Output Convergence Revisited: New Time Series Results on Industrialized Countries

[Running Title: *Output Convergence Revisited*]

ABSTRACT

The output convergence hypothesis is re-examined with a flexible concept of unit roots in time-series data. While the presence of a perfect unit root in cross-country output differentials implies non-convergence, the presence of a stochastic unit-root on the contrary implies convergence. By using the annual log per-capita output differences between U.S. and the other 14 OECD countries for the period 1900-1987, we find outputs divergence only for the U.S./U.K and U.S./Sweden country-pairs.

Keywords: convergence hypothesis, stochastic unit-root
JEL classification: C22, O47

I. Introduction

The cross-country output convergence hypothesis has been tested heavily after the birth of endogenous growth theories. Many empirical studies designed to test this hypothesis in a cross-sectional framework have found supportive evidence [e.g., Barro and Sala-i-Martin (1992, 1995)]. However, the convergence hypothesis has been challenged by time-series analyses of convergence.[1] These studies interpret the lack of common unit roots in output levels across countries as an indication of non-convergence. If, on the contrary, the output difference between any two economies is a stationary process, the observed output difference at any time tends to disappear in the long run. Based on this concept, studies such as Bernard and Durlauf (1995) and Quah (1992) find that there is no systematic convergence among a number of countries.

Studies on time-series tests of output convergence often assume a linear AR model and use the standard Augmented Dickey-Fuller unit root tests. Recent studies on unit root processes [e.g., Granger and Swanson (1997), Leybourne et al. (1996), and Ludlow and Enders (2000)] have argued that the linear decay in the linear time-series models fail to capture the asymmetric and/or time-varying adjustment of some macroeconomic variables. Leybourne et al. (1996) find that about half of the frequently employed data series in macroeconomic studies need to be considered nonstationary in a rather more general way.

Granger and Swanson (1997) propose a class of nonlinear processes which have a root that is stochastic and varying around unity. Their Stochastic Unit Root (STUR) processes can be stationary for some periods, and mildly explosive for others. They note that the STUR processes "are seen to arise naturally in economic theory, as well as in everyday macroeconomic applications." As to the statistical inference, since the standard unit root tests cannot easily distinguish between exact unit roots and stochastic unit roots, they propose to use an alternative test which has a null hypothesis of exact unit roots and an alternative of stochastic unit roots. They find that exact unit-root models are often rejected in favor of the STUR models.

The purpose of this paper is to model the cross-country output differences as a STUR process. It is motivated by the fact that standard unit root tests often exhibit low power against alternatives of near or stochastic unit root processes, and that the empirical studies using standard unit root tests often fail to reject the null of cross-country output divergence. We show that when the output difference follows a STUR process, the output paths of two economies starting at different levels actually tend to converge. Therefore, if an exact unit-root model for cross-country output differences is rejected in favor of a STUR model, the output convergence hypothesis implied by the neoclassical growth models is supported in the time-series framework. This can shed some light on previous contradicting empirical results from using cross-sectional and time-series methods.

The following section discusses the different implications on output convergence from the linear time-series models and the STUR models. Section III presents the empirical results from the ADF and STUR tests on per-capita real output differences among 15 OECD countries. Section IV concludes.

II. The Output Convergence Implication

In this section, we show that an exact unit-root and a stochastic unit-root in output differences yield opposite implications on output convergence. Define the cross-country output difference between country i and country j as $x_t \equiv y_{i,t} - y_{j,t}$, where $y_{i,t}$ is the log real per-capita GDP of country i. Bernard and Durlauf (1996) demonstrate in a proposition that a non-zero mean or a unit root in x_t would imply cross-country output divergence. The non-convergence hypothesis can be tested using standard tests such as the augmented Dickey-Fuller (ADF) test, which considers an exact unit root in x_t as the null hypothesis and a less-than-one root as the alternative.

It is intriguing to ask whether the non-convergence implication is still valid if x_t contains a *stochastic* unit root rather than an exact unit root. A process is said to be a stochastic unit root (STUR) process when it has a stochastic root that varies around unity. This concept was introduced in Granger and Swanson (1997) and also considered by Leybourne et al. (1996). The process of x_t is known as a STUR if it has the following form:

$$x_t = \alpha_t x_{t-1} + \varepsilon_t , \tag{1}$$

where $E(\alpha_t)$=1, ε_t is iid with zero mean, and α_t and ε_t are independent. When α_t= 1 for all t, x_t follows an exact unit-root process. Theoretically, stochastic unit roots may arise naturally in output variables. For example, consider a simple optimal growth model that allows for stochastic factor productivity while on average exhibiting constant returns to scale. Then output in logarithm can be derived as an autoregressive process with randomized coefficient [see Kelly (1992)].

