date *t*-1250 to date *t*-1001 (approximately a one-year period from four years ago). The use of the four-year horizon is driven by the empirical fact that the level of turnover is a very slowly mean reverting process. And because of this reason, for comparison, Lee and Swaminathan also compute results by using the *lagged* level of turnover ratios from four years ago.

Panel (A) of Table 3 compares the estimation results from the GJR model with three different measures of trading volume. The full sample is used. The first column repeats the results of the first column of Table 1 and the fourth column of Table2, which use the average turnover ratios from the past six month. The second column replaces the level of turnover ratios with the changes in turnover ratios over the past four years. The third column uses the lagged turnover ratios over a six-month period from four years ago. To control the return effect, in the third model we also lag the past 36-month average return by four years. The results are very consistent across three alternative measures of turnover, with only three noticeable differences from the first model to the second and the third models. First of all, the new definitions of trading volumes do not have a significant effect on the conditional variance anymore. Secondly, the estimated effects of past returns on market skewness (c_2) are now statistically insignificant. The sign even changes in the lagged-turnover model. Finally, and most importantly, the estimates of b_2, the effects of prior turnover ratios on market skewness, are negative for models with changes in turnover ratios and lagged turnover ratios. However, the estimates are not statistically significant. This observation is consistent with the findings of Chen, Hong, and Stein's (2001) time-series test: using aggregate market data, past turnover ratios have a negative effect on market skewness, but is statistically insignificant.

Finally, we experiment with models excluding past returns from the dynamics of the distributional parameters. This experiment is motivated by the fact that we find a negative relation between past returns and skewness. As suggested by models of stochastic bubbles, past returns pick up periods of overpricing. Therefore, the parameter b_2 can be interpreted as telling us whether past trading volume predicts a market crash after controlling for the level of overpricing. Dropping past returns allows us to answer the question of whether high past trading volume picks up when the aggregate market is overpriced due to increased opinion divergence and is hence more likely to crash. The results are shown in Panel (B) of Table3. All the estimates the effects of prior turnover ratios on skewness are negative, but are statistically insignificant. Therefore, even if we do not control the effects of past returns on skewness, we still cannot find statistically significant predicting power of past turnover ratios on future market crash.

In sum, we find that higher past turnover ratios either predict more positively skewed market returns, or have an insignificant negative effect on skewness of market returns. That is, the prediction of the Hong-Stein model that negative skewness will be most pronounced under high trading volume conditions is not supported in our time-series analysis with market data. The insignificantly negative effects of past turnover on skewness, though, are consistent with Chen, Hong, and Stein's time-series results using market data. Therefore, their claim that their insignificant result is due to the lack of statistical power is not legitimate because our model provides enough statistical power.

D. Estimation Results from a portfolio Excluding the S&P 500 Stocks

The Hong-Stein model predicts that, due to short-sale constraints, negative skewness will be most pronounced under high trading volume conditions. However, the literature on options trading argues that introducing options trading can potentially reduce or even eliminate the informational effect of short-sale constraints. For example, Figlewski and Webb (1993) present empirical evidence that trading in options allows investors who face short-sale constraints to take equivalent option positions. Therefore, one possible explanation for the lack of a negative relation between past turnover and market skewness is that the short-sale constraint may not be binding at the market level, given liquid future and option markets.

Even though our focus is on testing the Hong-Stein implication at the market level over time, it will be interesting to deviate from the market analysis and test for this conjecture by analyzing a sub-market portfolio where short-sale constraints are more likely to bind. Previous empirical studies suggest that small stocks with low institutional ownership are more expensive to short than large stocks with high institutional ownership. That is, our results using the market data may be dominated by economically important stocks in the market portfolio that are easy and cheap to short. Therefore, we form a portfolio by excluding the S&P 500 stocks from the market portfolio in an attempt to mimic a portfolio consisting of stocks that are hard for investors to short.[7] We use this new data set to repeat the tests parallel to those in Tables 3 and 4 and report the results in Tables 4 and 5. It is shown that, even if we exclude the S&P 500 stocks from

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[7] On each date of the sample period, we took out the stocks that were in the S&P 500 on that date from the market portfolio we formed in Section II.A.

the market portfolio, our conclusion is not changed – past turnover ratios still have no statistically significant predicting power on portfolio skewness in the time-series analysis.

III. Discussion and Conclusion

Chen, Hong, and Stein's (2001) time-series test on Hong-Stein's (2003) Differences-of-Opinion model is not able to generate statistically significant results, and they claim that it is because of the lack of statistical power. This paper proposes a model that can be used to conduct this time-series test with sufficient statistical power. We build a parametric model in which a distributional parameter controls the asymmetry of the distribution. This parameter is modeled as time-varying and depends on the daily observation of the forecasting variables. Therefore, daily observations of skewness are available and we are able to estimate the effects of the forecasting variables on asymmetry.

