Exploring Odd + Odd = Even

In class today, we considered the following problem:

If I add two numbers that are both _________,

then the resulting sum is _________.

We brainstormed different qualities of numbers to put in the blanks (e.g., even, odd, prime, etc.), and then tried to determine whether these qualities would make the statement true or false. Given the right environment, elementary students can and do explore similar ideas.

Consider the following conjecture proposed by Jennifer, a fourth grade student:

If I add two numbers that are both <u>odd numbers</u>,

then the resulting sum is an <u>even number</u>.

The students in Jennifer's class worked in class and at home to determine whether this is true. Below are 4 of the proofs that students proposed. Note that some students worked alone, and others worked in pairs. As you read each proposed proof, think about whether or not it convinces you that the conjecture is true, and whether or not it would convince the rest of the students in Jennifer's 4^{th} grade class. [Respond to the questions about these proposed proofs on the last page of WN-2 for you to respond.]

Proposed Proof #1 (Jeff)

1 + 5 = 6
13 + 9 = 22 So, odd + odd = even

Proposed Proof #2 (Brianna)

5 is odd, so I think of it as 2 pairs with 1 left over: ** ** *

13 is odd, so I think of it as 6 pairs with 1 left over: ** ** ** ** ** ** *

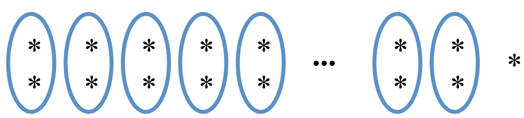
So, when you put them together, you put together 6 pairs and 2 pairs = 8 pairs, and then the 2 leftovers go together to make another pair, which makes 9 pairs. So, **5 + 13 is even**.

Proposed Proof #3 (Dean & his dad)

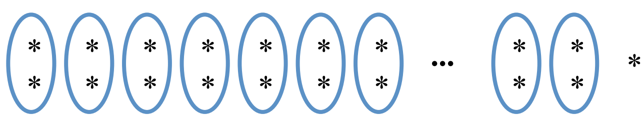
Let a and b be odd #s:	
$a = 2c + 1$ $b = 2d+1$	Definition of odd numbers
$a + b = 2c + 1 + 2d + 1$	Substitution
$a + b = 2c + 2d + 1 + 1$	Commutative property of addition
$a + b = 2c + 2d + 2$	Addition facts
$a + b = 2(c + d + 1)$	Distributive property of multiplication over addition
Let $e = c + d + 1$	Substitution
$a + b = 2e$	
$a + b$ is even	Definition of even numbers

Proposed Proof #4 (Jennifer & Sarah)

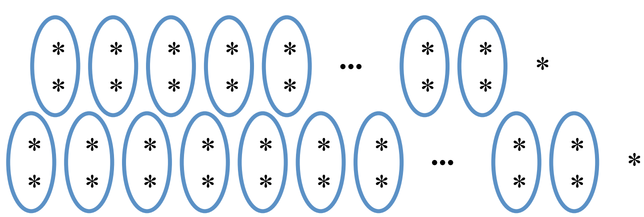
Here is the first odd number (which is a bunch of pairs with one leftover)



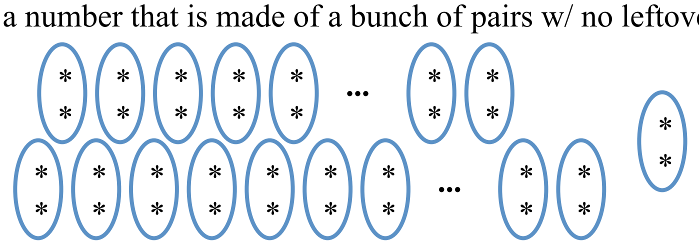
Here is the second odd number (which is also a bunch of pairs with one leftover)



To show what happens when you add the two numbers, push the piles together:



When I push them together, I can put the two leftovers together to make a new pair. Now I have a number that is made of a bunch of pairs w/ no leftovers, so it is an even number.



Analyzing the proposed proofs for Odd + Odd = Even

For each proposed proof, respond to the following (do so in the space provided below):

Is the proposed proof sufficient to prove that odd + odd = even is always true?

- **If yes,** what is it that makes this proof a reasonable one? For whom is this proof appropriate? (ex: the average 4^{th} grader, the average college student, …)
- **If no,** what is problematic about the argument?

Proposed Proof #1 (Jeff):

Proposed Proof #2 (Brianna):

Proposed Proof #3 (Dean and his dad):

Proposed Proof #4 (Jennifer and Sarah):