

**DO BUBBLES AND TIME-VARYING RISK PREMIUMS AFFECT
STOCK PRICES?
A KALMAN FILTER APPROACH^a**

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ABSTRACT

This paper separates the validity of the specification of the fundamental stock price model from the implications of bubbles. The time-varying risk premium model (Poterba and Summers, 1986) is used to explicitly derive the misspecification component. We construct a state-space model and use Kalman Filter to estimate the relationships between the observable price/dividend and the unobservable bubbles/misspecification. The model is applied to CRSP and S&P 500 data. The results show that the fundamental price model does not describe the market prices well. The time-varying risk premium is important in explaining stock price movements. No significant evidence of bubbles is found.

I. INTRODUCTION

This paper studies whether speculative bubbles and model-misspecification exist in the stock prices. We separate the validity of the specification of the fundamental stock price model from the implications of a bubble component to stock prices. The fundamental price model is constructed by Lucas (1978). He proposes that, based on the rational expectations hypothesis, the stock price should reflect the discounted values of future dividends and stock prices. This ideal specification, however, is not supported by many empirical studies [e.g. Shiller (1981); LeRoy and Porter (1981); Blanchard and Watson (1982)]. The failure of the fundamental price model has often been considered as evidence of the existence of rational bubbles. A rational bubble reflects a self-fulfilling belief that an asset's price depends on variables that are not parts of market fundamentals. The existence of bubbles causes discrepancies between an asset's price and its fundamental value.

However, studies such as Flood and Garber (1980) and Hamilton and Whiteman (1985) argue that the evidence presented in favor of the claim of self-fulfilling speculative bubbles might instead have arisen from rational agents responding to market fundamentals that the researchers cannot observe. One possibility for the unobservable fundamentals to exist is that the fundamental price model is misspecified. Tests for bubbles that take correct model-specification as given and interpret deviations from the fundamental price as evidence of bubbles may not be justified because they reject the joint hypothesis of no bubble and correct model-specification.

Explicitly expressing the misspecification in the model, however, is rarely seen in the literature. An exception is the study for the Cagan hyperinflation model by Durlauf and Hooker (1994). In their model, the price series is decomposed into components corresponding to fundamentals, bubbles, and model-misspecification. To be able to separate the model-misspecification from a bubble component, they propose two constraints, flow and stock, to uncover model noises. By testing the orthogonality of these constraints against the information set, they are able to indirectly test which noise component, bubbles or misspecification, is responsible for nonzero noises. They find that the empirical deviations from the fundamental price solution are primarily by virtue of misspecification rather than bubbles.

Unlike Durlauf and Hooker (1994), this paper directly models the misspecification component and retains the explicit reference to the model-misspecification and bubbles. The misspecification is modeled by using the time-varying risk premium framework proposed by Poterba and Summers (1986). We then express the model in a state-space form, treating the bubbles and the model-misspecification as unobservable state variables.

The Kalman filtering algorithm is applied to estimate the state-space model. The Kalman filter is an algorithm for sequentially updating a linear projection for the state-space model. It also opens the way to the maximum likelihood estimation of the unknown parameters in the model. In addition, the Kalman Filter is designed to work with nonstationary data, because the filter produces distributions of the state variables that are conditional on the previous realization of the states. Therefore, nonstationary in itself presents no problem [Bomhoff (1991)].¹

The rest of this paper is organized as follows. The next section demonstrates the theoretical model and its state-space form. Section 3 discusses the empirical results. The final section concludes the paper.

II. THEORETICAL MODEL

This section builds the theoretical foundation and the framework for detecting the bubbles and the model-misspecification contained in the stock prices. We explain how the speculative bubbles and model-misspecification are generated in the stock price model.

Specifically, under the assumptions of no-arbitrage and rational expectations, Lucas (1978) shows that the current stock price is equal to the discounted value of the sum of the expected stock price and the dividend next period:

$$P_t^g = \beta E_t [P_{t+1}^g + d_{t+1}], \quad (1)$$

where P_t^g is the stock price at period t , $\beta \in (0, 1)$ is the discount rate, d_{t+1} is the dividend at period $t+1$, E_t is the conditional expectation given the information set available at t .

