PHYS-115(3) (Kaldon-16406)
WMU - Winter 2001
Final Exam - 100,000 points

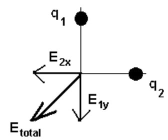
Name S O L U T I O N

Check-Out: ☐ Q ☐ X ☐ T ☐ WS ☐ ___
Section: **3a** A to F **3b** G to M **3c** N to S **3d** T to Z 04/16/2001•Rev.6

State Any Assumptions You Need To Make -- Show All Work -- Circle Any Final Answers
Use Your Time Wisely - Work on What You Can - Be Sure to Write Down Equations
BOLDFACE Variables Are Vectors - Feel Free to Ask Any Questions

***It's Game Show Physics Time! Jeopardy? Family Feud? No… it's…* (50,000 points)**

1.) Two charges, $q_1 = q_2 = +0.500\ C$, are located at $y = +2.00\ m$ and $x = +2.00\ m$ respectively. (a) Find the net Electric field, $\vec{E}$, at the origin ($x = y = 0$).



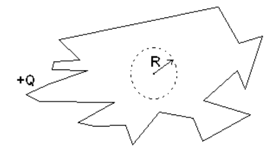
$$
\begin{aligned}
E_1 &= \frac{kq_1}{r^2} \\
&= \frac{\left(8.988\times10^9\ {}^{N\cdot m^2}/_{C^2}\right)(0.500C)}{(2.00m)^2} \\
&= 1.125\times10^9\ N/C \\
E_{1y} &= E_{2x} = -1.125\times10^9\ N/C \\
E_{total} &= \sqrt{E_{1y}^2 + E_{2x}^2} \\
&= \sqrt{\left(-1.125\times10^9\ N/C\right)^2 + \left(-1.125\times10^9\ N/C\right)^2} \\
&= 1.591\times10^9\ N/C \\
\vec{E}_{total} &= 1.591\times10^9\ N/C @ 235^\circ
\end{aligned}
$$

(b) Find the magnitude of the force of q_1 on q_2.

$$
\begin{aligned}
r &= \sqrt{x^2+y^2} = \sqrt{(2.00m)^2+(2.00m)^2} = 2.828m \\
F_E &= \frac{kq_1q_2}{r^2} \\
&= \frac{\left(8.988\times10^9\ {}^{N\cdot m^2}/_{C^2}\right)(0.500C)^2}{(2.828m)^2} \\
&= 281{,}300{,}000N
\end{aligned}
$$

Remember that 1 C of charge is an enormous amount of charge!

(c) Consider a solid hunk of metal that is very irregularly shaped. A charge $+Q = 5.00\ C$ is placed on the metal. Use Gauss' Law to find the magnitude of the electric field, E, on a Gaussian sphere of $r = R$, as shown.



Don't need to really calculate much here. If it's a hunk of metal, then all the charge is on the outside. The Gaussian sphere of radius r = R therefore encloses no charge and E = 0.

$$\Phi_E = EA = \frac{q_{inside}}{\varepsilon_0} = 0 \quad ; \quad E = 0$$

(d) Use Gauss' Law to find the magnitude of the electric field, E, of the hunk of metal at a point $r = 100\ R$ from the center.

$$
\begin{aligned}
\Phi_E &= EA = \frac{q_{inside}}{\varepsilon_0} \\
E\left(4\pi r^2\right) &= \frac{Q}{\varepsilon_0} \\
E &= \frac{Q}{4\pi r^2 \varepsilon_0} = \frac{kQ}{r^2} = \frac{kQ}{(100R)^2} \\
E &= \frac{kQ}{10{,}000R^2}
\end{aligned}
$$

Q = +5.00 C, but we don't have a value for R. However, r = 100R, so we are clearly *outside* and enclosing the entire +5.00 C this time.

(e) How do you make a charge of $+Q = 5.00\ C$ on a hunk of metal?

Short answer: You have to remove -5.00C worth of charge from a neutral block of metal.

