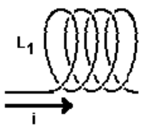
The Current Experiences A Death Defying Loop-The-Loop! (50,000 points)

1.) A steady current i = 0.685 A flows through a 400. loop inductor, L_1 = 0.0685 H. (a) Find the flux Φ_B of the magnetic field inside.



$$L = \frac{N\Phi_B}{I}$$
$$\Phi_B = \frac{LI}{N} = \frac{(0.0685H)(0.685A)}{400.turns}$$
$$= 0.0001173\,T\cdot m^2 = 1.173\times10^{-4}\,T\cdot m^2$$

(b) If the current was suddenly turned off in 0.0100 s, find the self-induced emf. *If you didn't get an answer to (a) use 0.685 T·m².*

$$\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t} = -(400.)\frac{(-0.0001173\,T\cdot m^2)}{0.0100\sec}$$
$$= +0.01173\,Volts$$

The positive sign means that the induced *emf*, induced current and induced B-field all are trying to support the dying flux.

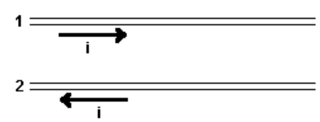
An RLC circuit contains the following elements: R = 685 Ω, C = 685 μF and L = 0.0685 H. It is driven by an AC generator with ω = 377 rad/s (f = 60.0 Hz). (c) Find the capacitive and inductive reactances, X_C and X_L.

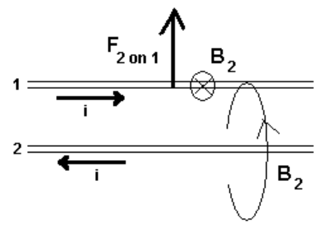
$$X_C = \frac{1}{\omega C} = \frac{1}{(377rad/\sec)(685\times10^{-6}F)} = 3.872\Omega$$
$$X_L = \omega L = (377rad/\sec)(0.0685H) = 25.82\Omega$$

(d) Find the impedance, Z, for the circuit. For what value of ω is the impedance a minimum? *If you didn't get an answer to (c), use $X_C = X_L = 50.5\ \Omega$.*

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(685\Omega)^2 + (25.82\Omega - 3.872\Omega)^2}$$
$$= 685.4\Omega$$
$$Z_{\min} = R \quad when \quad X_L = X_C$$
$$\omega L = \frac{1}{\omega C}\ ;\ \omega^2 = \frac{1}{LC}$$
$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.0685H)(685\times10^{-6}F)}}$$
$$= 146.0\,rad/\sec$$

(e) Two wires, marked 1 and 2 as shown below, are each 2.50 m long and are separated by a distance d = 0.130 m. They carry currents i_1 = 1.00 A and i_2 = 1.50 A. (a) Find magnitude and direction of the magnetic force $\vec{F}_{2on1}$.





$$B_2 = \frac{\mu_0 i_2}{2\pi d} = \frac{(4\pi\times10^{-7}\,\frac{T\cdot m}{A})(1.50A)}{2\pi(0.130m)}$$
$$= 2.308\times10^{-6}\,T$$
$$F_{2on1} = i_1\,l\,B_2 = (1.00A)(2.50m)(2.308\times10^{-6}T)$$
$$= 5.770\times10^{-6}\,N$$

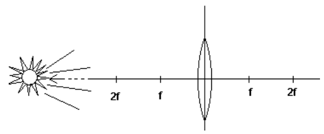
OR

$$F_{2on1} = \frac{\mu_0 i_1 i_2 l}{2\pi d}$$
$$= \frac{(4\pi\times10^{-7}\,\frac{T\cdot m}{A})(1.00A)(1.50A)(2.50m)}{2\pi(0.130m)}$$
$$= 5.769\times10^{-6}\,N$$

The force points "up" in the +y-direction as shown. Since the two currents are in opposite directions, the two current carrying wires repel each other.

It May Be A Gray Day In Michigan, But Somewhere The Sun Is Shining (50,000 points)

2.) (a) The light from the Sun shines on a lens with a focal length $f = 100.\ mm = 0.100\ m$. Although the Sun is huge, it is 93,000,000 miles away (150,000,000 km). Calculate where the image, q, of the Sun is located. *Drawing NOT to Scale.*



$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

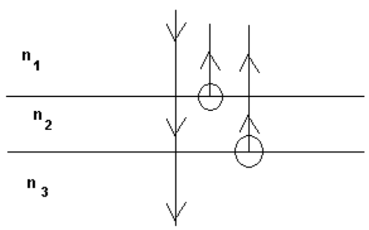
$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$= \frac{1}{0.100m} - \frac{1}{150{,}000{,}000{,}000m}$$

$$q = 0.100m \quad (= f)$$

But of course this is true – parallel rays from the Sun converge on the focal point at the right!

(b) This lens contains an anti-reflection coating ($n_{coating} = 1.27$) on glass ($n_{glass} = 1.57$) in air ($n_{air} = 1.00$). If the coating is 117 nm thick, for what wavelength of light in air is this an anti-reflection coating?



First, we note that the reflection from n_1 to n_2 is from a lower index of refraction to higher, as is n_2 to n_3 , so both have a ½ λ phase shift, so the phase shift doesn't matter.

Therefore we need the round-trip distance to be ½ λ and so the thickness is ¼ λ (quarter-wave coating).

$$d = 2t = 2(117nm) = 234.0nm = \frac{\lambda'}{2}$$

$$\lambda' = 2(234.0nm) = 468.0nm$$

$$\lambda' = \frac{\lambda}{n_m} \;;\quad \lambda = n_m\lambda' = (1.27)(468.0nm) = 594.4nm$$

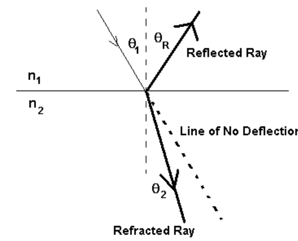
OR

$$t = \frac{\lambda'}{4}$$

$$\lambda' = 4(117nm) = 468.0nm$$

$$\lambda = n_m\lambda' = 594.4nm$$

(c) White light comes in to a flat piece of glass at an angle of incidence in air $\theta_1 = 25°$. Find the angle of the reflection.



$$\theta_1 = \theta_i = \theta_r = 25.0°$$

(d) If the index of refraction in the glass for red light is $n_{red} = 1.5500$ and violet light is it $n_{violet} = 1.5656$, find the refracted angles θ_2 for the red light and the violet light. *Give the angles to the one-hundredth of a degree.*

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

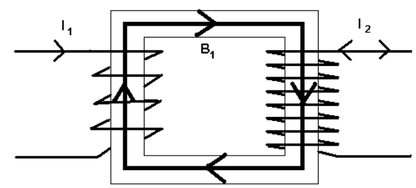
$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

$$\theta_{2,red} = \sin^{-1}\left(\frac{1.00}{1.5500}\sin 25°\right) = 15.82°$$

$$\theta_{2,violet} = \sin^{-1}\left(\frac{1.00}{1.5656}\sin 25°\right) = 15.66°$$

A transformer is shown at the right. (e) A constant primary current I_1 is going to the right as shown. Indicate whether the induced current of the secondary, I_2 , goes to the LEFT or to the RIGHT. You should sketch in the relevant magnetic field B_1 from the primary coil, and the induced magnetic field B_2 from the secondary coil to get full points.



The current is CONSTANT, so that means we are running DC into a transformer. Yes, there will be a magnetic field from the primary coil, but there is no change in the flux through the secondary coil, therefore there is NO induced magnetic field and NO induced current in the secondary coil.