Different from the cross-sectional results in Chen, Hong, and Stein, in which the effect of prior turnover on skewness is found to be negative and therefore supports the Hong-Stein model, this paper provides time-series evidence for the aggregate market that contradicts the prediction of the Hong-Stein model. Higher past turnover either predicts more positively skewed market returns or does not have a statistically significant predicting power on market skewness, depending on the different measures of past turnover being used. The results of this research, however, do not dispute the cross-sectional results obtained by Chen, Hong, and Stein using individual stocks. Instead, we argue that the insignificant result shown in their time-series analysis on aggregate market is not due to the lack of statistical power because our model provides enough statistical power.

Sharing the same concerns with those indicated in Chen, Hong, and Stein, our time-series analysis may not be a tight test of the Hong-Stein model. Specifically, it is still subject to debate whether our turnover variables are a good proxy for the intensity of disagreement among investors. The turnover variables may as well capture other factors that are not controlled in our parsimonious model, such as changes in trading costs. Nonetheless, we believe that we provide a theoretical framework which yields a time-series analysis comparable to the cross-sectional analysis conducted by Chen, Hong, and Stein. This paper identifies some issues associated with aggregate data which need to be addressed in the theory of return asymmetry.

The role of the effect of future and option markets on short-sale constraints in the Hong-Stein model is a topic of interest for future studies. Since our focus is on the time-series analysis of the aggregate market data, we only briefly analyze a sub-market portfolio where short-sale constraints are more likely to bind. Even though we show that this experiment does not change our conclusion, it is possible that our approximation to the portfolio facing short-sale constraints is not a good one. On the other hand, it is also possible that the effect of the short-sale constraints disappears in some way when a portfolio is formed. Further insight can be gained by analyzing comparative portfolios that contain stocks with and without future and option trading in both cross-sectional and time-series analyses.

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Table 1: Estimation Results

The parameters are shown in equations (1)--(5):

(1) $$y_t = \mu + \sum_{i=1}^{m} \theta_i y_{t-i} + \zeta \cdot h_t + \varepsilon_t\ , \qquad \varepsilon_t = \sqrt{h_t} v_t ,$$

(2) GJR $$h_t = \kappa + \alpha \cdot h_{t-1} + \beta \cdot \varepsilon_{t-1}^2 + \gamma \cdot I_{t-1}^{+} \cdot \varepsilon_{t-1}^2 + \delta \cdot TO_t + \phi \cdot MON_t\ ,$$

(2') EGARCH $$\log(h_t) = \kappa + \alpha \cdot \log(h_{t-1}) + \beta \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \gamma \cdot I_{t-1}^{+} \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \delta \cdot TO_t + \phi \cdot MON_t\ ,$$

(3) $$g_{ST}(v_t \mid \eta, \lambda) = \begin{cases} \sigma \cdot c \cdot \left[1 + \dfrac{1}{\eta - 2} \left(\dfrac{\sigma \cdot v_t + \mu}{1 + \lambda} \right)^2 \right]^{-\frac{\eta+1}{2}} & \text{for} \quad v_t \geq -\dfrac{\mu}{\sigma}, \\ \sigma \cdot c \cdot \left[1 + \dfrac{1}{\eta - 2} \left(\dfrac{\sigma \cdot v_t + \mu}{1 - \lambda} \right)^2 \right]^{-\frac{\eta+1}{2}} & \text{for} \quad v_t < -\dfrac{\mu}{\sigma}, \end{cases}$$

(4) $$\eta_t = 2.01 + \frac{30 - 2.01}{1 + \exp(-\omega_{1,t})}, \qquad \omega_{1,t} = a_1 + b_1 \cdot TO_t + c_1 \cdot RET_t + d_1 \cdot \sqrt{h_{t-1}} + e_1 \cdot \omega_{1,t-1},$$

(5) $$\lambda_t = -.99 + \frac{.99 - (-.99)}{1 + \exp(-\omega_{2,t})}, \qquad \omega_{2t} = a_2 + b_2 \cdot TO_t + c_2 \cdot RET_t + d_2 \cdot \sqrt{h_{t-1}} + e_2 \cdot \omega_{2,t-1}\ .$$

	Model	**GJR**	**EGARCH**	**GJR Excluding 10/1987**	**EGARCH Excluding 10/1987**
Mean Equation	μ (constant)	**0.021**	0.017	0.019	0.020
		(0.029)	(0.212)	(0.116)	(0.149)
	θ_1 (AR1)	**0.185**	**0.180**	**0.184**	**0.181**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**
	θ_2 (AR2)	**-0.028**	**-0.024**	**-0.028**	**-0.025**
		(0.009)	**(0.025)**	**(0.009)**	**(0.021)**
	θ_6 (AR6)			-0.020	-0.018
				(0.054)	(0.079)
	ζ (GARCH-in-Mean)	**0.044**	**0.051**	**0.053**	**0.050**
		(0.001)	**(0.022)**	**(0.010)**	**(0.037)**

The numbers in parentheses are P-values. A value of 0.000 indicates that the true value is smaller than 0.0005. All computations were performed using GAUSS MAXLIK module with the BHHH (Berndt, Hall, Hall, and Hausman's) algorithm using the default convergence criterion (10^{-5}) on the entries of the gradient vector. The estimated standard errors were calculated robust standard errors corresponding to results summarized in Greene (2003, P.520).