Next, we demonstrate that outputs from two economies tend to converge when x_t contains a stochastic unit root. Given that $\xi_t \equiv \dfrac{\varepsilon_t}{\alpha_t x_{t-1}}$ is relatively small, equation (1) can be written as

$$\log x_t = \log x_{t-1} + \beta_t + \xi_t = E(\beta_t) + \log x_{t-1} + [\beta_t - E(\beta_t) + \xi_t], \tag{2}$$

where $\beta_t = \log \alpha_t$. Jensen's inequality implies that

$$E(\beta_t) = E(\log \alpha_t) < \log(E(\alpha_t)) = 0 .$$

Therefore, the evolution of $\log x_t$ in equation (2) is equivalent to a random walk with a downward drift, namely, $\log x_t$ goes to $-\infty$ with probability one. Equivalently, x_t converges to zero with probability one. As a result, the output difference between country i and j, i.e. x_t, would disappear in the long run. In other words, output paths converge.

Therefore, when $\alpha_t = 1$ (or equivalently $\beta_t = 0$) for all t, cross-country outputs diverge, but if α_t is stochastic with mean one, cross-country outputs converge. To conduct a statistical test, we apply an LM test developed by Leybourne et al. (1996). We denote this test as the STUR test, in which the null hypothesis is an exact unit root (a linear I(1) process) and the alternative hypothesis is a stochastic unit root (a STUR process).

Let $\alpha_t \sim iid(1, w^2)$, and $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$. Under the null hypothesis $w^2 = 0$, x_t is an AR process with a unit root. Alternatively, if $w^2 > 0$, x_t is a STUR process. The test statistic for the STUR test is

$$\hat{Z}_T = T^{-3/2}\hat{\sigma}_\varepsilon^{-2}\hat{k}^{-1}\sum_{t=q+3}^{T}\left(\sum_{j=q+2}^{t-1}\hat{\varepsilon}_j\right)^2\left(\hat{\varepsilon}_t^2 - \hat{\sigma}_\varepsilon^2\right),$$

where $\hat{\varepsilon}_t$ is the residual from the regression of Δx_t on a constant, a trend, and q lags of Δx_t, $\hat{\sigma}_\varepsilon^2 = \frac{1}{T}\sum\hat{\varepsilon}_t^2$, and $\hat{k}^2 = \frac{1}{T}\sum\left(\hat{\varepsilon}_t^2 - \hat{\sigma}_\varepsilon^2\right)^2$. The optimal lag length q should be chosen according to some criterion so that the residuals are not autocorrelated. The critical values for various sample sizes are reported in Table 1 of Leybourne et al. (1996).

III. The Data and Results

The data used in the empirical exercise are taken from Bernard and Durlauf (1995). They are annual log real per-capita GDP in 1980 PPP-adjusted dollars for 15 OECD countries from 1900 to 1987. In testing for cross-country output convergence, we use the output differences between the U.S. and the other 14 OECD countries, a total of 14 country-pairs.

We first conduct the following Augmented Dickey-Fuller (ADF) tests:

$$x_t = \xi_1 \Delta x_{t-1} + \xi_2 \Delta x_{t-2} + \cdots + \xi_p \Delta x_{t-p} + \alpha + \rho x_{t-1} + \varepsilon_t ,$$

where x_t is the log real per-capita GDP difference between two countries and the lag length p is chosen by the AIC criterion. Table 1 reports the t-test statistics for the null hypothesis ρ=1; and the F-test statistics for the null hypothesis α=0 and ρ=1. At the 5% significance level, only one (U.S.-Netherlands) out of the 14 t-tests rejects the null of a unit root. For the other 13 country-pairs, the F-test also fail to reject the null that x_t is a unit root process without a drift. Based on the conventional argument in the time-series output convergence literature, outputs diverge for these 13 country-pairs.

Next, we conduct the STUR test on the output differences for these 13 country-pairs.[2] At the 5% significance level, only two out of the 13 tests fail to reject the null of an exact unit root. Therefore, the output differences of the other 11 country-pairs are better modeled as a STUR process, and outputs actually converge.