Substituting forward for (1) and imposing the transversality condition yields the fundamental price of the stock,

$$P_t^f = \sum_{i=1}^{\infty} \beta^i E_t[d_{t+i}]. \quad (2)$$

This is the famous assertion that the fundamental price of the stock is equal to the sum of the discounted flow of future dividends.

Note that P_t^f is only one of the solutions to (1). Adding to P_t^f any process B_t that satisfies the condition

$$E_t(B_{t+1}) = \beta^{-1} B_t \quad (3)$$

still solves the equation. Therefore, the stock price P_t^g can be parameterized by $P_t^g = P_t^f + B_t$, where B_t is called a rational bubble.

Now we turn to the model-misspecification aspect of the stock prices. Poterba and Summers (1986) argue that the stock prices are too volatile to be explained by the fundamental price model. Changes in risk are responsible for a significant part of the stock price volatility. Therefore, they suggest an alternative hypothesis of time-varying risk premiums.

Consider the following time-varying risk premium model:

$$P_t^{f'} = E_t \left[\sum_{i=1}^{\infty} \varphi_{t+i} d_{t+i} \right], \quad (4)$$

where $\varphi_{t+i} = \prod_{j=1}^i (1+r+\alpha_{t+j})^{-1}$, r is the constant risk-free interest rate, and

α_{t+j} is the risk premium. Assume that the mean value of the risk premium is a constant $\bar{\alpha}$. Using the first-order Taylor series expansion to expand α_t around $\bar{\alpha}$ yields

$$P_t^{f'} \cong \sum_{i=1}^{\infty} \frac{E_t d_{t+i}}{(1+r+\bar{\alpha})^{i+1}} + \sum_{i=1}^{\infty} \frac{\partial P_t^*}{\partial \alpha_{t+i}} \times (E_t \alpha_{t+i} - \bar{\alpha}), \quad (5)$$

where $\frac{\partial P_t^*}{\partial \alpha_{t+i}} = -(1+r+\bar{\alpha})^{-i} \times \sum_{k=0}^{\infty} \frac{E_t(d_{t+i+k})}{(1+r+\bar{\alpha})^{k+1}}$. The above derivation follows closely to that in Poterba and Summers (1986).

Assume the discount rate $\beta = (1+r+\bar{\alpha})^{-1}$. Then the first term on the right-hand side of (5) becomes the fundamental price, and the second term is the model-misspecification caused by the time-varying risk premium, which is an unobservable market fundamental. Denote the model-misspecification component as

$$N_t \equiv \sum_{i=1}^{\infty} \frac{\partial P_t^*}{\partial \alpha_{t+i}} \times (E_t \alpha_{t+i} - \bar{\alpha}). \quad (6)$$

By adding the speculative bubble B_t to (5), the stock price can be parameterized by

$$P_t = P_t^f + N_t + B_t. \quad (7)$$

Next we use the derivation in Hansen and Sargent (1980) to approximate the fundamental price P_t^f . Assume that the structure of the stochastic process of dividend d_t can be characterized by an AR(q) process:

$$d_t = \phi_0 + \phi_1 d_{t-1} + \phi_2 d_{t-2} + \dots + \phi_q d_{t-q} + e_t. \quad (8)$$

As shown in Note 2, the rational expectations hypothesis implies that the fundamental price can then be expressed as:

$P_t^f = h_0 + h_1 d_t + h_2 d_{t-1} + \dots + h_q d_{t-q+1}$, where h_j 's are functions of β and ϕ_j ($j = 0, 1, 2, \dots, q$).² Therefore, equation (7) relates the observable P_t and d_t to the unobservable B_t and N_t .