Table 1: Estimation Results (continued)

	Model	GJR	EGARCH	GJR Excluding 10/1987	EGARCH Excluding 10/1987
Variance Equation:	κ (constant)	**3.65E-07**	**-0.147**	3.66E-07	**-0.142**
		(0.004)	**(0.000)**	-------	**(0.000)**
	α (GARCH)	**0.924**	**0.982**	**0.920**	**0.983**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**
	β (ARCH)	**0.107**	**-0.189**	**0.104**	**-0.181**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**
	γ (asymmetry)	**-0.082**	**0.241**	**-0.079**	**0.233**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**
	δ (turnover)	**0.028**	**0.061**	**0.037**	**0.054**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**
	ϕ (Monday)	**0.016**	**0.171**	**0.021**	**0.166**
		(0.019)	**(0.000)**	**(0.008)**	**(0.000)**
Distribution Parameters: (η_t)	a_1 (constant)	0.028	-0.002	-0.017	-0.016
		(0.453)	(0.887)	(0.081)	(0.065)
	b_1 (turnover)	-0.871	-0.043	-0.049	-0.044
		(0.100)	(0.172)	(0.069)	(0.063)
	c_1 (return)	1.128	**0.083**	-0.003	-0.024
		(0.235)	**(0.046)**	(0.948)	(0.742)
	d_1 (GARCH)	-0.059	0.004	0.027	**0.024**
		(0.054)	(0.809)	(0.058)	**(0.028)**
	e_1 (lag)	**0.880**	**0.997**	**0.994**	**0.993**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**
(λ_t)	a_2 (constant)	-0.029	-0.021	-0.037	-0.026
		(0.069)	(0.201)	(0.186)	(0.195)
	b_2 (turnover)	0.119	0.090	0.160	0.131
		(0.091)	(0.120)	(0.050)	(0.097)
	c_2 (return)	**-0.687**	**-0.545**	**-0.792**	**-0.644**
		(0.004)	**(0.012)**	**(0.011)**	**(0.047)**
	d_2 (GARCH)	**0.042**	0.033	0.053	0.038
		(0.023)	(0.112)	(0.151)	(0.143)
	e_2 (lag)	**0.849**	**0.886**	**0.850**	**0.884**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**

The numbers in parentheses are P-values. All computations were performed using GAUSS MAXLIK module with the BHHH (Berndt, Hall, Hall, and Hausman's) algorithm using the default convergence criterion (10^{-5}) on the entries of the gradient vector. The estimated standard errors were calculated robust standard errors corresponding to results summarized in Greene (2003, P.520).

Table 2: Estimation Results with Different Horizons of Past Turnover

GJR model with full sample		Prior Daily Turnover	Prior Weekly Turnover	Prior Monthly Turnover	Prior 6-Month Turnover	Prior Annual Turnover	Prior 36-Month Turnover
Mean Equation:	μ (constant)	0.020	**0.020**	0.021	**0.021**	**0.021**	**0.021**
		(0.059)	**(0.049)**	(0.054)	**(0.029)**	**(0.029)**	**(0.028)**
	θ_1 (AR1)	**0.186**	**0.186**	**0.185**	**0.185**	**0.185**	**0.184**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	θ_2 (AR2)	**-0.027**	**-0.028**	**-0.028**	**-0.028**	**-0.028**	**-0.028**
		(0.012)	**(0.010)**	**(0.010)**	**(0.009)**	**(0.009)**	**(0.009)**
	ζ (GARCH-in-Mean)	**0.044**	**0.045**	**0.044**	**0.044**	**0.044**	**0.044**
		(0.009)	**(0.003)**	**(0.008)**	**(0.001)**	**(0.001)**	**(0.001)**
Variance Equation:	κ (constant)	3.7E-07	3.7E-07	3.7E-07	**3.7E-07**	**3.7E-07**	**3.7E-07**
		(0.260)	(0.513)	(0.199)	**(0.004)**	**(0.026)**	**(0.000)**
	α (GARCH)	**0.925**	**0.925**	**0.924**	**0.924**	**0.923**	**0.923**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	β (ARCH)	**0.106**	**0.106**	**0.106**	**0.107**	**0.107**	**0.108**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	γ (asymmetry)	**-0.084**	**-0.084**	**-0.083**	**-0.082**	**-0.083**	**-0.083**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	δ (turnover)	**0.028**	**0.028**	**0.028**	**0.028**	**0.029**	**0.030**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	ϕ (Monday)	**0.015**	**0.015**	**0.016**	**0.016**	**0.016**	**0.016**
		(0.041)	**(0.031)**	**(0.031)**	**(0.019)**	**(0.016)**	**(0.015)**
Distribution Parameters: (η_t)	a_1 (constant)	0.006	0.005	0.003	0.028	0.034	0.075
		(0.838)	(0.811)	(0.920)	(0.453)	(0.461)	(0.218)
	b_1 (turnover)	-0.192	-0.187	-0.255	-0.871	-1.059	-1.349
		(0.190)	(0.127)	(0.297)	(0.100)	(0.118)	(0.101)
	c_1 (return)	-0.046	-0.041	0.035	1.128	1.363	1.509
		(0.951)	(0.919)	(0.957)	(0.235)	(0.237)	(0.184)
	d_1 (GARCH)	-0.039	-0.037	-0.039	-0.059	-0.063	**-0.073**
		(0.321)	(0.151)	(0.327)	(0.054)	(0.056)	**(0.042)**
	e_1 (lag)	**0.941**	**0.944**	**0.932**	**0.880**	**0.861**	**0.850**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
(λ_t)	a_2 (constant)	-0.029	-0.028	-0.028	-0.029	-0.029	-0.031
		(0.075)	(0.075)	(0.086)	(0.069)	(0.068)	(0.053)
	b_2 (turnover)	**0.157**	**0.149**	**0.150**	0.119	0.122	**0.140**
		(0.037)	**(0.037)**	**(0.044)**	(0.091)	(0.066)	**(0.038)**
	c_2 (return)	**-0.797**	**-0.773**	**-0.782**	**-0.687**	**-0.673**	**-0.689**
		(0.004)	**(0.004)**	**(0.002)**	**(0.004)**	**(0.003)**	**(0.002)**
	d_2 (GARCH)	**0.038**	**0.038**	**0.038**	**0.042**	**0.040**	**0.041**
		(0.031)	**(0.031)**	**(0.042)**	**(0.023)**	**(0.025)**	**(0.018)**
	e_2 (lag)	**0.849**	**0.850**	**0.845**	**0.849**	**0.855**	**0.852**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**

