

How to find a radius of an electron:

1.) Start with a nucleus with Z protons. A single electron in orbit around this nucleus undergoes Uniform Circular Motion, with the Coulomb Force as the centripetal force.

$$F_E = \frac{kZe^2}{r^2} = m \frac{v^2}{r}$$

2.) Simplifying, we get:

$$\frac{kZe^2}{r} = mv^2$$

or

$$v^2 = \frac{kZe^2}{mr}$$

Note that r and v are now locked together.

3.) deBroglie said that matter has waves:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

4.) The Bohr atom quantizes the states – in this case, we can think of the electron deBroglie matter wave as a standing wave:

$$2\pi r = n\lambda$$

5.) Solve (4) for r and substitute (3) for λ , and we see that the angular momentum ($L = mvr$) is quantized.

$$mvr = n \frac{h}{2\pi} = n\hbar$$

6.) Square (5) and then substitute v^2 from (2):

$$m^2 v^2 r^2 = n^2 \hbar^2$$

$$\frac{m^2 kZe^2 r^2}{mr} = n^2 \hbar^2$$

$$mkZe^2 r = n^2 \hbar^2$$

$$r = \frac{n^2 \hbar^2}{mkZe^2}$$

7.) Note that the mass here is the mass of the electron, ($m_e = 9.11 \times 10^{-31}$ kg), so that the only variables in this equation are n and Z .

$$r_n = \frac{n^2 \hbar^2}{m_e kZe^2} = \frac{n^2}{Z} \left(\frac{\hbar^2}{m_e k e^2} \right) = \frac{n^2}{Z} a_0$$

8.) The stuff in the parentheses represents the radius of the $n = 1$ orbit of hydrogen ($Z = 1$):

$$a_0 = \frac{\hbar^2}{m_e k e^2} = \frac{(1.0538 \times 10^{-34} \text{ J}\cdot\text{s})^2}{(9.11 \times 10^{-31} \text{ kg})(9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2} = 0.528 \times 10^{-10} \text{ m}$$

9.) The Three-Body Problem says that we can't do this same kind of analysis with any atoms that have more than one electron. But... we can create a *hydrogenic* (means hydrogen-like) ion for all other elements by stripping off all but one of the electrons, and putting in the correct Z for the nuclear charge.

How to find the Energy of an electron:

10.) The kinetic energy is $\frac{1}{2} mv^2$. We can find this using (2) and (7):

$$\begin{aligned} \frac{1}{2} m_e v^2 &= \frac{kZe^2}{2r_n} = \frac{kZe^2 m_e kZe^2}{2n^2 \hbar^2} = \frac{k^2 Z^2 e^4 m_e}{2n^2 \hbar^2} = \frac{Z^2}{n^2} \left(\frac{k^2 e^4 m_e}{2\hbar^2} \right) \\ &= \frac{Z^2}{n^2} \left(\frac{(9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)^2 (1.602 \times 10^{-19} \text{ C})^4 (9.11 \times 10^{-31} \text{ kg})}{2(1.0538 \times 10^{-34} \text{ J}\cdot\text{s})^2} \right) \\ &= \frac{Z^2}{n^2} 2.18 \times 10^{-18} \text{ J} = \frac{Z^2}{n^2} 13.6 \text{ eV} \end{aligned}$$

(since 1 electron volt = 1 eV = 1.602×10^{-19} J).

11.) The total energy of the electron is the sum of the P.E. plus K.E. terms. The P.E. for Coulomb's Law is:

$$PE = -W = -Fd = -\frac{kZe^2}{r_n}$$

Or P.E. = -2K.E. This form has the advantage that the P.E. is zero at infinity, and is always negative. The total energy, $E_n = \text{K.E.} + \text{P.E.} = \text{K.E.} - 2\text{K.E.} = -\text{K.E.}$. Thus we can write the total energy as a function of n and Z as:

$$E_n = -\frac{Z^2}{n^2} 2.18 \times 10^{-18} \text{ J} = -\frac{Z^2}{n^2} 13.6 \text{ eV}$$

So for Hydrogen ($Z=1$), the first three levels are:

n	r_n	E_n
1	0.528 Å	-13.6 eV
2	2.11 Å	-3.40 eV
3	4.75 Å	-1.51 eV

(1 Å = 1×10^{-10} m)

How to find a transition photon:

12.) In hydrogen, for an $n=3$ electron to drop to the $n=2$ state, this is:

$$\Delta E = 3.4 \text{ eV} - 1.51 \text{ eV} = 1.89 \text{ eV} = 3.03 \times 10^{-19} \text{ J} = hf$$

$$f = \frac{E}{h} = \frac{3.03 \times 10^{-19} \text{ J}}{6.626 \times 10^{-34} \text{ J}\cdot\text{s}} = 4.57 \times 10^{14} \text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{4.57 \times 10^{14} \text{ Hz}} = 6.56 \times 10^{-7} \text{ m} = 656 \text{ nm}$$

This is red light.

NOTE: Don't confuse the wavelength λ here in (12) with the deBroglie wavelength λ in (3).