To express this model in a state-space form, we need to find the dynamics of the unobservable variables. Manipulating on (6) yields

$$\begin{aligned} & N_{t+1} - \beta^{-1} N_t \\ &= \left(\sum_{k=1}^{\infty} \beta^k E_t d_{t+k} \right) \times (E_t \alpha_{t+1} - \bar{\alpha}) + v_{t+1} = P_t^f \times E_t (\alpha_{t+1} - \bar{\alpha}) + v_{t+1}, \quad (9) \end{aligned}$$

where

$$\begin{aligned} v_{t+1} &= - \sum_{i=1}^{\infty} \beta^i \sum_{k=0}^{\infty} \beta^{k+1} E_{t+1} d_{t+1+i+k} \times (E_{t+1} \alpha_{t+1+i} - \bar{\alpha}) \\ &+ \sum_{i=2}^{\infty} \beta^{i-1} \sum_{k=0}^{\infty} \beta^{k+1} E_t d_{t+i+k} \times (E_t \alpha_{t+i} - \bar{\alpha}). \end{aligned}$$

By rational expectations, $E_t(v_{t+1}) = 0$. Furthermore, following Poterba and Summers (1986), assume that α_t follows an AR(1) process:

$$\alpha_{t+1} - \bar{\alpha} = \rho(\alpha_t - \bar{\alpha}) + \varepsilon_{t+1}, \quad (10)$$

where $E_t(\varepsilon_{t+1}) = 0$. Then equation (9) becomes

$$N_{t+1} - \beta^{-1}N_t = \rho P_t^f \times (\alpha_t - \bar{\alpha}) + v_{t+1}. \quad (11)$$

In addition, following Wu (1995), assume that the bubble term B_t that satisfies (3) follows an AR(1) process: $B_{t+1} = \beta^1 B_t + b_{t+1}$, where $E_t(b_{t+1}) = 0$. Then the time-varying risk premium model can be expressed as a state-space model with time-varying parameters:

$$P_t = P_t^f + [1 \quad 1 \quad 0] \begin{bmatrix} B_t \\ N_t \\ \alpha_t - \bar{\alpha} \end{bmatrix}, \quad (12)$$

$$\begin{bmatrix} B_{t+1} \\ N_{t+1} \\ \alpha_{t+1} - \bar{\alpha} \end{bmatrix} = \begin{bmatrix} \beta^{-1} & 0 & 0 \\ 0 & \beta^{-1} & \rho P_t^f \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} B_t \\ N_t \\ \alpha_t - \bar{\alpha} \end{bmatrix} + \begin{bmatrix} b_{t+1} \\ v_{t+1} \\ \varepsilon_{t+1} \end{bmatrix}. \quad (13)$$

Equation (12) is known as the *observation equation*, which relates the observable variables to the unobservable variables. Equation (13), known as the *state equation*, describes the dynamics of the unobservable variables. The state vector and the residual vector are assumed to be multivariate Gaussian with

$$\begin{bmatrix} b_t \\ v_t \\ \varepsilon_t \end{bmatrix} \sim N \left(\mathbf{0}, \begin{bmatrix} \sigma_b^2 & 0 & 0 \\ 0 & \sigma_v^2 & \sigma_{v\varepsilon} \\ 0 & \sigma_{\varepsilon v} & \sigma_\varepsilon^2 \end{bmatrix} \right). \quad (14)$$

Here we assume that b_t is uncorrelated with v_t and ε_t . Since bubbles are defined as the component that cannot be explained by the market fundamentals such as the dividends and the risk premium, this assumption seems to be reasonable enough. On the other hand, it can be shown that v_t and ε_t are correlated.³

III. THE EMPIRICAL RESULTS

The state-space model (12) and (13) is estimated by the Kalman filter. The Kalman filter is an algorithm for sequentially updating a linear projection for the state-space model. In addition, it allows us to use the maximum likelihood estimation to estimate the unknown parameters in the model. After the estimates are obtained, we derive the *smoothed* estimates of the state vector and its error covariance matrix, which are the estimated values based on the full set of data collected. For details about the Kalman filter and smoother, see Anderson and Moore (1979) and Hamilton (1994).