See the notes on Table 1.

Table 3: Estimation Results with Different Definitions of Past Turnover

GJR model with full sample		(A) Including returns in the dynamics of distributional parameters			(B) Excluding returns in the dynamics of distributional parameters		
		with past 6-month Turnover[a]	with changes in Turnover[b]	with lagged Turnover and Return[c]	with past 6-month Turnover[a]	with changes in Turnover[b]	with lagged Turnover
Mean Equation:	μ (constant)	**0.021**	**0.023**	**0.023**	**0.022**	**0.022**	0.022
		(0.029)	**(0.024)**	**(0.039)**	**(0.021)**	**(0.031)**	(0.082)
	θ_1 (AR1)	**0.185**	**0.182**	**0.168**	**0.185**	**0.183**	**0.168**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	θ_2 (AR2)	**-0.028**	**-0.028**	**-0.028**	**-0.027**	**-0.027**	**-0.029**
		(0.009)	**(0.012)**	**(0.012)**	**(0.011)**	**(0.015)**	**(0.015)**
	ζ (GARCH-in-Mean)	**0.044**	**0.039**	**0.042**	**0.044**	**0.040**	**0.042**
		(0.001)	**(0.004)**	**(0.003)**	**(0.001)**	**(0.003)**	**(0.010)**
Variance Equation:	κ (constant)	**3.7E-07**	3.7E-07	3.6E-07	**3.7E-07**	3.7E-07	3.6E-07
		(0.004)	--------	(0.721)	**(0.000)**	--------	(0.987)
	α (GARCH)	**0.924**	**0.928**	**0.932**	**0.923**	**0.928**	**0.933**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	β (ARCH)	**0.107**	**0.099**	**0.093**	**0.108**	**0.099**	**0.093**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	γ (asymmetry)	**-0.082**	**-0.074**	**-0.070**	**-0.082**	**-0.073**	**-0.069**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	δ (turnover)	**0.028**	0.021	0.007	**0.027**	0.023	0.008
		(0.000)	(0.062)	(0.320)	**(0.000)**	(0.055)	(0.346)
	ϕ (Monday)	**0.016**	**0.037**	**0.034**	**0.017**	**0.036**	**0.033**
		(0.019)	**(0.000)**	**(0.001)**	**(0.011)**	**(0.000)**	**(0.003)**
Distribution Parameters: (η_t)	a_1 (constant)	0.028	-0.022	0.035	0.012	-0.093	0.050
		(0.453)	(0.711)	(0.385)	(0.696)	(0.322)	(0.602)
	b_1 (turnover)	-0.871	0.476	-0.987	-0.547	-0.115	-0.719
		(0.100)	(0.511)	(0.361)	(0.175)	(0.762)	(0.642)
	c_1 (return)	1.128	-1.538	0.790			
		(0.235)	(0.370)	(0.672)			
	d_1 (GARCH)	-0.059	-0.099	-0.082	-0.068	-0.087	-0.075
		(0.054)	(0.206)	(0.144)	(0.060)	(0.071)	(0.518)
	e_1 (lag)	**0.880**	**0.861**	**0.864**	**0.871**	**0.870**	**0.885**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
(λ_t)	a_2 (constant)	-0.029	-0.024	-0.036	**-0.047**	**-0.047**	-0.027
		(0.069)	(0.202)	(0.143)	**(0.023)**	**(0.012)**	(0.257)
	b_2 (turnover)	0.119	-0.110	-0.193	-0.055	-0.191	-0.077
		(0.091)	(0.259)	(0.086)	(0.312)	(0.066)	(0.272)
	c_2 (return)	**-0.687**	-0.318	0.608			
		(0.004)	(0.073)	(0.119)			
	d_2 (GARCH)	**0.042**	**0.050**	**0.050**	**0.061**	**0.058**	0.047
		(0.023)	**(0.022)**	**(0.047)**	**(0.010)**	**(0.008)**	(0.063)
	e_2 (lag)	**0.849**	**0.827**	**0.805**	**0.815**	**0.808**	**0.804**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**

