

# ECE 2100

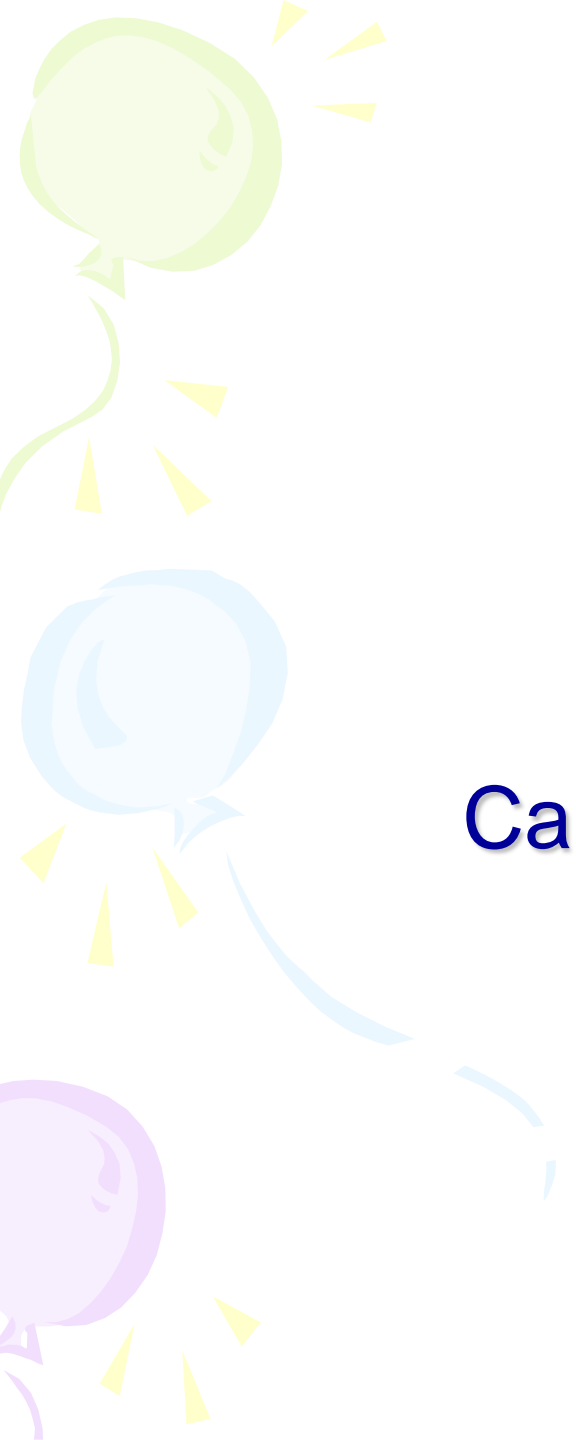
## Circuit Analysis

### Lesson 23

#### Chapter 9 & App B: Introduction to Sinusoids & Phasors

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A decorative graphic on the left side of the slide features three balloons: a green one at the top, a light blue one in the middle, and a purple one at the bottom. Each balloon is attached to a string and has several small yellow triangular shapes around it, resembling streamers or confetti.