The model is applied to two sets of stock price data. The first is the CRSP (Center for Research in Stock Prices) monthly data from 1926:1 to 1997:12, with a total of 864 observations. The NYSE/AMEX/NASDAQ Value-Weighted Market Index data are used. The stock price and dividend are constructed by using the “total return” (Series RET) and the “capital appreciation return” (Series RETX). The second data set is the quarterly S&P 500 index data from 1935:1 to 1997:4, with a total of 252 observations. The data are from the 1998 issue of the *Standard & Poor's statistical service: security price index record*. All prices and dividends are divided by the Consumer Price Index and seasonally adjusted.

We first estimate equation (8), the autoregressive process of the dividend. The lag length is chosen to be the shortest lag that renders the error term white noise. The Ljung-Box Q test is used to test the serial correlation in the residuals from the OLS regression. The lag length of thirteen is chosen to avoid the first-to-twelfth order autocorrelation for the monthly CRSP data, and three for the quarterly S&P 500 data to avoid first-to-sixth order autocorrelation in the residuals. After the lag lengths being selected, the lagged variables with insignificant estimated coefficients are dropped from the regressions. The AR processes are then re-estimated and the serial correlation in the residuals is checked again. After this step, the fundamental price is only a function of β .

After the stochastic process of the dividend is determined, the state-space model is estimated by maximum likelihood.⁴ The estimated parameters are shown in Table 1. Most of the estimates are significantly different from zero at the traditional significance level. As expected, the estimate of β is within its theoretical range, (0, 1), in both data set. They are also both very close to one, which implies that first, the time preference is not very strong in the stock market and second, the bubble is very persistent. In addition, the estimate of ρ , the first-order autocorrelation coefficient of the risk premium, is about -0.1 and the standard error is less than 20% of the estimate, which satisfies the assumption that the risk premium is stationary.

After the estimates are obtained by maximum likelihood estimation, we derive the smoothed estimates of the state vector. Figure 1 plots the market prices, the estimated fundamental prices, the smoothed estimates of the bubbles, and the smoothed estimates of the misspecification component for the CRSP data. Figure 2 plots those for the S&P 500 data.

For the CRSP data set (Figure 1), the volatility in the stock prices is mostly caused by time-varying risk premium, which can be seen from the fact that in Figures 1a and 1d, the market prices and the model-misspecification component generally move together. The fundamental price (Figure 1b) is relatively stable. The standard deviation of the estimated fundamental price is 4.36, while the market price and the misspecification component have a standard deviation of 34.33 and 33.65, respectively. On the other hand, the bubble component (Figure 1c) has a downward trend during the sample period, but its magnitude is very small compared to the other components. Indeed, the root mean squared forecasting errors of bubbles are all much bigger than the estimates of the bubbles, which indicates that there is no significant evidence of bubbles existing in the CRSP stock prices.⁵

For the S&P 500 data set (Figure 2), the fundamental price is more volatile compared to that in the CRSP data. However, it is still not as important as the model-misspecification component in explaining the volatility in stock prices. The standard deviation of the estimated fundamental price is about 41, while the market price and the misspecification component have a standard deviation of 129.22 and 63.98, respectively. The bubble component in the S&P 500 data set has an upward trend, but again it is very small and insignificant.

Another interesting result from Figures 1 and 2 is that, while the stock price has been rising sharply since 1990, the fundamental price was relatively stable during this period. That is, the recent booming stock price is mostly caused by the risk premium component, not the changes in dividends. This is consistent with the argument that investors in recent years have begun to understand that the stocks are not as risky as they thought [Glassman and Hassett (1999)]. Therefore, they require lower risk premium. As a result, they have bid up stock prices.

IV. CONCLUSION

The traditional hypothesis test on stock bubbles is based on the assumption that the fundamental price model is well specified. As a result, the finding of bubbles could be a result of model misspecification. This paper studies whether both speculative bubbles and model-misspecification exist in the stock prices. The time-varying risk premium model in Poterba and Summers (1986) is used to reveal the misspecification aspect in the stock prices. A state-space model and the Kalman Filter are used to derive the fundamental prices and the unobserved bubbles and misspecification contained in the stock prices. The model is applied to the CRSP monthly data from 1926 to 1997 and S&P 500 quarterly data from 1935 to 1997.

