

§ 7.7 Variation

... represents a relationship between two or more quantities, expressed by ONE single term.

Two kinds of variation:

1. DIRECT	2. INDIRECT
<ul style="list-style-type: none"> <li>- When one quantity <u>increases</u>, the other <u>increases</u> as well.</li> <li>- <i>Example:</i> For a mechanic who is paid hourly, the longer working hours mean more income - as the hours increase, so does the paycheck.</li> </ul>	<ul style="list-style-type: none"> <li>- When one quantity <u>increases</u>, the other <u>decreases</u>.</li> <li>- <i>Example:</i> The speed and the amount of travel time are indirectly related to each other; the greater the speed, the shorter the travelling time.</li> </ul>

We say that:

$y$ VARIES DIRECTLY with $x$	if	$y = kx$
$y$ VARIES DIRECTLY with $\sqrt{x}$	if	$y = k\sqrt{x}$
$y$ VARIES DIRECTLY with $x^2$	if	$y = kx^2$

$y$ VARIES INDIRECTLY with $x$	if	$y = \frac{k}{x}$
$y$ VARIES INDIRECTLY with $x^2$	if	$y = \frac{k}{x^2}$
$y$ VARIES INDIRECTLY with $x^{\frac{3}{2}}$	if	$y = \frac{k}{x^{\frac{3}{2}}}$

In indirect variation, the variable is underneath (in the denominator of a fraction)

$k$  - **CONSTANT OF VARIATION**

**Example** Which of the following equations shows that  $y$  VARIES DIRECTLY with  $x$ ?

1.  $y = ax + 1$

2.  $y = (a + 1)x$

3.  $y = \frac{a}{x^2}$

4.  $y = \frac{a}{x}$

5.  $y = 2x^2 + 3$

6.  $y = ax + 3$

7.  $y = \frac{2}{x}$

8.  $y = cx$

9.  $y = x^2$

10.  $y = 5a^2x$

11.  $y = -\frac{px}{a}$

12.  $y = 2x$

Answer: **2, 8, 10, 11, and 12.**

**We solve variation problems in four steps:**

1. Write the general form of the equation of variation
2. From the data, find the constant of variation ( $k$ )
3. Write your specific equation of variation
4. Calculate  $y$  for a given value of  $x$

**Examples**

(online sample test)

1. When a mass  $m$  hangs at the end of a spring, the distance  $d$  the spring is stretched VARIES DIRECTLY as the mass. A mass of 18 grams hanging from certain spring stretches the spring 12 cm . Which mass will stretch that spring 17 cm?

Solution:

Our labels:

$m$  - mass that stretches the spring

$d$  - distance of the spring from the position when it is not stretched

Since  $d$  varies directly with  $m$ , we have:

$$d = k \cdot m$$

To obtain the variation constant  $k$ , we substitute the data into our formula:

$$12 = k \cdot 18$$

$$\frac{12}{18} = k$$

$$k = \frac{2}{3}$$

Now we substitute the value of  $k$  into our variation equation.

$$d = \frac{2}{3} \cdot m$$

Now we can solve for the unknown  $m$  for given  $d = 17$ . So:

$$17 = \frac{2}{3} \cdot m$$

$$17 \cdot 3 = 2 m$$

$$51 = 2 m$$

$$m = 25.5$$

ANSWER:

**The mass which stretches the spring for 17 cm weighs 25.5 grams.**

2. The gravitational attraction  $G$  between two masses VARIES INVERSELY as the square of the distance  $d$  between them. If the attraction is 0.002 N when the masses are 2.5 m apart, find the attraction when the masses are 6 m apart.

Solution:

Our labels:

$F$  - gravitational force between two bodies  
 $d$  - distance of the bodies

Since  $G$  varies inversely with square of  $d$ , we have the following variation equation:

$$F = \frac{k}{d^2}$$

To find the variation constant  $k$ , we substitute the data into the variation equation for  $G$ :

$$0.002 = \frac{k}{2.5^2}$$

$$0.002 = \frac{k}{6.25}$$

$$6.25 \cdot 0.002 = k$$

$$k = 0.0125$$

That way the variation equation becomes:

$$F = \frac{0.0125}{d^2}$$

To find  $G$  when  $d = 6$ , we substitute  $d = 6$  into our variation equation:

$$F = \frac{0.0125}{6^2}$$

$$F = \frac{0.0125}{36}$$

$$F = 0.00034722$$

ANSWER:

**The gravitational attraction between the masses when they are 6 meters apart is  $F = 0.00034722$  N.**