See the notes on Table 1.

a. This is the same model as that in the first column of Table 1 and the fourth column of Table2.

b. This model replaces the level of turnover with the change in turnover defined as the average daily turnover from date t–1 to date t–125 (past six months) minus the average daily turnover from date t–1001 to date t–1250 (four years ago).

c. This model replaces the level of turnover with the lagged turnover defined as the average daily turnover from date t–1001 to date t–1125 (four-year-lagged six-month horizon). The return variable in the distributional parameter functions (RET_t) is changed to the average daily return from date t–1001 to date t–1750 (four-year-lagged 36-month horizon)

Table 4: Estimation Results with Different Horizons of Past Turnover from a Portfolio Excluding S&P 500 Stocks

GJR model with full sample		Prior Daily Turnover	Prior Weekly Turnover	Prior Monthly Turnover	Prior 6-Month Turnover	Prior Annual Turnover	Prior 36-Month Turnover
Mean Equation:	μ (constant)	**0.044**	**0.045**	**0.045**	**0.045**	**0.045**	**0.045**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	θ_1 (AR1)	**0.305**	**0.305**	**0.304**	**0.303**	**0.303**	**0.303**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	θ_2 (AR2)	**-0.050**	**-0.050**	**-0.049**	**-0.050**	**-0.049**	**-0.050**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	θ_3 (AR3)	**0.037**	**0.036**	**0.037**	**0.037**	**0.037**	**0.038**
		(0.001)	**(0.001)**	**(0.001)**	**(0.001)**	**(0.001)**	**(0.000)**
	ζ (GARCH-in-Mean)	**0.013**	0.012	0.013	0.013	0.013	**0.013**
		(0.048)	(0.054)	(0.066)	(0.065)	(0.073)	**(0.042)**
Variance Equation:	κ (constant)	3.7E-07	3.7E-07	3.7E-07	3.7E-07	3.7E-07	3.7E-07
		--------	--------	--------	--------	--------	--------
	α (GARCH)	**0.871**	**0.870**	**0.869**	**0.867**	**0.867**	**0.867**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	β (ARCH)	**0.182**	**0.182**	**0.183**	**0.185**	**0.185**	**0.185**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	γ (asymmetry)	**-0.140**	**-0.140**	**-0.139**	**-0.139**	**-0.139**	**-0.139**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	δ (turnover)	**0.019**	**0.020**	**0.021**	**0.020**	**0.021**	**0.021**
		(0.025)	**(0.019)**	**(0.015)**	**(0.024)**	**(0.024)**	**(0.031)**
	ϕ (Monday)	**0.053**	**0.053**	**0.053**	**0.056**	**0.056**	**0.055**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
Distribution Parameters: (η_t)	a_1 (constant)	-0.171	-0.195	-0.193	-0.205	-0.205	**-0.211**
		(0.217)	(0.159)	(0.145)	(0.116)	(0.128)	**(0.005)**
	b_1 (turnover)	**0.732**	**0.900**	0.810	0.679	0.663	0.461
		(0.035)	**(0.047)**	(0.062)	(0.093)	(0.097)	(0.228)
	c_1 (return)	**-3.445**	**-4.039**	-3.831	-3.474	-3.300	**-2.546**
		(0.049)	**(0.047)**	(0.060)	(0.071)	(0.075)	**(0.021)**
	d_1 (GARCH)	**-0.293**	**-0.328**	**-0.317**	**-0.314**	**-0.310**	**-0.307**
		(0.027)	**(0.023)**	**(0.031)**	**(0.035)**	**(0.038)**	**(0.000)**
	e_1 (lag)	**0.743**	**0.713**	**0.717**	**0.707**	**0.711**	**0.707**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
(λ_t)	a_2 (constant)	-0.020	-0.019	-0.019	-0.020	-0.021	-0.025
		(0.389)	(0.388)	(0.239)	(0.246)	(0.248)	(0.209)
	b_2 (turnover)	0.046	0.027	0.025	0.030	0.044	0.067
		(0.528)	(0.613)	(0.618)	(0.575)	(0.429)	(0.296)
	c_2 (return)	-0.872	-0.787	-0.703	-0.730	-0.748	-0.778
		(0.209)	(0.159)	(0.111)	(0.081)	(0.082)	(0.082)
	d_2 (GARCH)	0.040	0.040	0.039	0.040	0.039	0.039
		(0.216)	(0.233)	(0.097)	(0.105)	(0.128)	(0.118)
	e_2 (lag)	**0.825**	**0.834**	**0.851**	**0.846**	**0.848**	**0.843**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**

See the notes on Table 1.

