Inductive Reasoning Applied To The Capacitance Of Resistance (50,000 points)

1.) A series RLC circuit is connected to an A.C. power source with $V_{rms} = 125\ volts$ and *frequency* $f = 60.0\ Hz$, where $\omega = 377\ rad/sec$. The resistor has $R = 1000.\ \Omega$ and the capacitor $C = 1000.\ \mu F$. (a) Find the capacitive reactance X_C of this circuit.

$$X_C = \frac{1}{\omega C} = \frac{1}{(377 rad/\sec)(1000.\times 10^{-6} F)} = 2.653\Omega$$

(b) Find the inductance L which gives the minimum impedance Z.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}\ ;\ Z_{\min} = R \quad when \quad X_L = X_C$$
$$X_L = X_C = \omega L$$
$$L = \frac{X_C}{\omega} = \frac{2.653\Omega}{377 rad/\sec} = 0.007037H = 7.073mH$$

(c) Find the rms current I_{rms} for this circuit.

$$V_{rms} = I_{rms} Z$$
$$I_{rms} = \frac{V_{rms}}{Z} = \frac{125 volts}{1000.\Omega} = 0.1250A$$

(d) Does the impedance Z go up or down if the frequency is changed to $f = 25.0\ Hz$? Why? *Short answer!*

Since $Z = \sqrt{R^2 + (X_L - X_C)^2}$ **, and we had** $X_L - X_C = 0$ **, if *f* changes, then ω changes and the impedances change.**

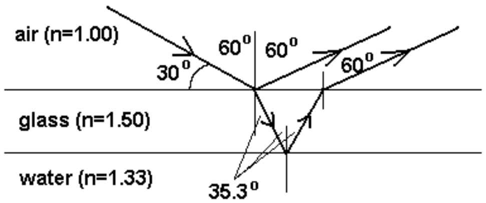
But then we won't have $X_L = X_C$ **, so Z must increase.**

(e) An inductor with $L = 40.0\ mH = 0.0400\ H$ has a length $l = 3.00\ cm$ long and 1000. turns. What's the diameter D of the coil?

$$L = \frac{\mu_0 N^2 A}{l}$$
$$A = \frac{l L}{\mu_0 N^2} = \frac{(0.0300m)(0.0400H)}{(4\pi \times 10^{-7} T\cdot m/A)(1000.)^2}$$
$$= 0.0009549m^2$$
$$A = \frac{\pi D^2}{4}\ ;\ D^2 = \frac{4A}{\pi}$$
$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.0009549m^2)}{\pi}} = 0.03487m$$

Upon Further Reflection… (50,000 points)

2.) (a) A perfectly flat piece of glass (known as parallel-plano in the optics business) has air above and water below as shown. Calculate and sketch on the diagram the light rays going back into the air from reflecting off the two interfaces.



$$\begin{aligned}
&\textit{First reflection}\\
&\theta_i = \theta_r \quad ; \quad 60° = 60°\\
&\textit{First refraction}\\
&n_1 \sin\theta_1 = n_2 \sin\theta_2\\
&\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}\\
&\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)\\
&\quad = \sin^{-1}\left(\frac{1.00 \sin 60°}{1.50}\right) = 35.3°\\
&\textit{Second reflection}\\
&\theta_i = \theta_r \quad ; \quad 35.3° = 35.3°
\end{aligned}$$

(b) A lens contains an anti-reflection coating ($n_{coating} = 1.44$) on glass ($n_{glass} = 1.50$) in air ($n_{air} = 1.00$). If the coating is 111 nm thick, for what wavelength of light is this an anti-reflection coating in air?

$$\begin{aligned}
&d = 2t = 2(111nm) = 222.0nm = \frac{\lambda'}{2}\\
&\lambda' = 2(222.0nm) = 444.0nm\\
&\lambda' = \frac{\lambda}{n_m} \quad ; \quad \lambda = n_m\lambda' = (1.44)(444.0nm) = 639.4nm
\end{aligned}$$

OR

$$\begin{aligned}
&t = \frac{\lambda'}{4}\\
&\lambda' = 4(111nm) = 444.0nm\\
&\lambda' = \frac{\lambda}{n_m} = 639.4nm
\end{aligned}$$

(c) Find the frequency of this light in air or vacuum. *If you didn't get an answer to (c), use* $\lambda = 555\ nm = 555 \times 10^{-9}\ m$.

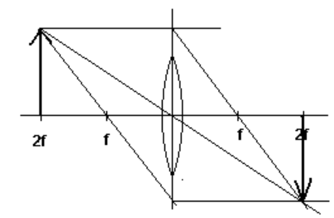
$$\begin{aligned}
&c = f\,\lambda\\
&f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\,m/s}{639.4\times10^{-9}\,m} = 4.689\times10^{14}\,Hz
\end{aligned}$$

(d) An object $h = 10.0\ cm$ high is located $p = 16.0\ cm$ from a positive, converging, biconvex lens with a focal length $f = +8.00\ cm$. Calculate the position q and the height h' of the real, inverted image produced. *You don't have to sketch anything below. Is there anything interesting about your answer?*

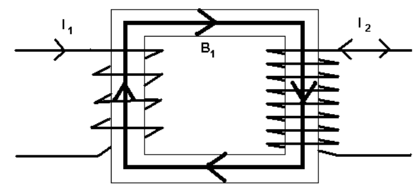
$$\begin{aligned}
&\frac{1}{f} = \frac{1}{p} + \frac{1}{q}\\
&\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{8.00cm} - \frac{1}{16.0cm} = 0.06250cm^{-1}\\
&q = \frac{1}{0.06250cm^{-1}} = 16.00cm\\
&M = -\frac{q}{p} = \frac{h'}{h}\\
&h' = -\frac{q}{p}h = -\frac{(16.00cm)}{(16.0cm)}(10.0cm) = -10.00cm
\end{aligned}$$

Ha! The image is the same size as the object. This is called 1:1 or "one to one" imaging.

You weren't asked to draw this, but this is what it would look like:



A transformer is shown at the right. (e) A constant primary current I_1 is going to the right as shown. Indicate whether the induced current of the secondary, I_2 , goes to the LEFT or to the RIGHT. You should sketch in the relevant magnetic field B_1 from the primary coil, and the induced magnetic field B_2 from the secondary coil to get full points.



The current is CONSTANT, so that means we are running DC into a transformer. Yes, there will be a magnetic field from the primary coil, but there is no change in the flux through the secondary coil, therefore there is NO induced magnetic field and NO induced current in the secondary coil.