The results show that the fundamental price model cannot explain the volatility of the stock prices. Most of the movements in the stock prices are caused by the time-varying risk premium. In addition, there is no significant evidence of bubbles in both data sets. Previous findings of bubbles may be a result of a time-varying risk premium.

Table 1: The estimation results of the parameters in the state-space model

<u>CRSP Data:</u>							
Parameter:	ϕ_0	ϕ_1	ϕ_3	ϕ_6	ϕ_9	ϕ_{12}	ϕ_{13}
Estimate:	0.006	0.099	0.279	0.149	0.088	0.452	-0.093
Std. Error:	0.003	0.030	0.031	0.032	0.032	0.031	0.030
Parameter:	β	ρ	σ_b	l	m	n	
Estimate:	0.992	-0.124	-1.001	0.229	0.874	-0.123	
Std. Error:	0.000	0.011	0.002	0.015	0.074	0.000	
Log-Likelihood:	-2087.37						
<u>S&P 500 Data:</u>							
Parameter:	ϕ_0	ϕ_1	ϕ_3				
Estimate:	0.096	1.240	-0.249				
Std. Error:	0.051	0.036	0.035				
Parameter:	β	ρ	σ_b	l	m	n	
Estimate:	0.964	-0.110	-0.262	-0.714	0.080	-0.225	
Std. Error:	0.001	0.021	0.593	0.135	0.222	0.001	
Log-Likelihood:	-1107.82						
<p>Note: The parameters ϕ's are the coefficients in the dividend generating process: $d_t = \phi_0 + \phi_1 d_{t-1} + \phi_2 d_{t-2} + \dots + \phi_q d_{t-q} + e_t$, where the insignificant lags are dropped from the regression; β is the discount rate; ρ is the first-order autocorrelation coefficient of the risk premium; and l, m, and n are the parameters in the following Cholesky decomposition:</p> $\begin{bmatrix} l & 0 \\ m & n \end{bmatrix}' \begin{bmatrix} l & 0 \\ m & n \end{bmatrix} = \begin{bmatrix} \sigma_V^2 & \sigma_{V\varepsilon} \\ \sigma_{\varepsilon V} & \sigma_\varepsilon^2 \end{bmatrix}.$ <p>This decomposition is used to guarantee that the variance-covariance matrices in the Maximum Likelihood estimation are all positive definite. The variance/covariance parameters σ's are defined in equation (14).</p>							

This paper provides a starting point for explicitly modeling the misspecification of the fundamental price model. The cause of the misspecification is limited to the time-varying risk premium here. Extensions of this study include other specifications for bubbles and model-misspecification component.