Table 5: Estimation Results with Different Definitions of Past Turnover from a Portfolio Excluding S&P 500 Stocks

GJR model with full sample		(A) Including returns in the dynamics of distributional parameters			(B) Excluding returns in the dynamics of distributional parameters		
		with past 6-month Turnover[a]	with changes in Turnover[b]	with lagged Turnover & Return[c]	with past 6-month Turnover[a]	with changes in Turnover[b]	with lagged Turnover
	μ (constant)	**0.045**	**0.043**	**0.034**	**0.042**	**0.040**	**0.035**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	θ_1 (AR1)	**0.303**	**0.306**	**0.301**	**0.306**	**0.308**	**0.302**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	θ_2 (AR2)	**-0.050**	**-0.051**	**-0.052**	**-0.051**	**-0.053**	**-0.053**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	θ_3 (AR3)	**0.037**	**0.033**	**0.030**	**0.037**	**0.035**	**0.030**
		(0.001)	**(0.003)**	**(0.009)**	**(0.001)**	**(0.001)**	**(0.008)**
	ζ (GARCH-in-Mean)	0.013	0.013	**0.033**	**0.019**	**0.020**	**0.032**
		(0.065)	(0.106)	**(0.001)**	**(0.007)**	**(0.011)**	**(0.001)**
Variance Equation:	κ (constant)	3.7E-07	3.7E-07	3.6E-07	3.7E-07	3.7E-07	3.7E-07
		--------	--------	--------	--------	--------	--------
	α (GARCH)	**0.867**	**0.876**	**0.889**	**0.865**	**0.877**	**0.887**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	β (ARCH)	**0.185**	**0.167**	**0.145**	**0.187**	**0.168**	**0.147**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	γ (asymmetry)	**-0.139**	**-0.123**	**-0.103**	**-0.137**	**-0.121**	**-0.105**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
	δ (turnover)	**0.020**	0.024	**0.010**	**0.022**	0.024	0.011
		(0.024)	(0.066)	**(0.300)**	**(0.009)**	(0.076)	(0.279)
	ϕ (Monday)	**0.056**	**0.074**	**0.056**	**0.053**	**0.069**	**0.055**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
Distribution Parameters: (η_t)	a_1 (constant)	-0.205	-0.042	0.215	**-0.311**	**-0.322**	0.105
		(0.116)	(0.645)	(0.217)	**(0.006)**	**(0.001)**	(0.278)
	b_1 (turnover)	0.679	1.950	**-1.218**	0.032	0.490	**-1.585**
		(0.093)	(0.170)	**(0.040)**	(0.820)	(0.177)	**(0.010)**
	c_1 (return)	-3.474	-5.339	-2.760			
		(0.071)	(0.148)	(0.200)			
	d_1 (GARCH)	**-0.314**	-0.348	**-0.172**	**-0.275**	**-0.280**	**-0.185**
		(0.035)	(0.144)	**(0.028)**	**(0.002)**	**(0.001)**	**(0.005)**
	e_1 (lag)	**0.707**	**0.674**	**0.767**	**0.708**	**0.707**	**0.754**
		(0.000)	**(0.001)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
(λ_t)	a_2 (constant)	-0.020	-0.023	**-0.072**	-0.051	**-0.071**	**-0.057**
		(0.246)	(0.252)	**(0.032)**	(0.090)	**(0.034)**	**(0.049)**
	b_2 (turnover)	0.030	-0.037	-0.211	-0.127	-0.249	-0.121
		(0.575)	(0.696)	(0.076)	(0.135)	(0.069)	(0.145)
	c_2 (return)	-0.730	**-0.607**	0.510			
		(0.081)	**(0.038)**	(0.101)			
	d_2 (GARCH)	0.040	0.047	**0.081**	0.064	**0.067**	**0.073**
		(0.105)	(0.054)	**(0.020)**	(0.056)	**(0.029)**	**(0.022)**
	e_2 (lag)	**0.846**	**0.838**	**0.817**	**0.812**	**0.807**	**0.814**
		(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**

See the notes on Table 3.

Figure 1: Excess returns (%)
[Mean: 0.054 Standard Deviation: 0.855]