# ECE 2100

## Circuit Analysis

Review  
Lesson 20-22  
Chapter 6:  
Capacitance & Inductance



# Capacitors and Inductors

## Chapter 6

6.1 Introduction


6.2 Capacitors

6.3 Series and Parallel Capacitors

6.4 Inductors

6.5 Series and Parallel Inductors

6.6 Applications

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# ECE 2100

## Circuit Analysis

Lesson 23  
Chapter 9 & App B:  
Introduction to Sinusoids & Phasors



# Sinusoids and Phasor

## Chapter 9

9.1 Motivation

9.2 Sinusoids' features

9.3 Phasors

9.4 Phasor relationships for circuit elements

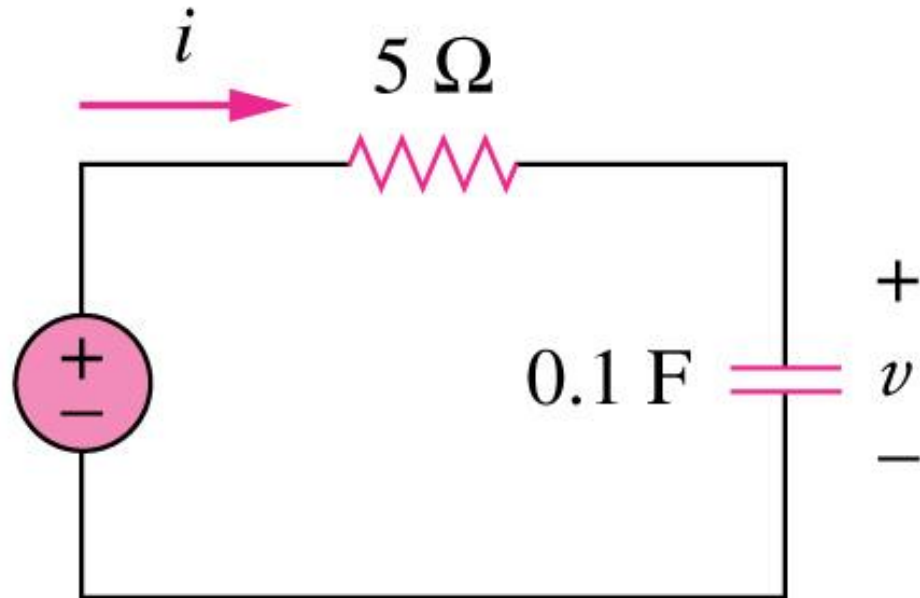
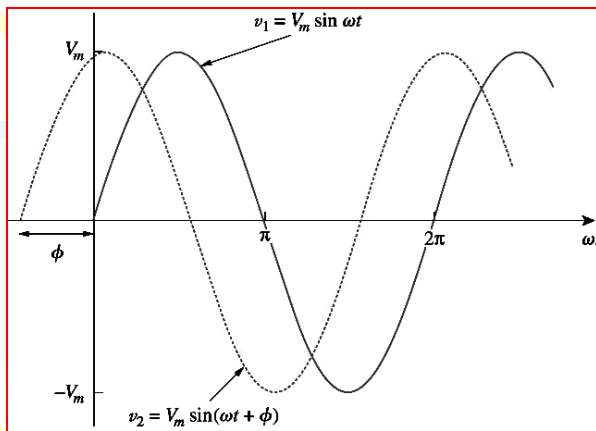
9.5 Impedance and admittance

9.6 Kirchhoff's laws in the frequency domain

9.7 Impedance combinations

# 9.1 Motivation (1)

How to determine  $v(t)$  and  $i(t)$ ?

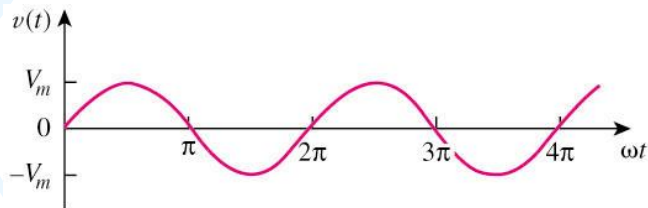


How can we apply what we have learned before to determine  $i(t)$  and  $v(t)$ ?

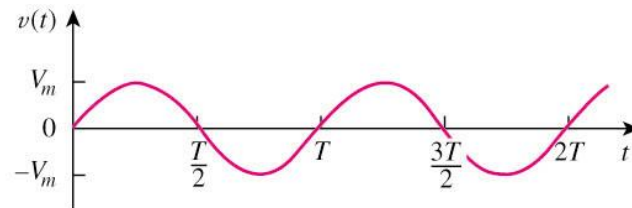
# 9.2 Sinusoids (1)

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi)$$



(a)



(b)

where

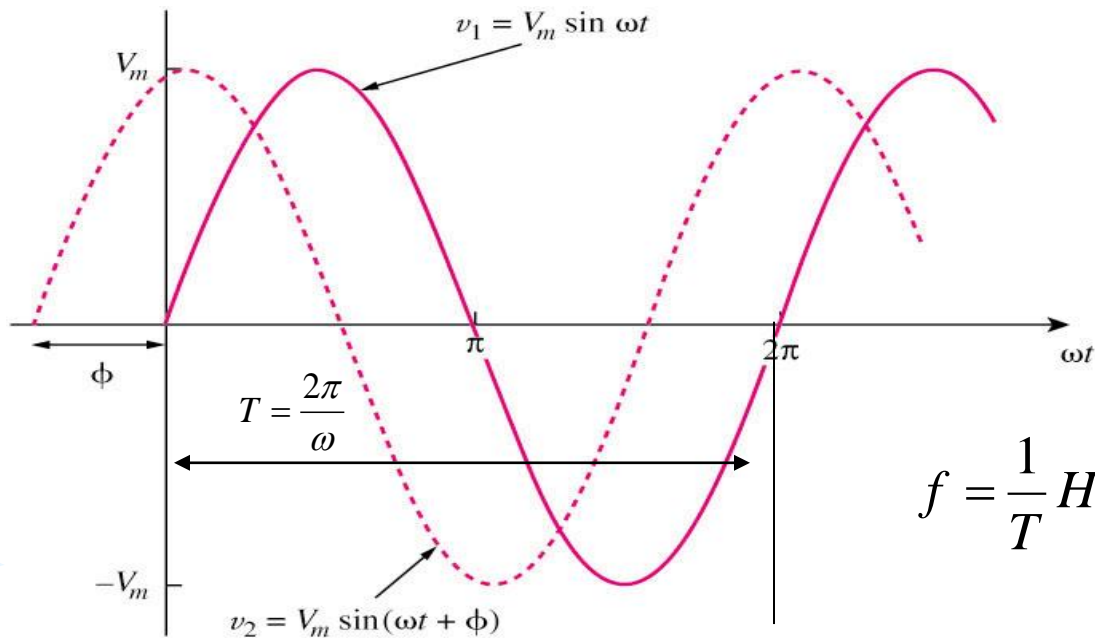
$V_m$  = the **amplitude** of the sinusoid

