Two Dangerous/Too Dangerous Equations

The Height Equation

As we saw in the ballistic problem in class, if one starts with the kinematic equation without time, $v_y^2 = v_{0y}^2 + 2\,a_y\,(y - y_0)$, we can easily find the height if one notes that at the turning point the speed in the y-direction must be zero.

$$v_y^2 = v_{0y}^2 + 2\,a_y\,(y - y_0)$$

but $v_y^2 = 0$ **,** $a_y = -g$ **and** $y - y_0 = h$ **, so**

$$0 = v_{0y}^2 - 2gh$$

$$v_{0y}^2 = 2gh$$

$$h = \frac{v_{0y}^2}{2g}$$

substituting in for the *y*-component of the initial velocity gives us The Height Equation:

$$h = \frac{v_0^2 \sin^2 \theta}{2g} \qquad (1)$$

Note that the height equation is always good near the surface of the earth. The term "$\sin^2 \theta$" means "sine squared" and literally means take the sine of theta, and then square the answer.

(The Time to Height Equation)

Since the time to "fall up" from the ground to the turning point is the same as the time to fall to the ground, from rest, at the same height, we have several ways of finding the time it takes to go from ground to the maximum height or turning point.

$$y = y_0 + v_{0y}\,t + \tfrac{1}{2}\,a_y\,t^2$$

but $y - y_0$ **is** h**, and** $a_y = -g$ **, so**

$$h = v_{0y}\,t - \tfrac{1}{2}\,g\,t^2$$

if we fall down, rather than fall up, then $v_{oy} = 0$ **and we get … oops, now we have to use** $y - y_0 = -h$ **(because we are going the other way, right?)**

$h = \tfrac{1}{2}\,g\,t^2$ **or** $t^2 = 2\,h / g$

$$t = \sqrt{\frac{2h}{g}} \qquad (2)$$

Or we can look at the change in speed:

$$v_y = v_{0y} + a_y\,t$$

put in gravity, and that v_y **is zero at the turning point**

$$0 = v_{0y} - g\,t$$

$g\,t = v_{oy}$ **or** $t = v_{0y} / g$

$$t = \frac{v_0 \sin \theta}{g} \qquad (3)$$

Note that this is the time from the ground to the maximum height (and vice versa) only.

(Substitute Equation (1) into (2) and you do get Equation (3), so they are equivalent.)

The Range Equation

Generations of Physics students, happy with knowing the height equation, manage to stumble across the similar looking Range Equation. They are then thrilled to think that they've discovered a way to do Physics problems without thinking. Ah, but PTPBIP... the Range Equation **is only good when the initial height and the final height are the same.** Having said that, let's see.

For motion in the *x*-direction, we use the kinematic equation:

$$x = x_0 + v_{0x}\,t + \tfrac{1}{2}\,a_x\,t$$

We choose, $x_0 = 0$ **and we know that there is no acceleration in the *x*-direction so** $a_x = 0$.

$$x = v_{0x}\,t$$

Substituting in for the *x*-component of the velocity and using <u>twice</u> Equation (3) for the time (why twice?), we get:

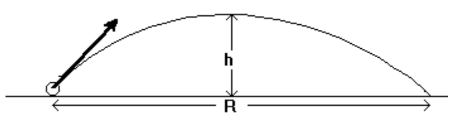
$$x = \frac{(v_0 \cos \theta)(2 v_0 \sin \theta)}{g}$$

Combining terms...

$$x = \frac{2 v_0^2 \sin \theta \cos \theta}{g}$$

There is a trig identity (take my word for it) that says that: $2 \sin \theta \cos \theta = \sin(2\theta)$

This then gives us the Range Equation, where *R* is the distance *downrange* from the launch site:



$$R = \frac{v_0^2 \sin 2\theta}{g} \qquad (4)$$

WARNING: You can only ever use the Range Equation if the initial and final heights are the same.
WARNING: The Range Equation and the Height Equation are very similar looking. Confuse them at your own peril.

If Dr. Phil thinks that these equations are dangerous, why did he make this handout? Because no one does Physics problems in a vacuum. You are likely to have run across these equations either from friends or in previous Physics courses. Knowing *where* these equations comes from and *when* you might be able to use them, is at least *useful* information – but remember that you can always start from the kinematic equations and get the same answer anyway!