ECE 2100
Circuit Analysis

Lesson 23
Chapter 9 & App B:
Introduction to Sinusoids & Phasors

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ECE 2100
Circuit Analysis

Review
Lesson 20-22
Chapter 6:
Capacitance & Inductance

Capacitors and Inductors
Chapter 6

6.1 Introduction

6.2 Capacitors

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6.5 Series and Parallel Inductors

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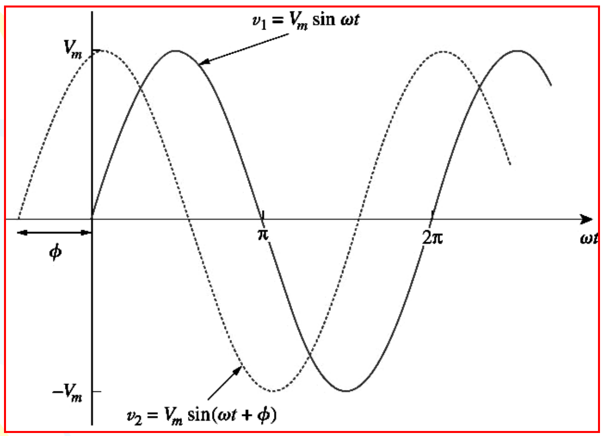
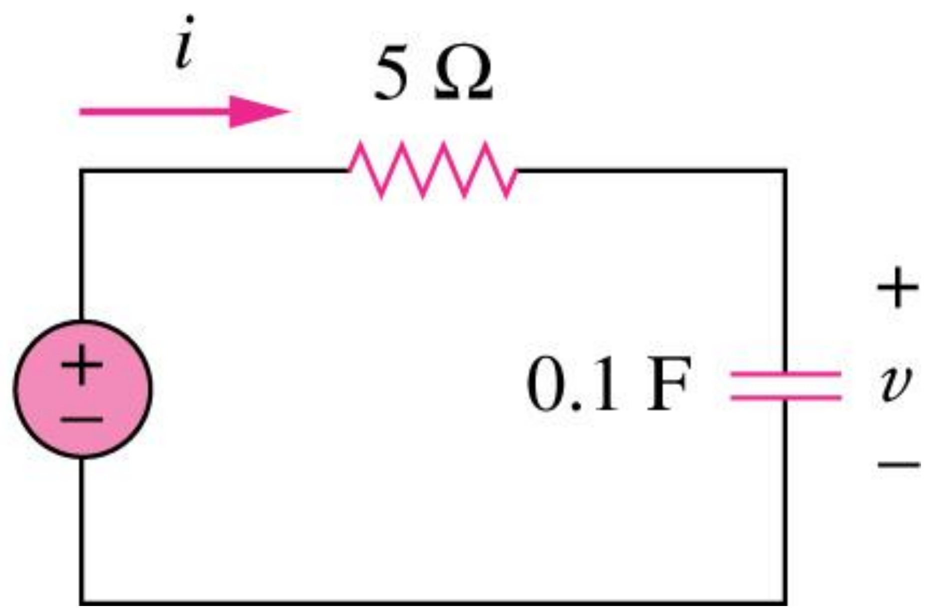
Lesson 23
Chapter 9 & App B:
Introduction to Sinusoids & Phasors

Sinusoids and Phasor
Chapter 9

9.1 Motivation (1)

How to determine *v(t)* and *i(t)*?



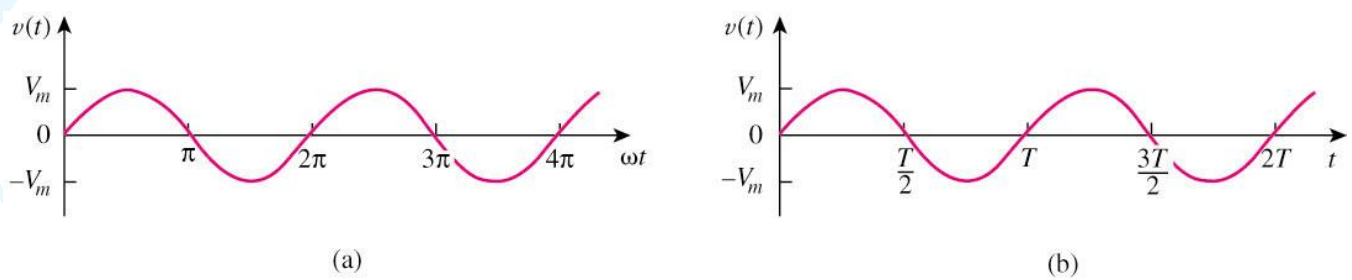


How can we apply what we have learned before to determine *i(t)* and *v(t)*?

