Beyond Arithmetic

Changing Mathematics in the Elementary Classroom

Jan Mokros
Susan Jo Russell
Karen Economopoulos

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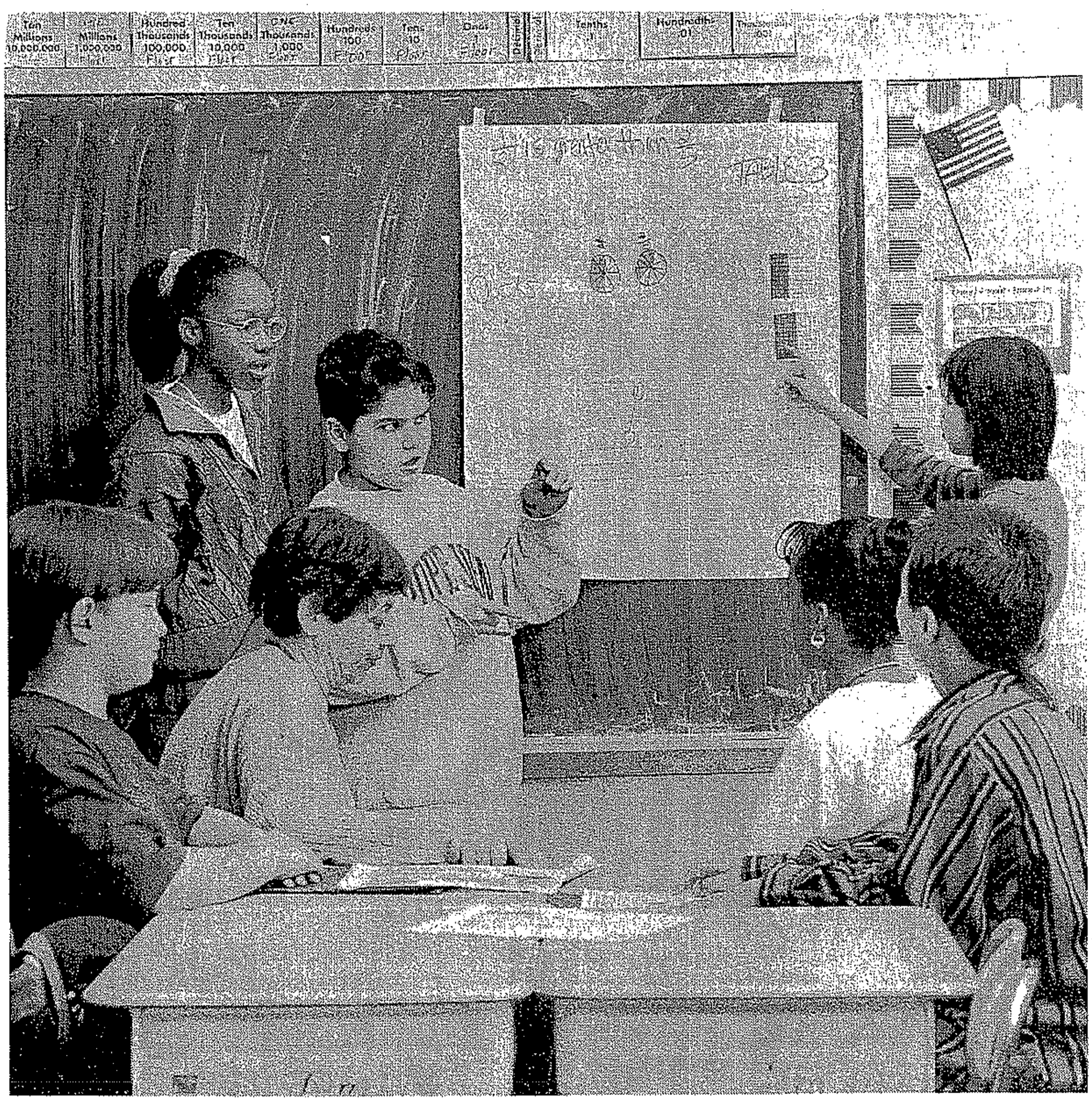
CHAPTER 1

Teaching Children Mathematics



The Results of Traditional Mathematics Education

The pedagogical point of view embodied in the traditional curriculum is simple: If you don't get it, do it again and again and again. Practice makes perfect. Only it doesn't. Despite years of training focused on calculation, American children enrolled in traditional mathematics programs have never become mathematically powerful. As results of national and international testing have repeatedly shown, our over-practiced students can apply procedures and formulas when problems are presented in the forms they recognize, but they run into trouble when they have to think. Numerous examples from national and international tests point to our students' weakness in number sense, estimation, and reasoning. For example, students can add up three given prices, but can't choose three items from a menu that total less than $4.00. They can find common denominators to add two fractions, such as $^5/_6$ and $^4/_5$, but don't know just by looking that the sum will be a little less than 2.

