PHYS-113(3) (Kaldon-10717)
WMU - Fall 2001
Exam 2 - 100,000 points

Name ______**S O L U T I O N**__________

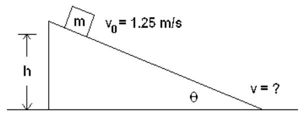
Book Title ______________________________

Section: **3a** (A to G) **3b** (H to M) **3c** (N to S) **3d** (T to Z) 10/14/2001•Rev.4

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Be Sure to Write Down Equations – Feel Free to Ask Any Questions
Use Your Time Wisely – Work on What You Can

Heading Down The Slippery Slope… (50,000 points)

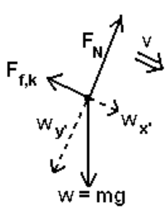
1.) In the following parts, we have a block of mass $m = 12.5\ kg$ moving down the ramp with an initial speed, $v_0 = 1.25\ m/s$, starting at a height $h = 1.25\ m$ and an angle $\theta = 28°$. (a) If there is no friction, find the speed v of the block at the bottom of ramp. Use any valid Physics method.



Conservation of T.M.E. is the fastest way to get a solution:

$$mgh_1 + \tfrac{1}{2}mv_1^2 = mgh_2 + \tfrac{1}{2}mv_2^2$$
$$mgh_1 + \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}mv_2^2$$
$$v_2^2 = v_1^2 + 2gh_1$$
$$v_2 = \sqrt{v_1^2 + 2gh_1} = \sqrt{(1.25m/s)^2 + 2(9.81m/s^2)(1.25m)}$$
$$= 5.108m/s$$

(b) Now we will add in friction. The coefficients of friction are 0.13 and 0.19. Use a F.B.D. and forces to find the acceleration, a, of the block down the ramp.



$$\mu_s = 0.19 \ ; \ \mu_k = 0.13$$
$$\sum F_{y'} = F_N - w_{y'} = 0 \ ; \ F_N = w_{y'} = mg\cos\theta$$
$$F_{f,k} = \mu_k F_N = \mu_k mg\cos\theta$$
$$\sum F_{x'} = w_{x'} - F_{f,k} = ma_{x'}$$
$$w_{x'} = mg\sin\theta$$
$$mg\sin\theta - \mu_k mg\cos\theta = ma_{x'}$$
$$a_{x'} = g\sin\theta - \mu_k g\cos\theta$$
$$= (9.81\text{m/s}^2)\sin 28° - (0.13)(9.81\text{m/s}^2)\cos 28°$$
$$= 3.479\ \text{m/s}^2$$

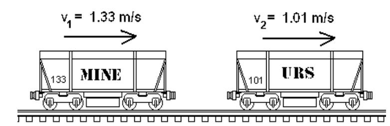
(c) Same setup as in (b), but this time use Work and Energy to find the final speed v at the bottom of the ramp. (In theory, knowing the acceleration from (b) and using the kinematic equations, we could get this v that way, too, but we won't take the time.)

$$h = L\sin 28°$$
$$L = \frac{h}{\sin 28°} = \frac{1.25m}{\sin 28°} = 2.663m$$
$$W_{net} = KE_f - KE_i$$
$$F_{f,k} = \mu_k F_N = \mu_k mg\cos\theta$$
$$mgh - F_{f,k}L = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$
$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 + mgh - \mu_k mgL\cos\theta$$
$$\tfrac{1}{2}v_f^2 = \tfrac{1}{2}v_i^2 + gh - \mu_k gL\cos\theta$$
$$v_f = \sqrt{v_i^2 + 2gh - 2\mu_k gL\cos\theta}$$
$$= \sqrt{(1.25m/s)^2 + 2(9.81\text{m/s}^2)(1.25\text{m}) - (0.13)(9.81\text{m/s}^2)(2.663m)\cos 28°}$$
$$= 4.805m/s$$

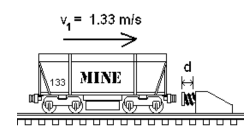
NOTE: that the answer to (c) *IS* less than (a).

(d) A railroad mine car, $m_1 = 40{,}200\ kg$, is moving to the right at $v_1 = 1.33\ m/s$, collides inelastically with a second car, $m_2 = 42{,}300\ kg$, moving to the right at $v_2 = 1.01\ m/s$. The two cars, now coupled and combined together, move with a speed V. Find V.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)V$$
$$V = \frac{m_1v_1 + m_2v_2}{(m_1 + m_2)}$$
$$= \frac{(40{,}200kg)(+1.33m/s) + (42{,}300kg)(1.01m/s)}{(40{,}200kg + 42{,}300kg)}$$
$$= 1.166m/s$$

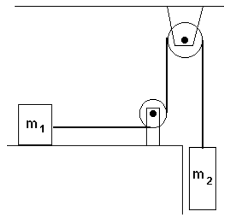


(e) On another day the railroad mine car, $m_1 = 40{,}200\ kg$, moving to the right at $v_1 = 1.33\ m/s$, is stopped by a safety bumper with a giant spring. The car comes to a stop in a distance $d = 1.75\ m$. Find the spring constant k of the giant spring.