Figure 1: CRSP Data

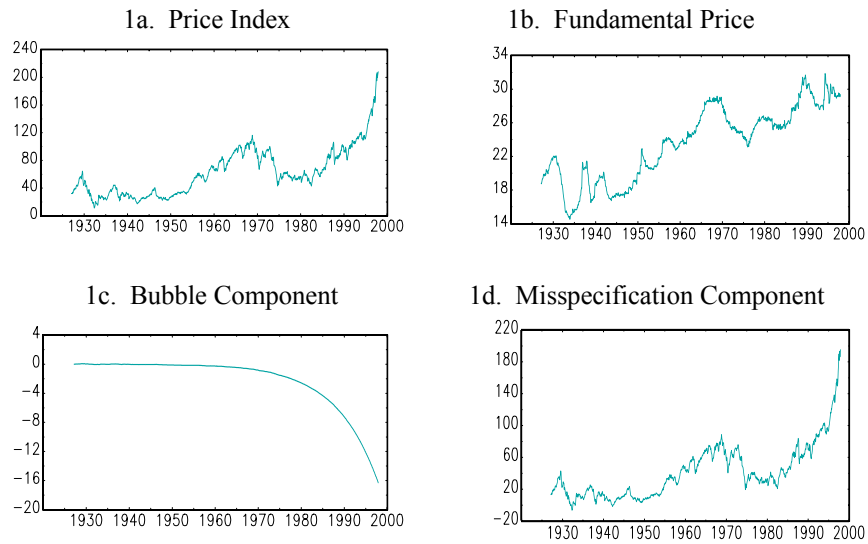
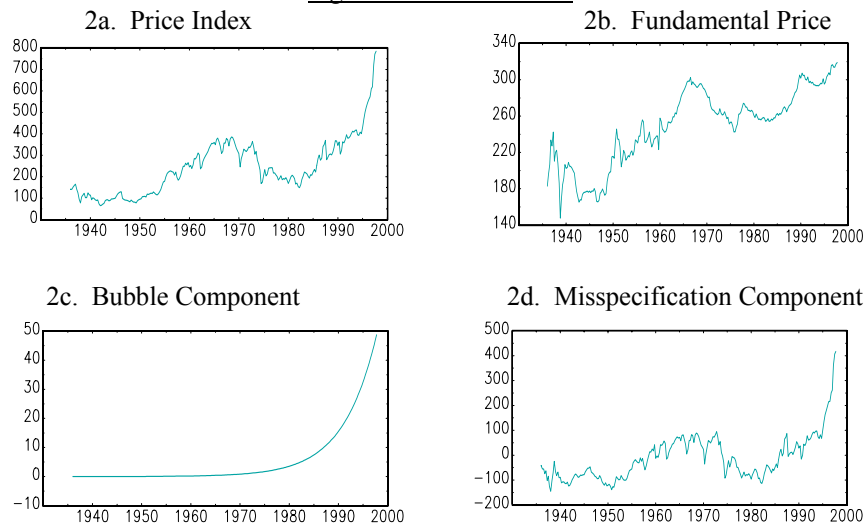


Figure 2: S&P 500 Data



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ENDNOTES

1. The Kalman filter algorithm was used by Burmeister and Wall (1982) to test price-level bubbles of the German hyperinflation, and by Wu (1995) to test for exchange-rate bubbles.
2. Rewrite the stochastic process of dividend d_t in a matrix form:

$D_t = \Gamma D_{t-1} + e_t$, where

$$D_t = \begin{bmatrix} 1 \\ d_t \\ d_{t-1} \\ \vdots \\ d_{t-q+1} \end{bmatrix}, \quad e_t = \begin{bmatrix} 0 \\ e_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \text{and} \quad \Gamma = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \phi_0 & \phi_1 & \phi_2 & \phi_3 & \cdots & \phi_q \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 1 & 0 & 0 \\ 0 & \cdots & \cdots & 0 & 1 & 0 \end{bmatrix}.$$

Therefore, $E_t(D_{t+i}) = \Gamma^i D_t$. Then P_t^f can be written as

$$\begin{aligned} P_t^f &= E_t \left[\sum_{i=1}^{\infty} \beta^i g D_{t+i} \right] = \sum_{i=1}^{\infty} \beta^i g \Gamma^i D_t = g \beta \Gamma (I - \beta \Gamma)^{-1} D_t \\ &= h_0 + h_1 d_t + h_2 d_{t-1} + \cdots + h_q d_{t-q+1}, \end{aligned}$$

where $g = [0 \ 1 \ 0 \ 0 \ \cdots \ 0]$ and h_j 's are functions of β and ϕ_j 's ($j = 1, 2, \dots, q$).