American students often rank near the bottom in international testing (Stevenson and Stigler, 1992). While a number of factors make it difficult to compare children from different cultures, two striking findings emerge from international comparisons: (1) Students from other countries exhibit better performance on "higher order" mathematical skills such as reasoning, conceptualization, and problem solving, and (2) in many other countries, the mathematics curricula are broader and focus more on problem solving than does the traditional U.S. curriculum.

The age-old mathematics curriculum simply no longer serves the needs of students, schools, or society as a whole. The NCTM, in its *Curriculum and Evaluation Standards for School Mathematics* (1989), calls for a shift from teaching students *procedures* to teaching them to *think and reason* mathematically. This shift is required by the more complex demands of today's society. Employers no longer look for employees who can apply memorized procedures to do rote calculations—everyone's pocket calculator takes care of these quite efficiently. Rather, they look for employees who can face an unfamiliar problem and think of a number of possible strategies to solve it. They look for sound estimation and number sense, skills in spatial visualization, competence in using and interpreting data, and a familiarity with technological tools and processes. This means that students must learn a great deal more mathematics than what we considered sufficient in the past, and that we must make room for more and deeper mathematics in the curriculum. In the past, the elementary curriculum focused almost entirely on paper-and-pencil work with arithmetic, while the study of geometry, data, number theory, and other important aspects of mathematics were relegated to a few special exercises or to chapters that were never reached. The pages in the NCTM *Standards* (1989, pp. 20–21, 70–74) that describe what needs increased and decreased attention in the curriculum must be taken seriously. If students are going to pursue mathematical ideas in depth, we have to make real choices about how they will spend their time in the mathematics curriculum.

What Do We Need to Teach?

The current consensus among mathematics educators and mathematicians is that we can't continue to focus on basic arithmetic, as in traditional curricula, and ignore the much bigger mathematical world that envelops us. A spate of recent books by mathematicians decry Americans' lack of familiarity with large numbers, geometry, percentages, statistics, and logic (Dewdney, 1993; MacNeal, 1994; Paulos, 1988). These books point out that most of us lack even the basic mathematical skills needed to make informed decisions about political candidates, consumer products, and health care. As a result, advertisers and politicians can easily take advantage of our "innumeracy" and present us with information that is at best misleading and at worst downright dangerous.

Our children deserve more from mathematics education. They deserve a curriculum that emphasizes many forms of mathematical thinking and that teaches them to use technological tools. If students are to become powerful and confident users of mathematics, the mathematics curriculum in the elementary school must contain a balance of study in at least four areas: number, statistics or data analysis, geometry, and what we call the mathematics of change. Each of these is described in turn below.

Number

The very first item on the NCTM *Standards* list of what should receive "increased attention" is number sense (NCTM, 1989, p. 20). To have "sense" about number means to understand how numerical quantities are constructed and how they relate to each other. A "numerically literate" person brings a rich web of interconnected knowledge and experience to an encounter with a number such as 5, or 100, or 0.98, or $^3/_4$. A person with number sense knows how these numbers are related to other numbers, where they fall in the number system, and how they can be transformed into other numbers to make calculation or comparison more manageable.

Consider a practical example of number sense in action:

> I have $^1/_2$ cup of flour and need $1^1/_4$ cups of flour; how much more flour do I need?

If I have a good sense of these familiar fractions, their magnitudes, and their relationships to each other and to 1, I would be unlikely to use the traditional subtraction algorithm ($1^1/_4 - ^1/_2$), which requires that I find common denominators, transform the mixed number into an improper fraction, then subtract. Rather, I immediately "see" that if I needed 1 cup of flour, I would need $^1/_2$ cup more, but I need $^1/_4$ cup more than 1, so in fact I need $^1/_2$ cup *and* $^1/_4$ cup, or $^3/_4$ cup.

Here's another example: Suppose we ask you to add 58 and 57 in your head. Some people will try to use the traditional "carrying" algorithm, but people with good number sense are more likely to do it another way: "Well, 50 and 50, that's 100, and 8 and 7 is 15, so it's 115." Others with good number sense might think: "Both numbers are close to 60. I know 60 and 60 is 120, then I subtract the 5 that I added on, and I get 115." People who use these methods can mentally figure this problem quickly and efficiently. They keep in mind the whole quantities involved rather than breaking them up into meaningless digits ("8 and 7 is 15, put down the 5, carry the 1"). Their calculations are based on sound knowledge of the number system, and they typically have good strategies for estimating and for double-checking for accuracy.

We want students coming out of elementary school to easily calculate two-digit addition and subtraction problems, problems using familiar fractions and decimals, and many other numerical problems mentally—not because they know a memorized procedure, but because they have a deep, rich, connected understanding of how our number system works.