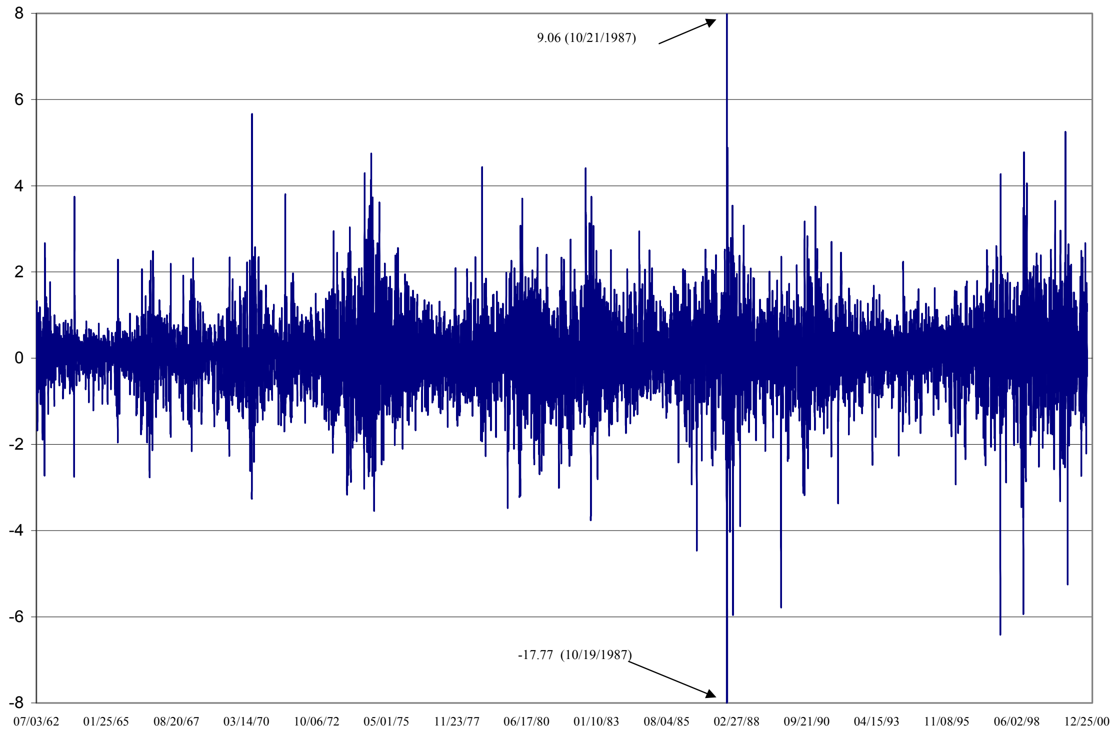


Figure 2: Turnover ratios (%)
[Mean: 0.185 Standard Deviation: 0.113]

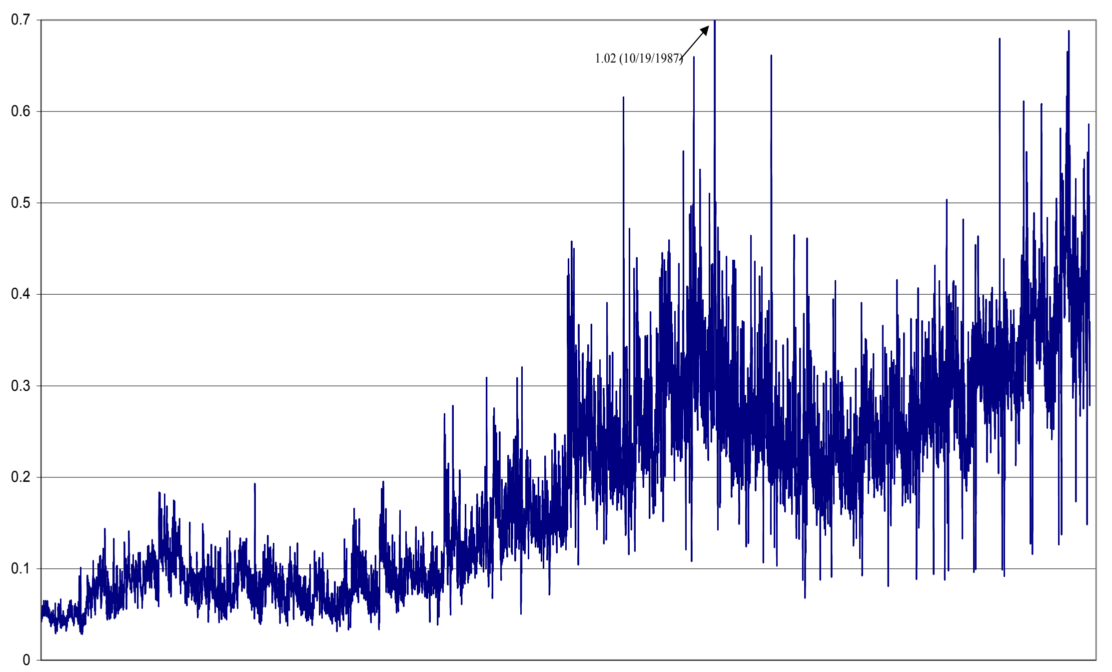
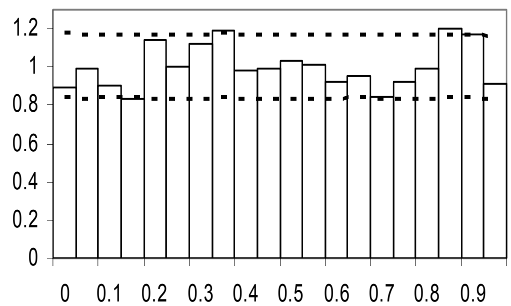
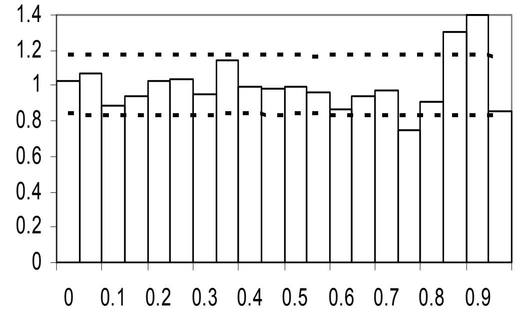