$\omega$  = the angular frequency in radians/s

$\Phi$  = the phase

## 9.2 Sinusoids (2)

A **periodic function** is one that satisfies  $v(t) = v(t + nT)$ , for all  $t$  and for all integers



$$f = \frac{1}{T} \text{ Hz} \quad \omega = 2\pi f$$

- Only two sinusoidal values with the **same frequency** can be compared by their amplitude and phase difference.
- If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.



## 9.2 Sinusoids (3)

### **Example 1**

Given a sinusoid  $5 \sin(4\pi t - 60^\circ)$ , calculate its amplitude, phase, angular frequency, period, and frequency.

### **Solution:**

Amplitude = 5, phase =  $-60^\circ$ , angular frequency =  $4\pi$  rad/s, Period = 0.5 s, frequency = 2 Hz.

## 9.2 Sinusoids (4)

### Example 2

Find the phase angle between  $i_1 = -4\sin(377t + 25^\circ)$  and  $i_2 = 5\cos(377t - 40^\circ)$ , does  $i_1$  lead or lag  $i_2$ ?

### Solution:

Since  $\sin(\omega t + 90^\circ) = \cos \omega t$

$$i_2 = 5\sin(377t - 40^\circ + 90^\circ) = 5\sin(377t + 50^\circ)$$

$$i_1 = -4\sin(377t + 25^\circ) = 4\sin(377t + 180^\circ + 25^\circ) = 4\sin(377t + 205^\circ)$$

therefore,  $i_1$  leads  $i_2$   $155^\circ$ .

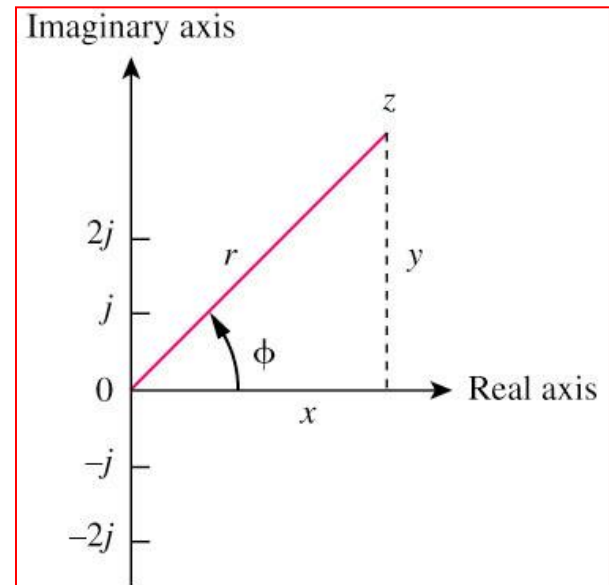
# 9.3 Phasor (1)

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.
- It can be represented in one of the following three forms:

Rectangular  $z = x + jy = r(\cos \phi + j \sin \phi)$

Polar  $z = r \angle \phi$

Exponential  $z = re^{j\phi}$



where

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

## 9.3 Phasor (2)

### Example 3

- Evaluate the following complex numbers:

a.  $[(5 + j2)(-1 + j4) - 5 \angle 60^\circ]$

b.  $\frac{10 + j5 + 3 \angle 40^\circ}{-3 + j4} + 10 \angle 30^\circ$

### Solution:

a.  $-15.5 + j13.67$

b.  $8.293 + j2.2$

## 9.3 Phasor (3)

Mathematic operation of complex number:

1. Addition

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

2. Subtraction

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

3. Multiplication

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$$

4. Division

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$$

5. Reciprocal

$$\frac{1}{z} = \frac{1}{r} \angle -\phi$$

6. Square root

$$\sqrt{z} = \sqrt{r} \angle \phi/2$$

7. Complex conjugate

$$z^* = x - jy = r \angle -\phi = r e^{-j\phi}$$

8. Euler's identity

$$e^{\pm j\phi} = \cos \phi \pm j \sin \phi$$