As students grow in their understanding about the number system, they become familiar with very large as well as very small numbers. They get a sense of the size of a number like 3989 by relating it to important "landmarks" in the number system. For example, fourth and fifth graders develop mental images of larger and larger units: 100 is a familiar "landmark" to them, and ten 100's make a thousand, and a hundred 100's make ten thousand. Of course, simply being able to recite this information is not enough. Students must have mental images of how these numbers are built from each other. Once those images are firm, students can use these units to make sense of an even larger number, like 39,989—it is close to 40,000, which is the same as 4 ten thousands.

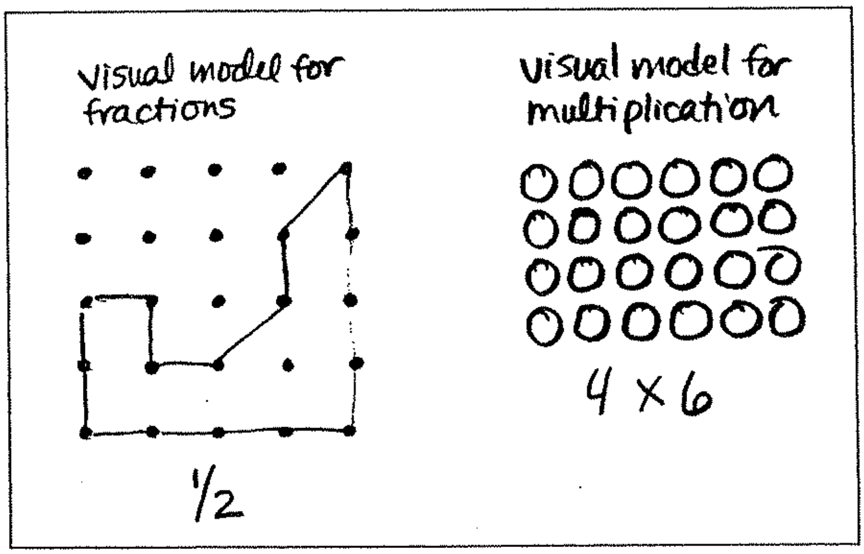
Students must also become comfortable with small numbers, a task that is in some ways more difficult. Traditionally, students have learned to calculate with fractions and decimals in the elementary grades without necessarily understanding how fractions and decimal numbers represent quantities less than 1. For example, if an athlete scores 5.73 on an event, is her performance better or worse than that of someone who scores 5.8? Many students believe that 5.73 is the better score, because 73 is bigger than 8. These same students may be correctly adding, subtracting, and multiplying with such numbers because they have learned memorized procedures. However, they are not able to apply their knowledge, because they have little idea about what these numbers mean. Finding one's way around the number system and understanding its intricacies is an essential goal of elementary school mathematics. It is clearly a goal that transcends arithmetic.

As students work with numbers, they also think about the characteristics of the numbers themselves. They describe, compare, and classify numbers. For example, a third grade class might investigate odd and even numbers. What makes a number *odd?* How do these numbers behave? When an odd number is added to or subtracted from another odd number, the result is an even number. Why is this true? Students can study many categories of numbers as they move through the elementary grades: primes, factors, and multiples; even and odd numbers; square numbers. As they build their own theories about these numbers and their relationships, students begin to think mathematically and to see and analyze patterns in all the mathematics they do.

Geometry and Measurement

Although much neglected in school, geometric and measuring skills are essential to our everyday lives. Think about how you use these skills when you are reading a map, giving directions to a friend, building a fort with your child, or ordering a window box for your plants. Think about how often you figure out the relationship of shape to area (will the new rug fit properly in the living room?) or to volume (will the leftovers fit in the container you've chosen?). Many people believe that spatial skills are innate—that you are either good at spatial visualization or you aren't. But the truth is, many of us have had very little opportunity in school to work on developing our spatial visualization skills. As in all learning situations, we won't get better at something unless we have enough opportunities to do it.

Much of mathematics and science depends on geometric models for understanding. When students develop visual models for fractions or for multiplication, they are using geometric relationships to help them think about number. Some students have a very good spatial sense, but have been labeled "not good at math" because they lack strong computational skills. These students can make powerful connections between what is to them the *familiar* spatial world and the more confusing world of number.



Geometric models can extend and support students' understanding of number.