3. Specifically,

$$\begin{aligned} E(v_t \varepsilon_t) &= E \left\{ \left[\sum_{i=1}^{\infty} -\beta^i \left(\sum_{k=0}^{\infty} \beta^{k+1} E_t d_{t+i+k} \times (E_t \alpha_{t+i} - \bar{\alpha}) \right) \right] \times \varepsilon_t \right. \\ &\quad \left. + \beta^{-1} \left[\sum_{i=2}^{\infty} -\beta^i \left(\sum_{k=0}^{\infty} \beta^{k+1} E_{t-1} d_{t-1+i+k} \times (E_{t-1} \alpha_{t-1+i} - \bar{\alpha}) \right) \right] \times \varepsilon_t \right\} \\ &= E \left\{ \left[\sum_{i=1}^{\infty} -\beta^i \left(\sum_{k=0}^{\infty} \beta^{k+1} (E_t d_{t+i+k}) \rho^i (\alpha_t - \bar{\alpha}) \right) \right] \times \varepsilon_t \right\} \\ &= E \left\{ \left[\sum_{i=1}^{\infty} -\beta^i \left(\sum_{k=0}^{\infty} \beta^{k+1} (E_t d_{t+i+k}) \rho^i (\rho (\alpha_{t-1} - \bar{\alpha}) + \varepsilon_t) \right) \right] \times \varepsilon_t \right\} \\ &= - \sum_{i=1}^{\infty} \beta^{i-1} \rho^i E [P_{t+i-1}^f] \sigma_{\varepsilon}^2. \end{aligned}$$

Therefore, if $\rho \neq 0$, we get $E[v_t \varepsilon_t] \neq 0$.

4. Ideally, the coefficients in the dividend processes should be jointly estimated with other parameters in the maximum likelihood estimation. However, this would result in a bad fit of the model. Therefore, since the OLS estimation of the dividend process is consistent, we adopt the current two-step estimation. In the first step we obtain the estimates of ϕ 's and express the fundamental price as a function of β only. In the second step, β is estimated, with the other parameters in the model, by the maximum likelihood estimation.

5. The root mean squared prediction errors of the bubbles and misspecification are too big to be plotted clearly with the estimates. Therefore, we do not show them in the paper. They are available from the authors upon requests.

REFERENCES

- Anderson, B.D.O. and Moore, J.B. (1979) *Optimal Filtering*, Prentice-Hall: Englewood Cliffs.
- Blanchard, O.J. and Watson, M.W. (1982) "Bubbles, Rational Expectations, and Financial Markets," in Paul Wachtel, ed., *Crises in the Economic and Financial Structure*, Lexington Books, 295-315.
- Bomhoff, E. J. (1991) "Stability of Velocity in the Major Industrial Countries: A Kalman Filter Approach," *IMF Staff Working Papers*, Vol. 38, No. 3, 626-642.
- Burmeister, E. and Wall, K.D. (1982) "Kalman Filtering Estimation of Unobserved Rational Expectations with an Application on the German Hyperinflation," *Journal of Econometrics*, 20, 255-284.
- Durlauf, S.N. and Hooker, M.A. (1994) "Misspecification versus Bubbles in the Cagan Hyperinflation Model," in Colin Hargreaves ed., *Nonstationary Time Series Analysis and Cointegration*, Oxford University Press.
- Flood, R.P. and Garber, P.M. (1980) "Market Fundamentals versus Price-Level Bubbles: The First Tests," *Journal of Political Economy*, 88, 745-770.
- Glassman, J.K. and Hassett, K.A. (1999) *DOW 36,000, The New Strategy for Profiting from the Coming Rise in the Stock Market*, Times Business, New York.
- Hamilton, J.D. (1994) *Time Series Analysis*, Princeton University Press.
- Hamilton, J.D. and Whiteman C.H. (1985) "The Observable Implications of Self-fulfilling Expectations," *Journal of Monetary Economics*, 16, 353-373.
- Hansen, L.P. and Sargent, T.J. (1980) "Formulating and Estimating Dynamic Linear Rational Expectation Models," *Journal of Economic Dynamics and Control*, 2, 7-46.
- LeRoy, S.F. and Porter, R.D. (1981) "The Present-Value Relation: Tests Based on Implied Variance Bounds," *Econometrica*, 49, 555-574.
- Lucas, R.E. (1978) "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1429-1446.
- Poterba, J.M. and Summers, L.H. (1986) "The Persistence of Volatility and Stock Market Fluctuations," *American Economic Review*, 76, 1141-1151.

Shiller, R.J. (1981) "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review*, 71, 421-436.

Wu, Y. (1995) "Are there Rational Bubbles in Foreign Exchange Markets? Evidence from an Alternative Test," *Journal of International Money and Finance* 14, 27-46.