In sum, the outputs of the U.S. and Netherlands converge over the period 1900-1987 because their difference is stationary. The outputs between the U.S. and the U.K., and between the U.S. and Sweden diverge because the output differences should be modeled as an exact unit root process. The outputs between the U.S. and the other 11 OECD countries converge over this period because the output differences follow a stochastic unit root process. Using the traditional ADF unit root tests would falsely conclude that the outputs between U.S. and those 11 OECD countries diverge over the sample period.

IV. Conclusion

The Stochastic Unit Root (STUR) processes naturally describe certain macroeconomic variables. When applied to the cross-country output differences, it has very different implications on the time series test of output convergence as opposed to the constant unit root process. Failing to reject the null of a constant unit root by the traditional ADF tests is usually considered as evidence of non-convergence in outputs. However, this rejection does not rule out the possibility that the underlining process indeed contains a stochastic unit root, and we show that when the output difference contains a stochastic unit root, the output paths of the two economies actually converge.

By using the annual log per-capita output differences between the U.S. and the other 14 OECD countries for the period 1900-1987, we find non-convergent output paths only in the U.S./U.K and the U.S./Sweden country-pairs. Although the traditional ADF tests suggest that 13 out of the 14 country-pairs have divergent outputs, 12 of them actually have convergent outputs because these output differences should be modeled as a STUR process instead of an exact unit root process.

Notes

1. Discussion about the distinction between cross-sectional convergence and time-series forecast convergence can be found in Durlauf and Quah (1998).

2. Before conducting the STUR test, we use the Kalman filter approach to estimate the time-varying coefficient model (1), with the coefficient being modeled as an AR(1) process. The mean of the absolute values of the estimated $\xi_t \equiv \frac{\varepsilon_t}{\alpha_t x_{t-1}}$ ranges from 0.056 to 0.119. Therefore, the approximation in (2) is reasonable. The mean of the estimated coefficient α_t ranges from 0.990 to 1.000.

References

Barro, R.J. and Sala-i-Martin, X. (1992) Convergence, Journal of Political Economy, 100, 223-251.

Barro, R.J. and Sala-i-Martin, X. (1995) Economic Growth, McGraw-Hill Publication.

Bernard, A.B. and Durlauf, S.N. (1995) Convergence in international output, Journal of Applied Econometrics, 10, 97-108.

Bernard, A.B. and Durlauf, S.N. (1996) Interpreting tests of the convergence hypothesis, Journal of Econometrics, 71, 161-174.

Durlauf, S.N. and Quah, D.T. (1998) The new empirics of economic growth, NBER Working Paper 6422.

Granger, C.W.J. and Swanson, N.R. (1997) An introduction to stochastic unit-root processes, Journal of Econometrics, 80, 35-62.

Kelly, M. (1992) On endogenous growth with productivity shocks, Journal of Monetary Economics, 30, 47-56.

Leybourne, S.J., McCabe, B.P.M. and Tremayne, A.R. (1996) Can economic time series be differenced to stationarity? Journal of Business and Economic Statistics, 14, 435-446.

Ludlow, J. and Enders, W. (2000) Estimating non-linear ARMA models using Fourier coefficients, International Journal of Forecasting, 16, 333-47.

Quah, D. (1992) International patterns of growth: I. Persistence in cross-country disparities, Working paper, London School of Economics.

Table 1: Unit root tests log per-capita output differences

Country Pair	(A) Augmented Dickey-Fuller		(B) Stochastic Unit Root test statistics
	t-test	F-test	
U.S. – U.K.	-2.590	3.500	0.093
U.S. – Sweden	-1.717	1.712	0.160
U.S. – Norway	-1.846	2.004	0.239*
U.S. – Netherlands	-3.279*	5.379*	-------
U.S. – Japan	-0.235	0.709	0.245*
U.S. – Italy	-2.278	2.652	0.905*
U.S. – Germany	-0.879	0.706	0.931*
U.S. – France	-2.693	3.660	0.435*
U.S. – Finland	-1.222	1.174	0.385*
U.S. – Denmark	-2.898	4.240	0.308*
U.S. – Canada	-0.480	0.847	0.400*
U.S. – Belgian	-2.781	3.868	0.246*
U.S. – Austria	-1.798	1.624	0.240*
U.S. – Australia	-2.326	2.759	0.232*

The asterisk denotes statistical significance at the 5% significance level.