Math 1710
Review Problems for Final Exam
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No calculators on problems marked with (*).

1. Let
$$\vec{r}(t) = (t, \cos t, \sin t).$$
Find the velocity and acceleration vectors, the tangential and normal components of acceleration, curvature, radius of curvature, and center of curvature, all when $t = 0$.

2. A particle moves around a circle. At a given instant, its velocity is $(3, 2, 1)\,\mathrm{m/s}$ and $\|\vec{a}_N\| = 5\,\mathrm{m/s^2}$. Find the radius of the circle.

3. Show that the curvature for the parabola $y = ax^2$ attains its maximum at the parabola's vertex.

4. Do 4 steps using Euler's method with $h = 0.25$ to obtain approximate values for $F(0.25)$, $F(0.5)$, $F(0.75)$, $F(1.0)$, where
$$F(x) = \int_0^x \frac{1}{1+t^2}\,dt.$$

5. Use Simpson's rule with $N = 2$ to approximate $F(1)$, where
$$F(x) = \int_0^x \frac{1}{1+t^2}\,dt.$$

6. (*) Compute the derivative of $\ln|\sec x + \tan x|$.

7. (*) Compute the anti-derivatives:
$$\int \tan x\,dx, \qquad \int \frac{x+1}{x^2+2x+25}\,dx.$$

8. (*) Compute the derivatives of the following functions:
$$e^{(x^2)}, \quad (e^x)^2, \quad e^x \ln|\sin x|, \quad e^{\pi \ln(2x)}.$$

9. Solve the differential equation:
$$y' = xy, \quad y(0) = 2.$$

10. Express $z = 2 - i$ as $e^{\alpha+i\beta}$.

11. Express e^z as $\alpha + i\beta$, where $z = 3 + i\pi/4$.

12. Simplify the expression $\sin(\arctan x)$.

13. (*) Evaluate the integrals:
$$\int \frac{x}{4x^2+1}\,dx, \quad \int \frac{1}{\sqrt{1-9x^2}}\,dx, \quad \int \frac{e^x}{1+e^x}\,dx, \quad \int \frac{e^x}{1+e^{2x}}\,dx.$$

14. Use the given conditions to find k and solve the equation:
$$y'(t) = ky(t), \quad y(0) = 2, \quad \text{and } y(2) = 4.$$

15. If the half-life of a radioactive material is 100 yr, what percent of the original material would be left in 6 mo?

16. An object has been heated, and at time $t = 0$ it is placed in a room that is held at a temperature of 70°F. At time $t = 5$, its temperature is 258°F, and at time $t = 10$, the temperature of the object is 224°F. Given that its temperature function T satisfies the equation $T'(t) = k(T(t) - 70)$, what was the object's original temperature?

17. A 2 kg ball is shot straight up with an initial speed of 50 m/s. The constant of resistance due to the atmosphere is known to be $k = 0.02$. Determine how high the ball will go.

18. Solve the separable differential equation:
$$y'(x) = 2y(x) + 3y^2(x), \quad y(0) = 1.$$

19. Find the limits (with justification):
$$\lim_{x\to\infty} \frac{\sin x}{x}, \quad \lim_{x\to 0} \frac{\sin x}{x}, \quad \lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x, \quad \lim_{x\to\pi/2^-} (\sin x)^{\tan x}.$$

20. Calculate the improper integrals:
$$\int_0^\infty \frac{1}{1+x^2}\,dx, \quad \int_0^\infty xe^{-x}\,dx, \quad \int_0^1 \frac{1}{\sqrt{x}}\,dx.$$

21. Determine (with justification) whether the series converge or converge absolutely:

$$\sum_{n=0}^{\infty} \frac{1}{n^{1.5}}, \quad \sum_{n=0}^{\infty} (1.5)^n, \quad \sum_{n=0}^{\infty} \frac{10^5}{n+n^2}, \quad \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}, \quad \sum_{n=0}^{\infty} \frac{n^2}{n!}.$$

22. Find the radius and set of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} x^n, \quad \sum_{n=1}^{\infty} \frac{nx^n}{1 \cdot 3 \cdot 5 \cdots (2n-1)}.$$

23. Find the Maclaurin series expansions for:

$$f(x) = \frac{x}{1-x}, \quad g(x) = \sin(2x), \quad h(x) = \frac{e^x - 1}{x}.$$

24. Find a lower bound for the radius of convergence for the Taylor series expansion of

$$f(x) = \frac{x}{1+x^2}, \quad \text{at } c = 2.$$

25. (*) Evaluate the following integrals:

$$\int x \cos x \, dx, \quad \int \arcsin x \, dx, \quad \int \sqrt{25 - x^2} \, dx, \quad \int \frac{x^2}{\sqrt{x^2 - 9}} \, dx, \quad \int x\sqrt{x^2 + 9} \, dx.$$

26. (*) Evaluate the following integrals:

$$\int \frac{1}{x^3 + x^2} \, dx, \quad \int \frac{x^2 + 4x + 2}{x^2 + x} \, dx, \quad \int \frac{x}{(x^2 + 4)^2} \, dx.$$

27. Find $x'(t)$ when:

$$x''(t) = -[x(t)]^{-2}, \quad x(0) = 2, \quad x'(0) = -1.$$

28. A spherical tank with radius R is full of water. Compute the work done in pumping the water out the top of the tank.

29. A chain of length 1 m and density 1 kg/m is hanging (by its top end) from the top of a table. Find the work done in pulling (the top end of) the chain across the table until the chain is entirely on the table, assuming that the magnitude of the frictional force opposing the motion of the chain is 1/4 the weight of that portion of the chain on the table.

30. One end of a chain of length 1 m and density 1 kg/m is hanging off a table, so that half of the chain is on the table and half is not. The chain is then allowed to slide off the table. Determine the velocity of the chain the instant that it leaves the table, assuming the frictional force opposing the motion of the chain is 1/4 the weight of that portion of the chain on the table.

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