As students study increasingly sophisticated mathematics and science in school, those who are able to construct and analyze physical and mental models will be at an advantage. An increasing number of jobs are also dependent on these skills. Consider the use of spatial reasoning in the burgeoning field of computer networking, in communications technology, in architecture, and in the development of CAD/CAM (computer-aided design/computer-aided manufacturing) for designing machinery. With the use of computers as modeling tools, we can represent and manipulate 3-D objects on the screen. Translating back and forth between 3-D objects and 2-D representations of those objects—maps, drawings, diagrams, plans—is a central part of many high-tech and skilled-trade vocations. It's not just that more and more jobs require geometric skills; it's also the case that the disproportionately higher-paying jobs demand these skills. By emphasizing geometry in elementary school math, we are giving all our students—not just those who build models and play with computer graphics at home—a wider range of options.

Statistics

It is shocking that Americans learn so little about statistics in school, given the fact that so much of daily life depends upon our being able to interpret and use data. Looking at the morning newspaper one day, for example, we find data on the percentages of children of various ethnic backgrounds enrolled in city schools, with an accompanying statistics-packed article about the disparity in minority enrollment between suburban and city schools. There is also an article comparing the risk of amniocentesis with that of a new procedure, chorionic villi sampling. And, just as there has been nearly every day for a month, there is another article about the relative costs of health care and who would pay those costs under different plans proposed by government officials. In these articles, the reader is expected to interpret data from maps, charts, and graphs; to understand the notion of *median*, and how it differs from *mean;* to make sense of cost-benefit ratios; and to compare different kinds of risks.

Statistics are prevalent in our lives, and critical personal decisions are often dependent on being able to interpret and apply statistical data. But what do we offer in school to help students use and interpret data? Most students learn how to identify the tallest or shortest bar on a graph, how to calculate an average, and how to put data on a line graph, then "connect the dots." But there are serious gaps in their understanding.

Most children and many adults do not know how an average represents the data that it describes. While students can use the add-'em-up-and-divide algorithm for finding the mean, they typically have little idea what this number tells them. When we give students $1.69 as the average price of a bag of chips and ask them to figure out what the prices for 9 different brands might be, they are stumped—they don't have an idea of the prices this average could reflect. When we read about an average in the newspaper, we need to be able to imagine what possible data sets might result in that average. For example, if we read that the average person holds 8 jobs in his or her lifetime, what might this average mean? Does it mean that a large percentage of the population holds, say, 7 to 9 jobs, while smaller percentages hold fewer or more? Or is it the case that the largest percentage holds only 2 or 3 jobs, while another group that holds between 15 and 20 jobs "pulls up" the average? Statistically literate citizens need to ask questions like these and need to integrate information from a variety of sources in order to interpret what the average conveys about the data.

Students must acquire a deep, firsthand understanding of how to ask questions, how to collect data, how to represent these data, and how to make sense of them. Students need to learn to tell the story of the data—the true story, not a misleading simplistic version—and they need to learn how to interpret the statistical stories that they read in the print media and hear on TV. They need to learn about probability if they are to make informed decisions about cigarette smoking, the effects of exercise, or whether they should play the lottery. Of course, many subjective factors, too, affect the decisions we make—but if we don't help students learn some basic statistical ideas, we are sending them into a risky world without the tools they need to evaluate risk.

The Mathematics of Change

Most students do not learn about the mathematics of change until they are in high school or college. During algebra or calculus, students learn to describe and compare rates of growth and to examine patterns of change. In algebra, students begin to learn what a function is. They learn to describe how one variable changes in direct relationship to another—for example, the area of a circle is related to the length of its radius. They make graphs and write equations to describe these functions. In calculus, students learn about more complicated rates of change. They learn that rates of change may not be constant. For example, a baby grows fastest right after it is born, then growth slows down until adolescence, at which point it speeds up. Describing the rate at which something is speeding up or slowing down is an important element of calculus.

Figuring out how quickly something grows or declines is essential in the sciences and social sciences. Despite our mathematics courses, most of us have learned little about the idea of *rate of change.* We are confused by claims and counterclaims about how big something is versus how fast it is growing. For example, during the 1992 presidential debates, Bill Clinton claimed that Arkansas had experienced one of the biggest increases in high school graduation rates of any state in the country. George Bush refuted this, claiming that Arkansas had one of the lowest percentages of high school graduates. Who was right? Both were. Clinton was talking about *rate of change* (and Arkansas had indeed posted impressive gains), whereas Bush was describing the absolute percentages of high school graduates from various states.

Recent research suggests that students should be learning about change many years before they enter high school (Nemirovsky, 1993; Tierney and Nemirovsky, 1991). Even quite young children can and should begin to describe and understand the mathematics of change. As students construct bigger and bigger squares out of tiles, they can describe how the area of the square changes in relation to the increasing length of its side. By keeping track of how tall their bean plants are, students can begin describing in their own words not only the *height* of a graph but its *slope* (the rate at which change occurs): "it suddenly slowed down," "it's gradually growing faster" or "it is growing faster and faster all the time." As elementary grade students examine and talk about change, they are developing important ideas about mathematical representations and relationships.