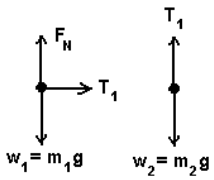


$$PE_{spring} = \tfrac{1}{2}kx^2 \ ; \ KE = \tfrac{1}{2}mv^2$$
$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2$$
$$k = \frac{mv^2}{x^2} = \frac{(40{,}200kg)(1.33m/s)^2}{(1.75m)^2} = 23{,}220N/m$$

Cables and Pulleys and Hanging Weights (Oh My!) (50,000 points)

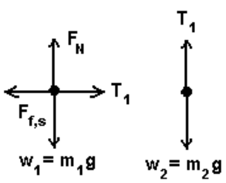


2.) Block 1 has a mass $m_1 = 2.50\ kg$ and rests on a frictionless table. Block 2 has a mass $m_2 = 1.25\ kg$ and hangs by a cable that is connected by pulleys to Block 1. Both blocks start at rest. Frictionless perfect pulleys are used. (a) Find the acceleration of Block 1.



$$\sum F_{2y} = T_1 - m_2 g = m_2 a_y = -m_2 a$$
$$\sum F_{1y} = F_N - m_1 g = 0$$
$$\sum F_{1x} = T_1 = m_1 a_x = +m_1 a$$
$$T_1 - m_2 g = -m_2 a$$
$$m_1 a - m_2 g = -m_2 a$$
$$m_1 a + m_2 a = m_2 g$$
$$a = \frac{m_2 g}{m_1 + m_2} = \frac{(1.25kg)(9.81m/s^2)}{2.50kg + 1.25kg}$$
$$= 3.27m/s^2$$

(b) If the table was *not* frictionless, find the friction force F_f needed to keep both blocks at rest. What kind of friction would this be? And does it point to the LEFT or the RIGHT ?

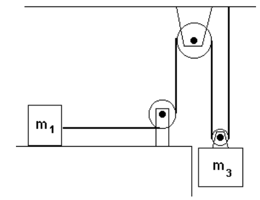


$$\sum F_{2y} = T_1 - m_2 g = 0$$
$$T_1 = m_2 g$$
$$\sum F_{1y} = F_N - m_1 g = 0$$
$$\sum F_{1x} = T_1 - F_{f,s} = 0$$
$$T_1 = F_{f,s}$$
$$F_{f,s} = m_2 g$$
$$= (1.25kg)(9.81m/s^2)$$
$$= 12.26N$$

Need static friction pointing to the LEFT to keep both blocks at rest.

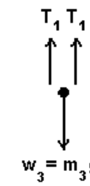
Although not asked for, the minimum coefficient of static friction for this problem is $\mu_s = 0.500$. Why?

(c) Block 3 replaces Block 2 and adds an extra pulley. Find the mass m_3 that would provide the same tension T to Block 1 as in (a) and (b).



For part (b), where blocks are at rest, it is relatively easy to see:

$$\sum F_{3y} = 2T_1 - m_3 g = 0$$
$$m_3 g = 2T_1 \ \ but\ \ T_1 = m_2 g$$
$$m_3 g = 2m_2 g$$
$$m_3 = 2m_2 = 2(1.25kg) = 2.500kg$$



For part (a), where blocks are accelerating, it gets more complicated, but in the end the answer is what you expect it to be:

$$a = 3.27m/s^2$$
$$T_1 = m_1 a$$
$$\sum F_{3y} = 2T_1 - m_3 g = m_3 a_y = -m_3 a$$
$$2T_1 = m_3 g - m_3 a$$
$$2m_1 a = m_3 (g - a)$$
$$m_3 = \frac{2m_1 a}{(g-a)} = \frac{2(2.50kg)(3.27m/s^2)}{(9.81m/s^2 - 3.27m/s^2)} = 2.50kg$$

Tom shoves a crate, $m = 85.5\ kg$, a distance $d = 5.00\ m$ across a shop floor at a constant speed $v = 1.00\ m/s$. There is friction. The force that Tom supplies is $F_{1x} = 419.\ N$. Find (d) the Work that Tom is doing to the crate and (e) the Power that Tom needs to do this work.

(d) $W = Fd = (419N)(5.00m) = 2095J$

(e) $P = Fv = (419N)(1.00m/s) = 419.0W$