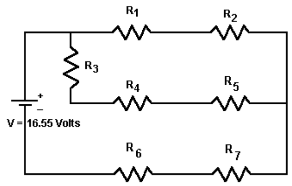
Medium answer: Remove this many electrons from a neutral block.

$$
\begin{aligned}
Q &= \pm Ne = -5.00C \\
N &= \frac{-5.00C}{-e} = \frac{-5.00C}{-1.602\times10^{-19}C} = 3.121\times10^{19}\,electrons
\end{aligned}
$$

Long answer: Bring a strong negative charge near a neutral block of metal. Positive charges will be attracted as the electrons flee to the far side. Attach a grounded wire to the far side, and drain off -5.00 C worth, 3.121×10^{19} electrons.

Who Wants to Win a Million Dollars? (50,000 points)

2.) All of the resistors in the circuit shown are $R = 100.\ \Omega$. Consider the three resistors along the right hand side (R_2, R_5, R_7) which, for the record, are *not* in parallel with each other. (a) Which of the three resistors sees the largest current? Why?



R_7 sees the largest current.

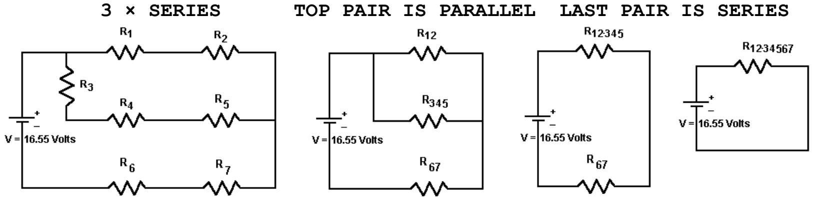
R_6 and R_7 are in series with the battery, so R_7 sees the entire current. R_2 and R_5 are in two legs of a parallel circuit, so they share the total current.

(b) Which of these three resistors generates the most heat? Why?

Since R_7 sees the largest current, it will also generate the greatest heat, since one form of the power equation is: $P = I^2R$ and all the resistors are the same.

(c) Find the equivalent resistance of this circuit. *Note: you don't have to do part (c) first to answer (a) or (b). If you want more room, use the back of page 7.*

3 × SERIES TOP PAIR IS PARALLEL LAST PAIR IS SERIES

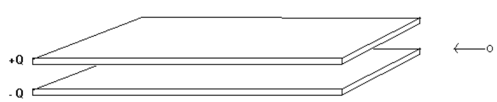


$$R_{12} = R_1 + R_2 = 100.\Omega + 100.\Omega = 200.\Omega$$
$$R_{345} = R_3 + R_4 + R_5 = 3(100.\Omega) = 300.\Omega$$
$$R_{67} = R_6 + R_7 = 100.\Omega + 100.\Omega = 200.\Omega$$

$$\frac{1}{R_{12345}} = \frac{1}{R_{12}} + \frac{1}{R_{345}} = \frac{1}{200.\Omega} + \frac{1}{300.\Omega}$$
$$R_{12345} = 120.0\Omega$$

$$R_{1234567} = R_{12345} + R_{67} = 120.0\Omega + 200.\Omega = 320.0\Omega$$

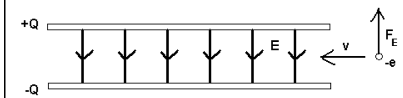
(d) A parallel plate capacitor consists of plates that are 0.100 m × 0.275 m, separated by 0.0350m. If this capacitor is charged up with a 12.0 V car battery, what is the charge on the plates, ±Q ?



$$C = \varepsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\,F/m\right)\frac{(0.100m)(0.275m)}{(0.0350m)} = 6.954\,pF$$
$$Q = CV = \left(6.954\times10^{-12}\,F\right)(12.0volts) = 8.345\times10^{-11}\,C$$

(e) An electron is shot in between the plates as shown, moving to the left. Find the magnitude of the electric force, F_E, and tell whether the electron is accelerated *Left, Right,* **Up**, *Down, Front* or *Back*.