Figure 3: Estimate of the Density of z_t

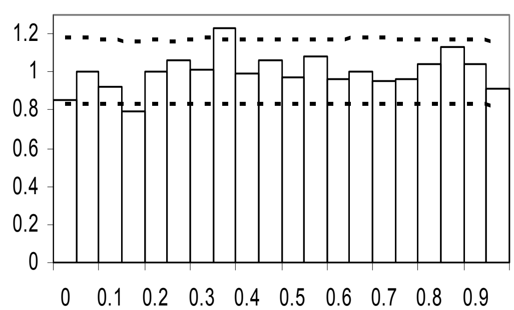
(a) GJR model including October 1987
(Kolmogorov-Smirnov test statistic = 0.020*)



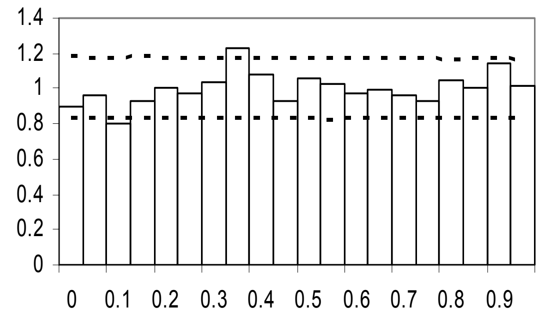
(b) EGARCH model including October 1987
(Kolmogorov-Smirnov test statistic = 0.030*)



(c) GJR model excluding October 1987
(Kolmogorov-Smirnov test statistic = 0.029*)



(d) EGARCH model excluding October 1987
(Kolmogorov-Smirnov test statistic = 0.026*)



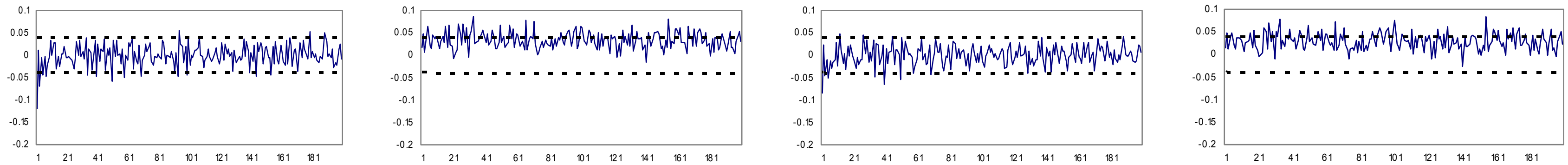
See text for details of the definition of the probability integral transform series z_t.

* The 10%, 5%, and 1% critical values for the Kolmogorov-Smirnov test statistics are 0.024, 0.027, and 0.032, respectively, for (a) and (b), and 0.024, 0.027, and 0.033, respectively, for (c) and (d).

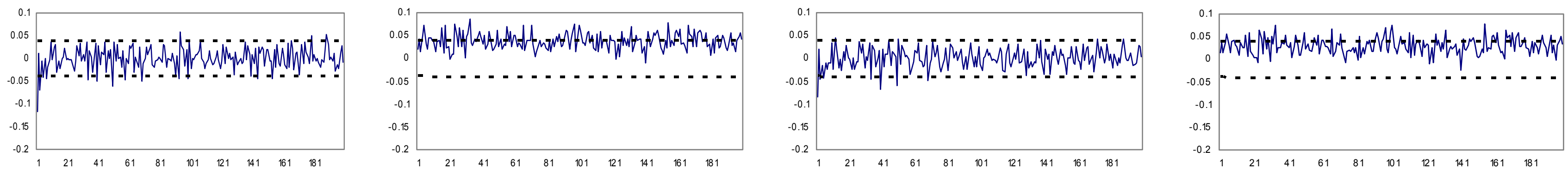
Figure 4: Estimates of the autocorrelation functions of powers of z_t

$(z_t - \overline{z_t})$ | $(z_t - \overline{z_t})^2$ | $(z_t - \overline{z_t})^3$ | $(z_t - \overline{z_t})^4$

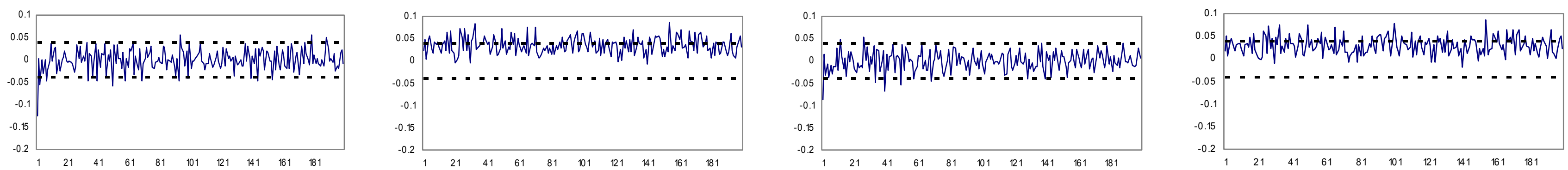
(a) The GJR model including October 1987 observations



(b) The EGARCH model including October 1987 observations



(c) The GJR model excluding October 1987 observations



(d) The EGARCH model excluding October 1987 observations