

Poor Richard's Book of Physics (50,000 points)

1. Benjamin Franklin is flying his kite into an electrical storm. A thunderstorm cloud 2000. meters overhead has a net charge of $Q = +113 \text{ C}$. Assuming it acts like a point charge, (a) what is the strength of the electric field, E , that Ben sees on the ground?

$$E = \frac{kq}{r^2} = \frac{(8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(113\text{C})}{(2000.\text{m})^2} = 253,900 \text{ N/C}$$

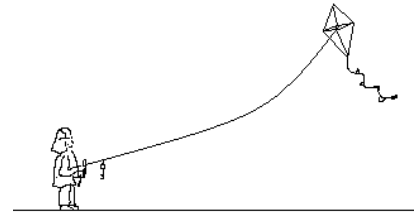
(b) What is the potential difference, ΔV , between the charged cloud and the neutral ground?

The only way to easily do this problem: assume we can use $\Delta V = E d$.
(Why might this be a bad assumption otherwise?)

$$\Delta V = Ed = (253,900 \text{ N/C})(2000.\text{m}) = 507,800,000 \text{ volts}$$

(c) If the metal key that is hanging from the wire has a charge $q = +1.04 \times 10^{-3} \text{ C}$, then find the electric force, F_E , on the key from the overhead thunderstorm cloud.

$$F_E = \frac{kq_1q_2}{r^2} = \frac{(8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(113\text{C})(1.04 \times 10^{-3} \text{ C})}{(2000.\text{m})^2} = 264.1 \text{ N} \quad \text{OR} \quad F_E = qE = (1.04 \times 10^{-3} \text{ C})(253,900 \text{ N/C}) = 264.1 \text{ N}$$



(d) Ben Franklin would have said that his single electric fluid would have been added to the metal key to reach $q = +1.04 \times 10^{-3} \text{ C}$. In truth, how many electrons would have been added or subtracted to make this q ?

$$q = +Ne \quad N = \frac{1.04 \times 10^{-3} \text{ C}}{1.602 \times 10^{-19} \text{ C}} = 6.492 \times 10^{15} \text{ electrons subtracted}$$

(e) Underneath the thunderstorm, a charge of $Q = -113 \text{ C}$ is spread out onto the ground, covering an area of 100. meters \times 100. meters. Use Gauss's Law to find the magnitude of the electric field, E , treating the ground as a sheet of charges.

$$\sigma = \frac{Q}{A} = \frac{-113\text{C}}{(100.\text{m})^2} = 0.01130 \text{ C/m}^2 \quad \Phi_E = E(2A) = \frac{q_{\text{inside}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \quad E = \frac{\sigma}{2\epsilon_0} = \frac{0.01130 \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} = 63,840,000 \text{ N/C}$$

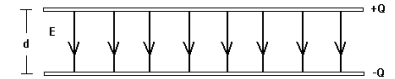
This Problem Should Cap Off The Start of the New Year of the Snake (50,000 points)

2.) A parallel plate capacitor consists of plates with an area $A = 0.100 \text{ m}^2$ and a separation of $d = 7.00 \text{ cm}$.
(a) Find the capacitance, C . $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12} \text{ F/m}) \frac{(0.100 \text{ m}^2)}{(0.0700 \text{ m})} = 1.265 \times 10^{-11} \text{ F} = 12.65 \text{ pF}$$

(b) If this capacitor is hooked up to a 512 volt battery, find the magnitude of the constant electric field, E , between the plates, using the " $V = Ed$ " equation. Sketch some of the E-field lines between the plates in the diagram. You do not need the answer to (a) for this part.

$$\Delta V = Ed \quad E = \frac{\Delta V}{d} = \frac{512 \text{ volts}}{0.0700 \text{ m}} = 7314 \text{ V/m}$$



(c) Find the magnitude of the electric field between the plates, using the equation $E = \frac{\sigma}{\epsilon_0}$. The capacitor equation, $C = Q/V$, will get you the charge on one of the plates. If you did not get an answer to (a), use $C = 12.6 \text{ pF}$.

$$Q = CV = (1.265 \times 10^{-11} \text{ F})(512 \text{ volts}) = 6.477 \times 10^{-9} \text{ C} \quad \sigma = \frac{Q}{A} = \frac{6.477 \times 10^{-9} \text{ C}}{0.100 \text{ m}^2} = 6.477 \times 10^{-8} \text{ C/m}^2 \quad E = \frac{\sigma}{\epsilon_0} = \frac{6.477 \times 10^{-8} \text{ C/m}^2}{8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = 7319 \text{ N/C}$$

(d) How much energy is stored in this capacitor when fully charged?

$$\text{Energy} = U = \frac{1}{2} CV^2 = \frac{1}{2} (1.265 \times 10^{-11} \text{ F})(512 \text{ V})^2 = 1.658 \times 10^{-6} \text{ J}$$

(e) If half the capacitor is filled with water ($\kappa = 80$, Dielectric Strength = ∞), then this becomes two capacitors in series.

Find the equivalent capacitance of this waterlogged capacitor.



$$C_1 = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12} \text{ F/m}) \frac{(0.100 \text{ m}^2)}{(0.0350 \text{ m})} = 2.529 \times 10^{-11} \text{ F} = 25.29 \text{ pF} \quad C_2 = \kappa \epsilon_0 \frac{A}{d} = (80)(8.85 \times 10^{-12} \text{ F/m}) \frac{(0.100 \text{ m}^2)}{(0.0350 \text{ m})} = 2.023 \times 10^{-9} \text{ F} = 2023 \text{ pF} \quad \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{25.29 \text{ pF}} + \frac{1}{2023 \text{ pF}} \quad C_{\text{eq}} = 24.98 \text{ pF}$$