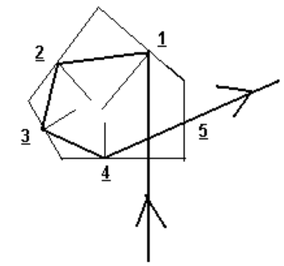
$$V = Ed$$
$$E = \frac{V}{d} = \frac{12.0volts}{0.0350m} = 342.9V/m = 342.9N/C$$
$$F_E = qE = eE = \left(1.602\times10^{-19}C\right)(342.9N/C) = $$
$$= 5.493\times10^{-17}\,N$$



"The Tribe Has Spoken" **(50,000 points)**

SURVIVOR the AUSTRALIAN OUTBACK

3.) (a) A light ray in the air comes down the perpendicular at the bottom of the five-sided glass prism ($n = 1.53$). The reflection angles are approximately as follows: 1. 43° 2. 47° 3. 52° 4. 65°. Which of those four surfaces requires a silver mirror on the surface to force the reflection (circle them) and which don't need to be silvered (X them)?

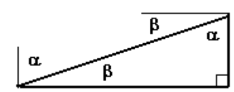


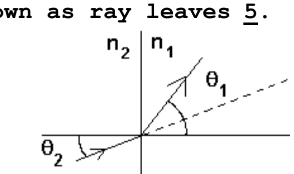
$$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right) = \sin^{-1}\left(\frac{1.00}{1.53}\right) = 40.8°$$

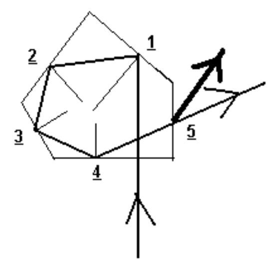
All four given angles are > 40.8°, so all four are T.I.R. – no silvering required.
A "pentaprism" is used to redirect the light in an SLR (single lens reflex) camera, though not in the way shown here.

(b) There's something wrong with the drawing at 5. Correctly describe the problem for 5000 pts. Show the correct direction of the ray for 8000 pts. Or calculate the correct angle in air for 10,000 points. *If you assume that the lower right corner of the prism is a right-angle, you have enough information to solve the puzzle, Pat.*

No refraction shown as ray leaves 5.

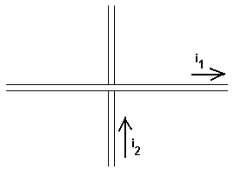






$$\alpha = 65° \quad \beta = 90° - 65° = 25°$$
$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$\sin\theta_1 = \frac{n_2\sin\theta_2}{n_1}$$

$$\theta_1 = \sin^{-1}\left(\frac{n_2\sin\theta_2}{n_1}\right)$$
$$= \sin^{-1}\left(\frac{1.53\sin 25°}{1.00}\right) = 40.3°$$



(c) Two 1.25 m long segments of wire cross over each other as shown, separated by a distance $d = 0.0250\ m$ (about an inch). If $i_1 = 1.25\ A$ and $i_2 = 2.50\ A$, then find the magnitude of the magnetic force of wire 1 on wire 2.

Perpendicular wires have no magnetic force between them, because B-field from Wire 1 is parallel to the current in Wire 2.

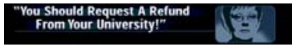
(d) A thin film of oil ($n = 1.47$) is sitting on water ($n = 1.33$), above the oil is air. If the thin film is a "half-wave" thickness, then if you were directly above the surface looking down ($\theta = 0°$), then do you see glare or no reflection?

Going from air to oil is low to high, so the reflection is shifted by half a λ. However, going from oil to water is high to low, so the second reflection is not shifted. That means the two reflections are already phase-shifted by ½ λ, so the round-trip distance, d = 2t = 2(½λ)= λ , therefore for this problem, the half-wave coating is an anti-reflection coating.