9.2 Sinusoids (1)

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi)$$



where

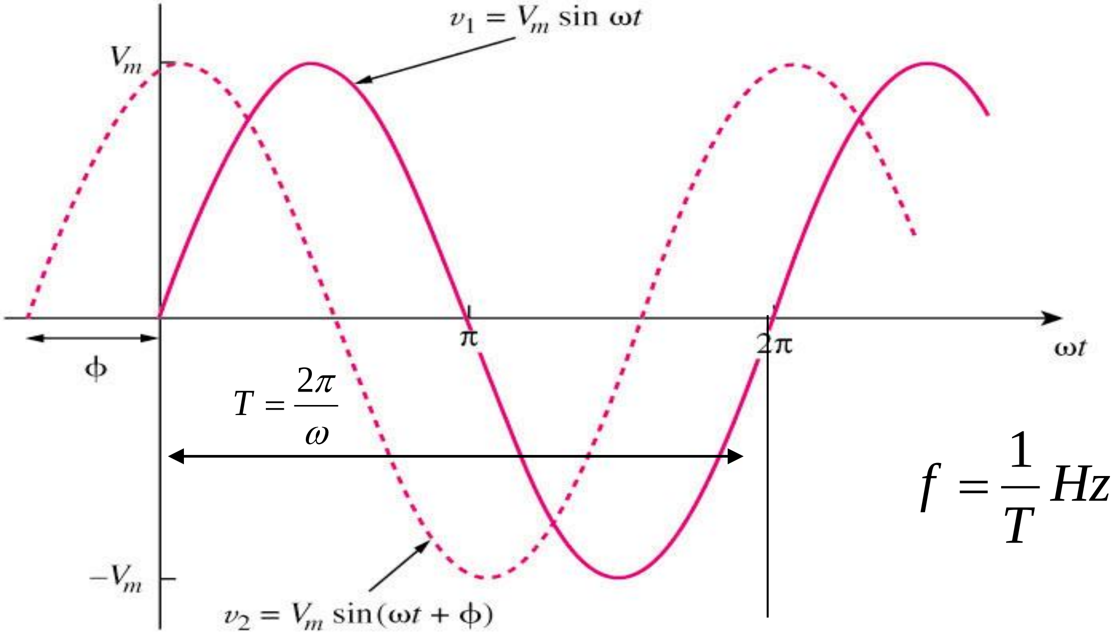
V_m = the **amplitude** of the sinusoid

ω = the angular frequency in radians/s

Φ = the phase

9.2 Sinusoids (2)

A **periodic function** **is one that satisfies** ***v(t)*** **=** ***v(t + nT)*****, for all t and for all integers**



$$f = \frac{1}{T} Hz \quad \omega = 2\pi f$$

- Only two sinusoidal values with the same frequency can be compared by their amplitude and phase difference.
- If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.

9.2 Sinusoids (3)

Example 1

Given a sinusoid $5\sin(4\pi t - 60^o)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Solution:

Amplitude = 5, phase = –60^o, angular frequency = 4π rad/s, Period = 0.5 s, frequency = 2 Hz.

9.2 Sinusoids (4)

Example 2

Find the phase angle between $i_1 = -4\sin(377t + 25^o)$ and $i_2 = 5\cos(377t - 40^o)$, does i_1 lead or lag i_2?

Solution:

Since $\sin(\omega t + 90^o) = \cos \omega t$

$$i_2 = 5\sin(377t - 40^o + 90^o) = 5\sin(377t + 50^o)$$

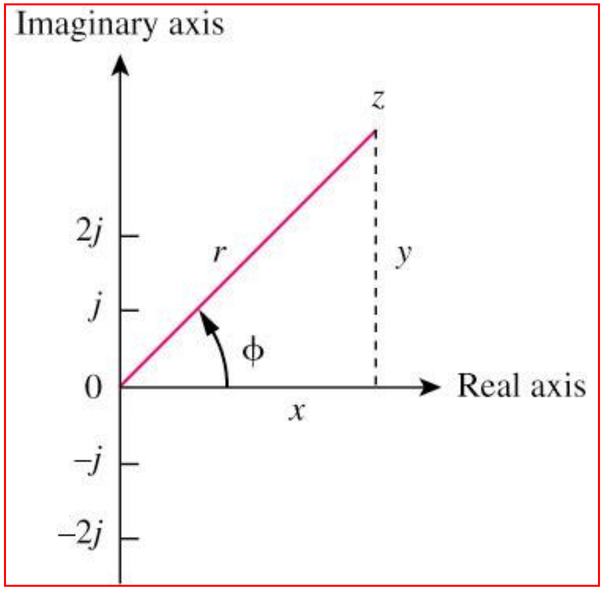
$$i_1 = -4\sin(377t + 25^o) = 4\sin(377t + 180^o + 25^o) = 4\sin(377t + 205^o)$$

therefore, i_1 leads i_2 155^o.

9.3 Phasor (1)

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.

- It can be represented in one of the following three forms:



Rectangular $z = x + jy = r(\cos\phi + j\sin\phi)$

Polar $z = r\angle\phi$

Exponential $z = re^{j\phi}$

where $r = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\dfrac{y}{x}$

9.3 Phasor (2)

Example 3

- Evaluate the following complex numbers:

a. $$[(5+j2)(-1+j4)-5\angle 60^{o}]$$

b. $$\frac{10+j5+3\angle 40^{o}}{-3+j4}+10\angle 30^{o}$$

Solution:

a. –15.5 + j13.67

b. 8.293 + j2.2

9.3 Phasor (3)

Mathematic operation of complex number:

1. Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
2. Subtraction $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. Multiplication $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$
4. Division $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \angle \phi_1 - \phi_2$
5. Reciprocal $\dfrac{1}{z} = \dfrac{1}{r} \angle -\phi$
6. Square root $\sqrt{z} = \sqrt{r} \angle \phi/2$
7. Complex conjugate $z^* = x - jy = r \angle -\phi = re^{-j\phi}$
8. Euler's identity $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$