(e) A microwave has a wavelength $\lambda = 20.0\ cm = 0.200\ m$. Find the frequency f of this wave.

$$c = f\lambda$$
$$f = \frac{c}{\lambda} = \frac{2.998\times10^8\ m/s}{0.200m} = 1.499\times10^9\ Hz = 1.499 GHz$$

"You Are The Weakest Link – Goodbye." (50,000 points)



4.) (a) Most calculators work display ten digits. If your calculator does so, then $\gamma = 1.000\ 000\ 001$ is the smallest *gamma* you can use to make any sort of a relativistic calculation. Solve the γ equation for β, and find the speed v that $\gamma = 1.000\ 000\ 001$ corresponds to.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$
$$\sqrt{1-\beta^2} = \frac{1}{\gamma}$$
$$1-\beta^2 = \frac{1}{\gamma^2}$$
$$\beta^2 = 1-\frac{1}{\gamma^2}$$
$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{(1.000\,000\,001)^2}} = 0.000\,044\,72$$
$$v = \beta c = 0.000\,044\,72c = (0.000\,044\,72)(3.00\times10^8\ m/s)$$
$$= 13{,}410 m/s$$

NOTE: Actual answers vary based on the brand/model of calculator used. Many display 10 digits, but calculate to 10, 11 or 12.

(b) If $\gamma = 1.000\ 000\ 001$, and the proper length L_p = *one meter exactly* then the Lorenz length contraction of the improper length would result in a change of length on the order of: a *football field*, a *meter stick*, the *width of this dot* • , the *width of an atom* or the *width of the nucleus* of an atom. Pick one and defend your answer.

$$L = \frac{L_0}{\gamma} = \frac{1.00m}{1.000000001} = 1.000m$$

(c) An electron in the helium ion He^+ jumps from the $n = 2$ orbit to the $n = 3$ orbit. Find the radius of the new orbit and indicate whether a photon of light was ABSORBED or EMITTED by the electron to make the jump.

$$r_n = \frac{n^2}{Z}a_0\ ;\ r_3 = \frac{3^2}{2}(5.28\times10^{-11}m) = 2.376\times10^{-10}m$$

Photons are absorbed to gain energy and jump to a higher orbit – or to be ionized and removed completely from the system.

24
Cr
Chromium
51.9961

(d) Given the information from the Periodic Table shown here, what isotope of chromium is likely to be the most common? Write the isotope's complete symbol down, then indicate how many electrons, protons and neutrons that an atom of that isotope should have.

We didn't talk about this at the end, though I might've mentioned earlier in the semester that rounding off the atomic mass number gives the most likely isotope. In this case. 51.9961 becomes A = 52. With Z = 24, and A = Z + N, then we get N = A - Z = 52 - 24 = 28. (Both Z and N are even, which confers some stability, and 28 is a "magic number" in nuclear physics, which means a nuclear shell or sub-shell is completed, much like Z = 2, 10, 18, 36, 54, 86 are the Nobel gasses of the Periodic Table.)

$$^{52}_{24}Cr$$

(e) The kinetic energy of a hydrogenic electron is $KE = +\frac{Z^2}{n^2}E_0$, where $E_0 = 2.18 \times 10^{-18}\,J$. For what element, with what Z, is the innermost electron ($n = 1$) have a KE that is just relativistic? (Use $KE = \frac{1}{2}mv^2$.)

$$v = \beta c = (0.417)(2.998 \times 10^8\,m/s) = 1.250 \times 10^8\,m/s$$

$$KE = +\frac{Z^2}{n^2}E_0 = \tfrac{1}{2}mv^2$$

$$Z^2 = \frac{n^2 m v^2}{2E_0}$$

$$Z = \sqrt{\frac{n^2 m v^2}{2E_0}} = \sqrt{\frac{(1)^2(9.11 \times 10^{-31}\,kg)(1.250 \times 10^8\,m/s)^2}{2(2.18 \times 10^{-18}\,J)}}$$

$$= 57.18 = 57$$

57
La
Lanthanum
88.9059

Host Anne Robinson – the rudest


The smash U.K. game show hit *The*